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STOCHASTICITY IN GEOPHYSICAL AND HYDROLOGICAL TIME SERIES*

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Most time processes within the atmosphere and oceans, and along the water cycle are periodic-stochastic processes, with the physical basis of the periodicity derived from astronomic cycles and the stochasticity produced by various sources of randomness within the environments of the earth. The atmosphere is the major source of stochasticity in all those geophysical time processes which are connected to the incoming solar radiation, though the oceans, the earth's surface, and the earth's crust also produce the stochasticity. The main influence of the oceans and the continental surfaces as well as the underground water is, however, to attenuate the high stochasticity produced by the atmosphere. It is concluded that without stochastic meteorology a good understanding of many geophysical stochastic processes will be difficult.

It is convenient to divide the earth's environments for the purpose of studying the stochasticity (noise, randomness) in geophysical processes into four parts: atmosphere, oceans and seas, continental surfaces, and the earth's upper crust.

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Each of them is the product of a long historic development of the earth with the approximately time-frozen boundaries and other properties at present excluding the natural and man-made disruptions. Therefore, stochasticity is discussed in this paper under the conditions that for several past and future centuries the properties of these environments are approximately constant, and the annual time series of various geophysical processes are approximately stationary stochastic processes. The stochasticity is defined as those properties of geophysical processes which are definitely and predominantly governed by the laws of chance, with some of them periodic-stochastic processes; for them the outlooks are nearly nil for developing the classical cause-effect relationships.

Most time processes within the atmosphere and oceans, and along the water cycle are periodic-stochastic processes, with the physical basis of the periodicity derived from astronomic cycles and the stochasticity produced by various sources of randomness within the environments of the earth. The stochasticity of some space properties of the earth's environments is conceived as being the accumulated results of past time periodic-stochastic processes. Some examples are the stochastic variations in boundaries of environments, geomorphological forms, and internal environmental properties such as the porosity of the upper part of the earth's crust. These various stochastic environmental factors often produce responses to time inputs of a periodic-stochastic nature, thus making outputs also periodic-stochastic processes, even when the inputs do not follow such properties.

Some of the primary sources of stochasticity in geophysical processes are: the turbulence and large-scale vorticities associated with movements of fluids; the randomness in heat transfers, or generally, in energy transfers; the random effects on processes by various dissolved, suspended, floating or otherwise transported materials in fluids; the randomness connected with the passage of water through critical points of freezing, melting, evaporating, condensing and sublimating; and the effects of random factors in approximately time-frozen states of environments including biological changes. Stochasticity reduced to independent or dependent stochastic components of various processes or environmental properties, after the periodic components in various parameters are removed, best defines the sources and measures the types of randomness they produce.

An analysis of various geophysical time processes leads to the following basic conclusions of how the four parts of the earth create or affect the stochasticity of these processes. The atmosphere has a dual role in the formation of the stochastic part of geophysical periodic-stochastic processes. First, it is the major source of stochasticity of processes in all environments. Second, it is the principal environment which disrupts the high time-dependence in the stochastic components of those periodic-stochastic processes which other environments produce and which interact with the atmosphere. Furthermore, the atmosphere transforms the periodic components either of the periodic solar radiation input, or of the periodic components of the major periodic-stochastic geophysical processes. Air makes up the most non-conservative environment on the earth as it concerns the energy and the water vapor storage. It is the primary source of stochasticity though not the only source, a filter for the periodicity of input periodic-stochastic processes, and the major disruptor of high stochastic dependence produced by the other three environments.

The oceans and seas also accumulate heat and kinetic energy, but conserve them for a much longer time than the air. They usually transform the input perodic-stochastic processes with the nearly time-independent stochastic component into highly attenuated periodic-stochastic processes, with the stochastic component now highly time-dependent. But the oceans and seas are also the environments which produce the stochasticity by various types of processes. The simplest example is the stored heat in the upper layers of oceans and seas. The solar radiation input into these layers is a periodic-stochastic process with a very small time-dependence in its stochastic component, while the stored heat measured by the temperatures is a periodic-stochastic process with a highly time-dependent stochastic component, as will be demonstrated later.

The retentions of water on the continental surfaces in bodies of water such as snow and ice, as overland and channel water flow, and as water in the shallow and deep groundwater formations, transform a periodic-stochastic component into the periodic-stochastic process of runoff and/or surface evaporation, with highly time-dependent stochastic components. However, the randomness of several climatic variables makes the evaporation and evapotranspiration a periodic-stochastic process with a smaller time-dependent stochastic component than is the case with the surface or particularly the groundwater runoff.

All four of earth's major environments are sources of various types of stochasticity because of the high random components in the turbulence, vorticity, heat and energy transfers, various random aspects of chemical and biological processes, and similar. However, a distinction must be made between a major impact on the stochasticity of geophysical processes and all other minor effects of each of these four parts of the earth.

The most general description of the sources of stochasticity in the geophysical processes should be as follows. The solar energy input into the uppermost atmosphere is a deterministic-periodic process, for all practical purposes. Figures 1 and 2 illustrate the average daily solar radiation input into several catchments with different latitudes, but particularly, with different orientation and

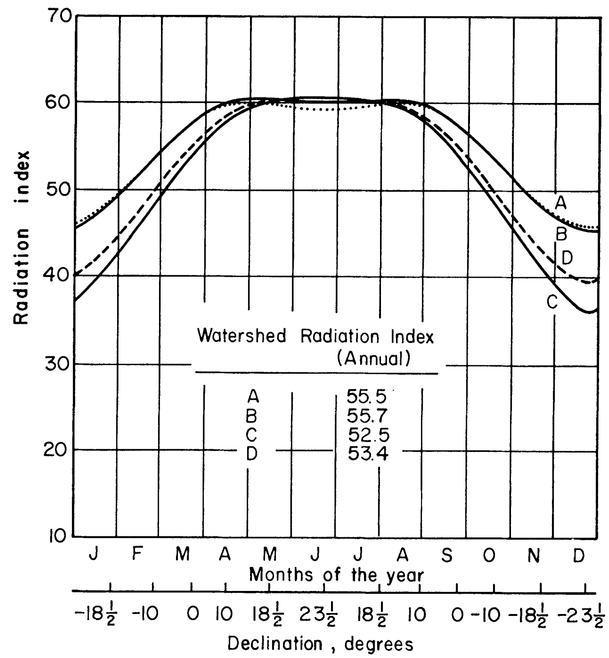


Fig. 1.
Radiation indexes for four experimental catchments, giving variation with time, Sierra Ancha Experimental Forest, Arizona, U.S.A. (after Richard Lee, Colorado State University Hydrology Paper, No. 2, 1963).

slopes, provided there is no atmospheric effect on the incoming solar radiation. However, the periodic-stochastic process of the opacity (or in the opposite context, of the transparency) of the atmosphere for the incoming and outgoing radiation transforms (filters) the astronomic deterministic-periodic process of solar energy input into a periodic-stochastic process of energy received by any area of a given latitude and orientation on the earth's surface. Similarly, the outgoing radiation into the space from the liquid and solid earth surface is further changed by the opacity of the atmosphere for energy transmission, so that the outgoing radiation is a periodic-stochastic process with a stronger stochastic component than that of the incoming radiation on the earth's surface. Then the periodic-stochastic distribution of energy in time and the stochastic distribution of energy in space of each environment act as the prime movers of all other periodic-stochastic geophysical time processes connected in any way with the solar energy input.

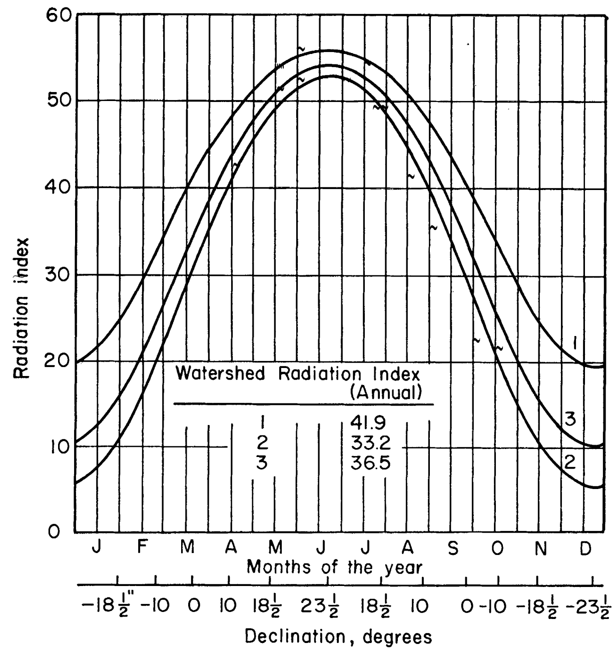


Fig. 2.

Radiation indexes for three experimental catchments, giving variation with time, Andrews Experimental Forest, Oregon, U.S.A. (after Richard Lee, Colorado State University Hydrology Paper, No. 2, 1963).

Instead of studying the detailed effects of various individual sources of stochasticity, the approach in this paper is to study the effects of various environments as sources of stochasticity. The effects of randomness in the transition of ice and snow into water or water vapor, and water into water vapor, and vice versa, but particularly the formation and dissipation of clouds, would be by itself an important subject. Similarly, the turbulence, energy transfers, and all other physical, chemical, and biological random processes as the sources of stochasticity in geophysical processes would each require a special study. This global or "lumped" approach to the effects of individual of earth's environments on this stochasticity in geophysics seems to be somewhat neglected but deserving of attention.

The stochasticity in the incoming solar radiation, measured at the continental earth surface, is first presented from the point of view of determining the degree and the type of stochasticity created by the atmospheric opacity for the solar

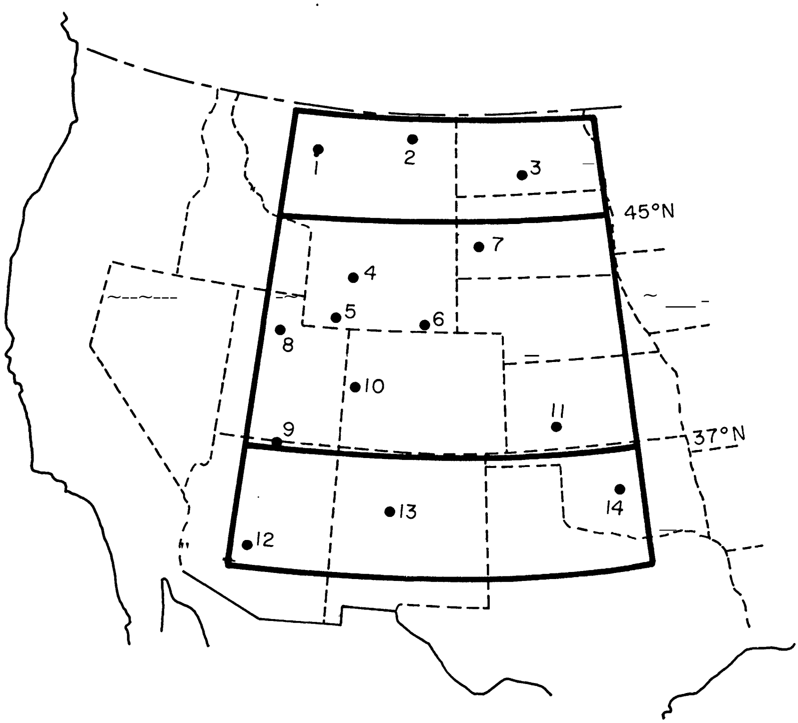


Fig. 3.
Stations used in the study of incoming solar radiation series in the Western United States: 1. Great Falls, 2. Glasgow, 3. Bismarck, 4. Lander, 5. Flaming Gorge, 6. Laramie, 7. Rapid City, 8. Salt Lake City, 9. Page, 10. Grand Junction, 11. Dodge City, 12. Phoenix, 13. Albuquerque, and 14. Oklahoma City.

energy input. Then a discussion is made of stochasticity in time series of ocean surface temperature, and in time series of monthly precipitation along the coastal areas. Finally, the stochasticity of daily river flows is presented, considering the runoff as an integrated process of various stochastic effects produced in all four earth's environments.

STOCHASTICITY IN MEASURED RADIATION

The time series of recorded incoming solar radiation in the form of average daily energy in langleys at 14 points at the earth's surface in the Western United States are used in this paper to demonstrate the degree and type of stochasticity created by the atmosphere in this most basic geophysical process. Figure 3

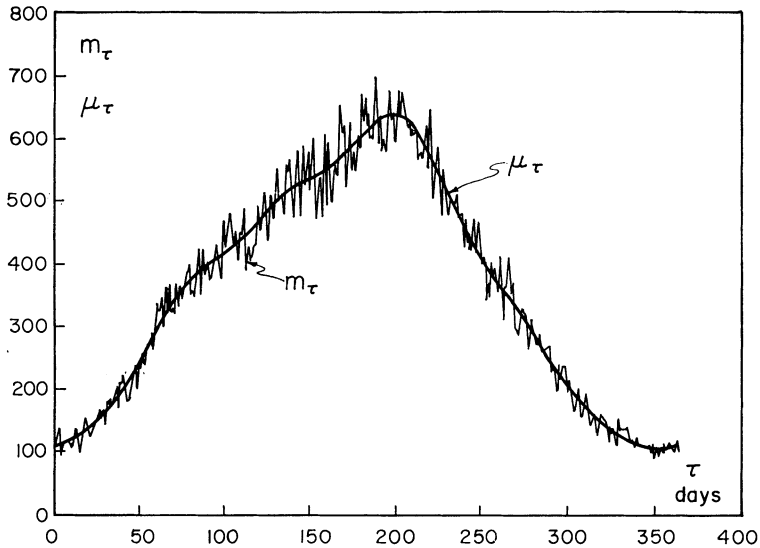


Fig. 4.
The mean daily incoming solar radiation (m_τ) and the fitted periodic function (μ_τ) for the Great Falls station (no. 1), for 14 years, in langleys.

shows the positions and the names and numbers of these 14 stations. The analysis of these 14 series is as follows. For each day (date) of the year, designated by $\tau = 1, 2, \ldots, 365$, the daily mean and the daily standard deviation, designated respectively by m_τ and s_τ, are computed. Figures 4 and 5 present these computed values for the Great Falls station (no. 1 of Fig. 3), together with the fitted periodic functions μ_τ and σ_τ to the computed values m_τ and s_τ.

These two figures show a much smaller sampling variation of the estimated mean daily values m_τ about the fitted periodic function μ_τ than the sampling variation of the estimated daily standard deviation s_τ about the fitted periodic function σ_τ. This should be expected because the second moment of the sample values has a much larger sampling variation than the first sample moment. The second peak in the variation of the standard deviation might or might not be the product of the sampling variation: 14 years is for all practical purposes a small sample; and the second peak in σ_τ, not present in μ_τ, may be a chance occurrence. Figure 6 gives the 14 curves of fitted periodic functions to the estimated daily means. The differences between these curves result from three factors: (1) differences in latitude of each of the 14 stations; (2) climatic dif-

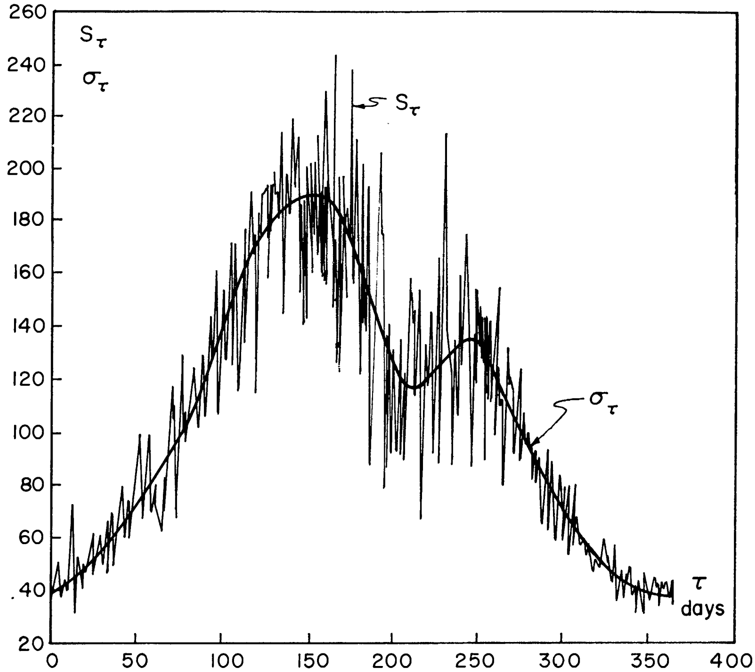


Fig. 5.
The standard deviation of daily incoming solar radiation (s_τ) and the fitted periodic function (σ_τ) for the Great Falls station (no. 1), for 14 years, in langleys.

ferences, or the effects of differences in the opacity of atmosphere because of variations in climate at these 14 stations; and (3) sampling errors, because of only 14 years of data, which inevitably affect the accuracies of the estimated amplitudes and phases of harmonics in the periodic functions. Figure 7 similarly gives the 14 curves of fitted periodic functions to the estimated standard deviations of daily values. Here all three factors of differences between curves are present, but it is a logical hypothesis to assume that the sampling errors must be large, though the climatic variations may also be decisive. Without the atmosphere the fitted periodic functions μ_τ of Fig. 6 would look rather like those of Fig. 2, while the functions σ_τ of Fig. 7 would be zero values.

The standard deviations s_τ, and σ_τ of Fig. 5 and σ_τ of Fig. 7, being greater than zeros, represent the main measures of stochasticity produced by the opacity of atmosphere to incoming and outgoing solar radiation. This may be considered the basic impact of atmosphere in transforming a pure deterministic-periodic process into a clear periodic-stochastic process with a significant

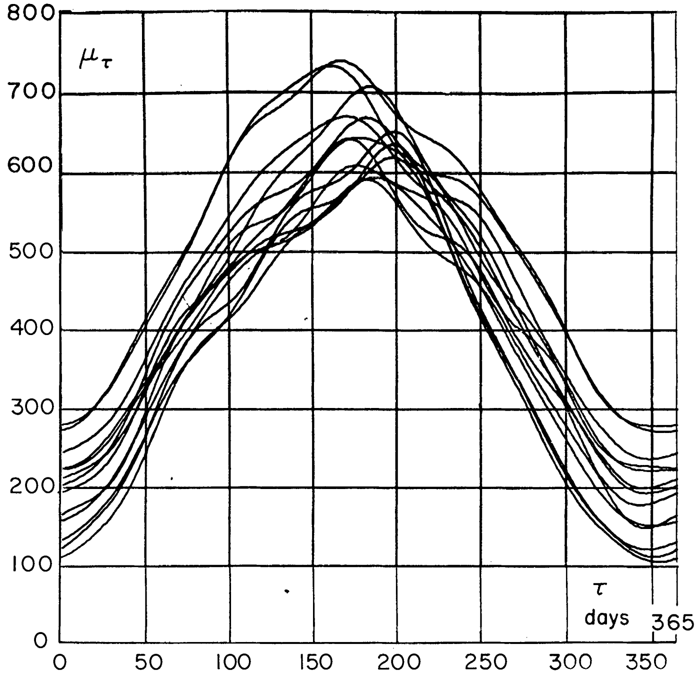


Fig. 6.
Fitted periodic functions to the computed means of daily incoming solar radiation for 365 days and the 14 stations of Fig. 3, in langleys.

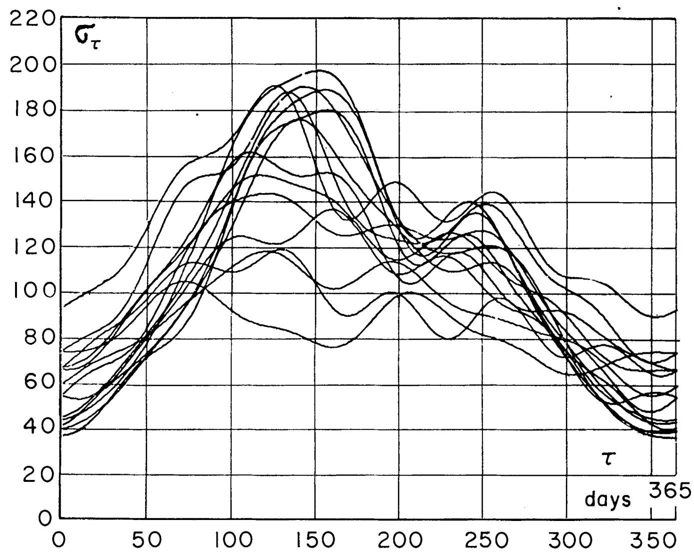


Fig. 7.
Fitted periodic functions to the computed standard deviations of daily incoming solar radiation for 365 days and the 14 stations of Fig. 3, in langleys.

stochastic component. The coefficients of variation s_τ/m_τ as a function of τ measure the importance of the stochastic part in comparison with the periodic part in this periodic-stochastic process, though these measures are somewhat unprecise. These coefficients, on the average, are about 0.20–0.30, which are relatively large values taking into account that this is the first "lumped" source of stochasticity in form of the opacity of atmosphere for solar radiation. These coefficients, as can be inferred from Figs. 6 and 7, are subject to large sampling variation. Therefore, the variation of these coefficients measure in some way the climatic conditions which affect the transparency or opacity of the atmosphere.

The removal of periodicities μ_τ and σ_τ in the mean and standard deviation of daily solar radiation $x_{p,\tau}$ by

$$\varepsilon_{p,\tau} = \frac{x_{p,\tau} - \mu_\tau}{\sigma_\tau}, \tag{1}$$

in which p = 1, 2, . . ., n, with n the number of annual cycles in data (or the number of years) and τ the days inside the year as τ = 1, 2, . . ., 365, produces $\varepsilon_{p,\tau}$, the stochastic component of the series, provided there is no periodicity in

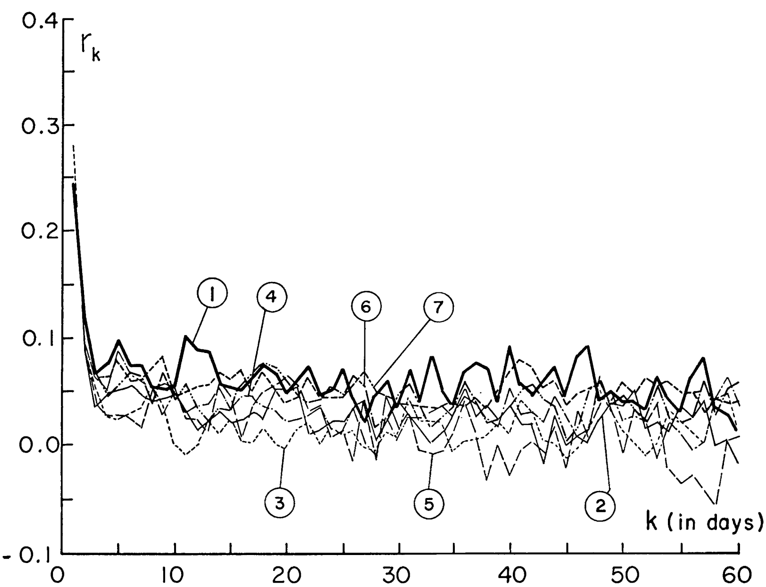


Fig. 8.

The correlograms of the dependent stochastic components ($\varepsilon_{p,\tau}$) of the daily incoming solar radiation of the first 7 stations of Fig. 3.

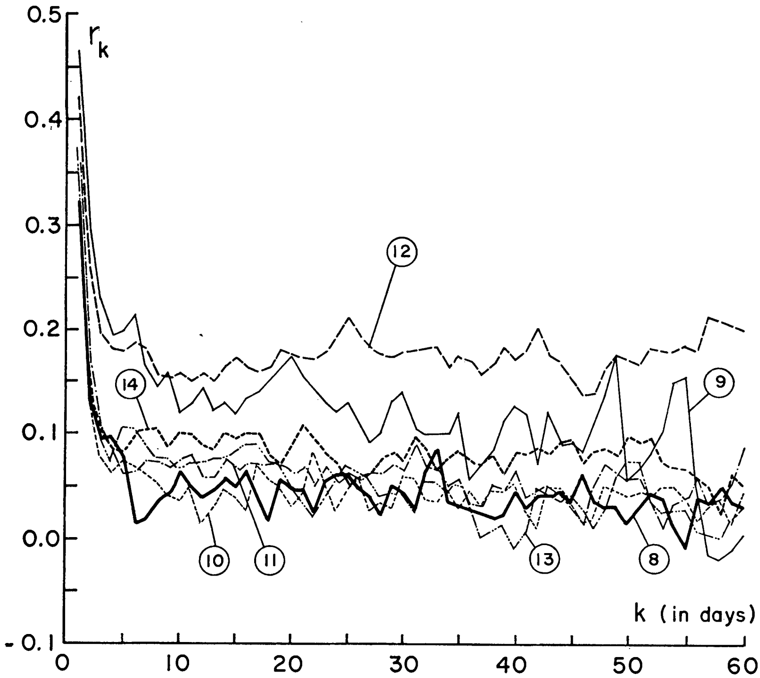


Fig. 9.
The correlograms of the dependent stochastic components ($\varepsilon_{p,\tau}$) of daily incoming solar radiation of the second 7 stations of Fig. 3.

the covariances (autocorrelation coefficients) and in the higher-order moments.

The sample correlogram $r_k = f(k)$ of the $\varepsilon_{p,\tau}$-series, with k the time lags in days and r_k the autocorrelation coefficients, are given for the 14 series up to k = 60 (60 days) in Figs. 8 and 9, each for seven series, with the numbers of correlograms in these figures corresponding to the numbers of stations of Fig. 3. The average correlogram of the 14 correlograms of Figs. 8 and 9 is given in Fig. 10. The correlograms of Figs. 8 through 10 show the following general patterns of time-dependence of the stochastic component $\varepsilon_{p,\tau}$: (1) the first autocorrelation coefficient, r_1, of this component is on the order of 0.25–0.45, with the average value of the 14 series being about 0.32; this shows the degree of persistence in the character of atmospheric opacity or transparency for the incoming solar radiation; the r_2 and r_3 are much smaller than r_1, so that an autoregressive dependence model may be indicated; and (2) from r_3 on, the r_k-values fluctuate in the range 0–0.20, and the average r_k-values of the cor-

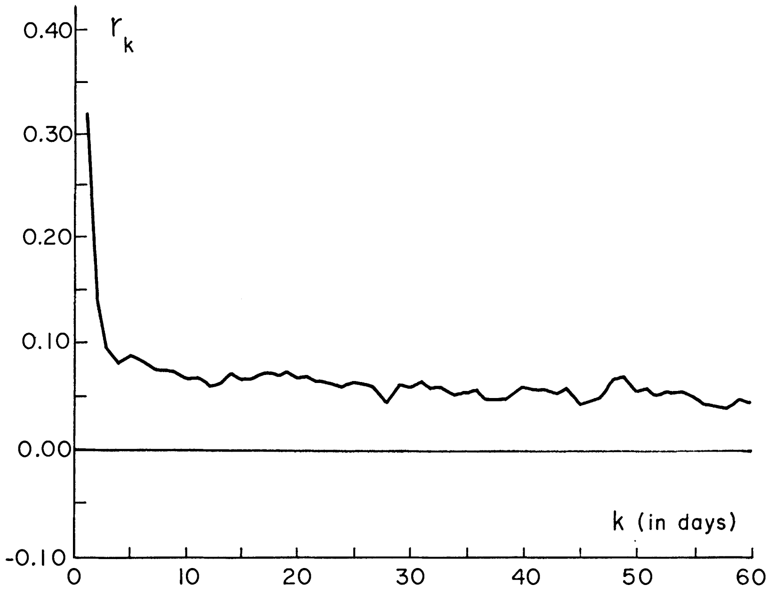


Fig. 10.
The mean correlogram of the dependent stochastic components (after the periodicity in the mean and standard deviation are removed) of daily incoming solar radiation of the 14 stations of Fig. 3.

relogram in Fig. 10 decrease from a value of 0.09 at k = 3 to a value of 0.05 at k = 60, a very slow decrease; this second property may indicate a small periodic movement in the correlogram. The r_1-values of 0.25–0.45 for $\varepsilon_{p,\tau}$ are of the same order of magnitude as the r_1-values of daily precipitation series. The explained dependence in $\varepsilon_{p,\tau}$ by the eventual periodic movement is small.

The autocorrelation patterns in the $\varepsilon_{p,\tau}$-series point out that the stochastic dependence or the "memory" in the series is very short, on the order of several days, while the small periodic movement in the correlogram may be interpreted as a non-removed periodicity in one or in both parameters, the mean and the standard deviation of daily radiation, or some non-significant harmonic in the periodic component has been inferred as being significant. By neglecting the periodic part of the correlogram, and by assuming the first-order linear auto-regressive model to be sufficient first approximation for the true physical dependence model, then the stochastic dependence of $\varepsilon_{p,\tau}$ is

$$\varepsilon_{p,\tau} = r_1 \, \varepsilon_{p,\tau-1} + \sqrt{1 - r_1^2} \;\; \xi_{p,\tau} \, , \qquad (2)$$

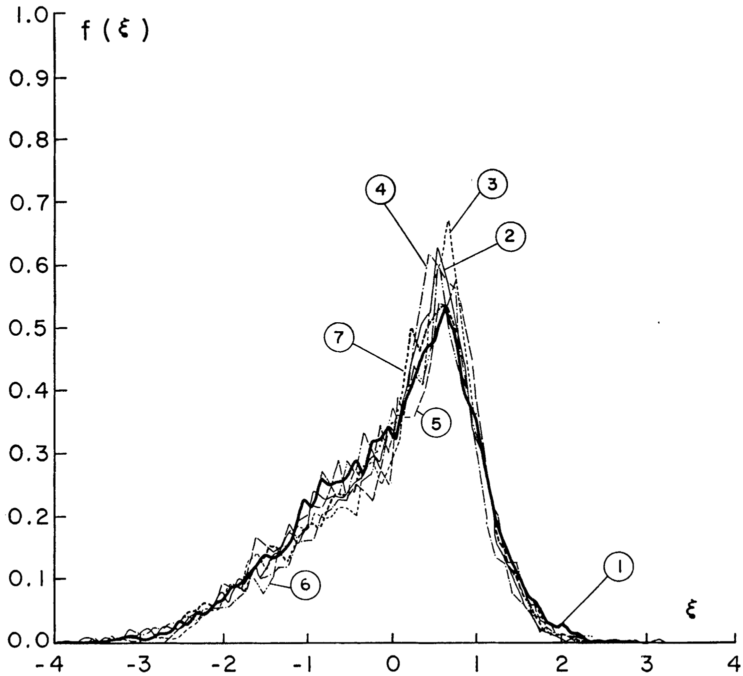


Fig. 11.
Frequency density distributions of the independent stochastic components of daily incoming solar radiation of the first 7 stations of Fig. 3.

in which $\xi_{p,\tau}$ is approximately the second-order stationary and independent component. By using Eq. 1 the $\varepsilon_{p,\tau}$-series is computed, and by using Eq. 2 and the estimated r_1, the independent stochastic component $\xi_{p,\tau}$ is obtained for each of the 14 series of incoming solar radiation. The frequency distributions of these $\xi_{p,\tau}$-series are presented in Figs. 11 and 12, each with distributions for seven stations.

These 14 empirical frequency density distributions of approximately independent stochastic components, or the white noise in the incoming daily solar radiation produced by the atmosphere, are evidently negatively skewed. They should be bounded on both tails. The lower bound can be estimated approximately by putting $x_{p,\tau}$ in Eq. 1 to be zero giving thus $\varepsilon_{p,\tau} = -\mu_\tau/\sigma_\tau$; then by using Eq. 2 the approximately lower bound for $\xi_{p,\tau}$ is

$$\text{Min } (\xi_{p,\tau}) \equiv \frac{-(1-r_1)\,(\mu_\tau/\sigma_\tau)_{\min}}{\sqrt{1-r_1^2}} \qquad (3)$$

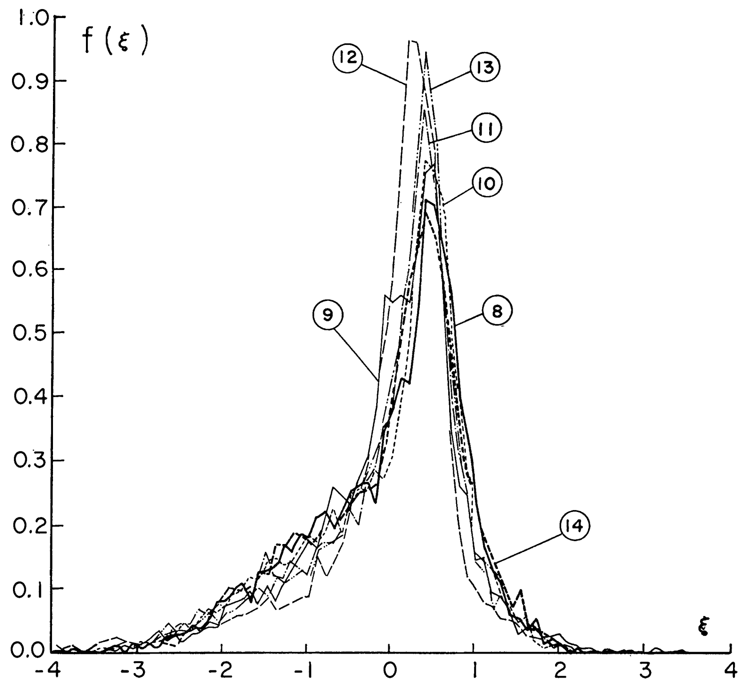


Fig. 12.
Frequency density distributions of the independent stochastic components of daily incoming solar radiation of the second 7 stations of Fig. 3.

The upper bound is less easy to determine, though there is an absolute maximum for $x_{p,\tau}$ at any place, as well as the corresponding values of μ_τ and σ_τ which produce a Max $(\xi_{p,\tau})$.

A very consistent pattern in the distributions of $\xi_{p,\tau}$ among 14 series, though there are some differences in them because of differences in station latitude, climate, and sampling variation, points out clearly that the atmosphere is a large noise generator in all geophysical time processes which are related to solar energy inputs. This stochasticity has particular characteristics created mainly by the periodic-stochastic filter of the atmospheric opacity for the incoming solar radiation. Though the processes of energy transfer from the atmosphere to other of earth's environments are much more complex than might be implied by the above discussion, the simple recordings of measured incoming solar radiation give good insight into the effects of atmosphere on the stochasticity in geophysical processes.

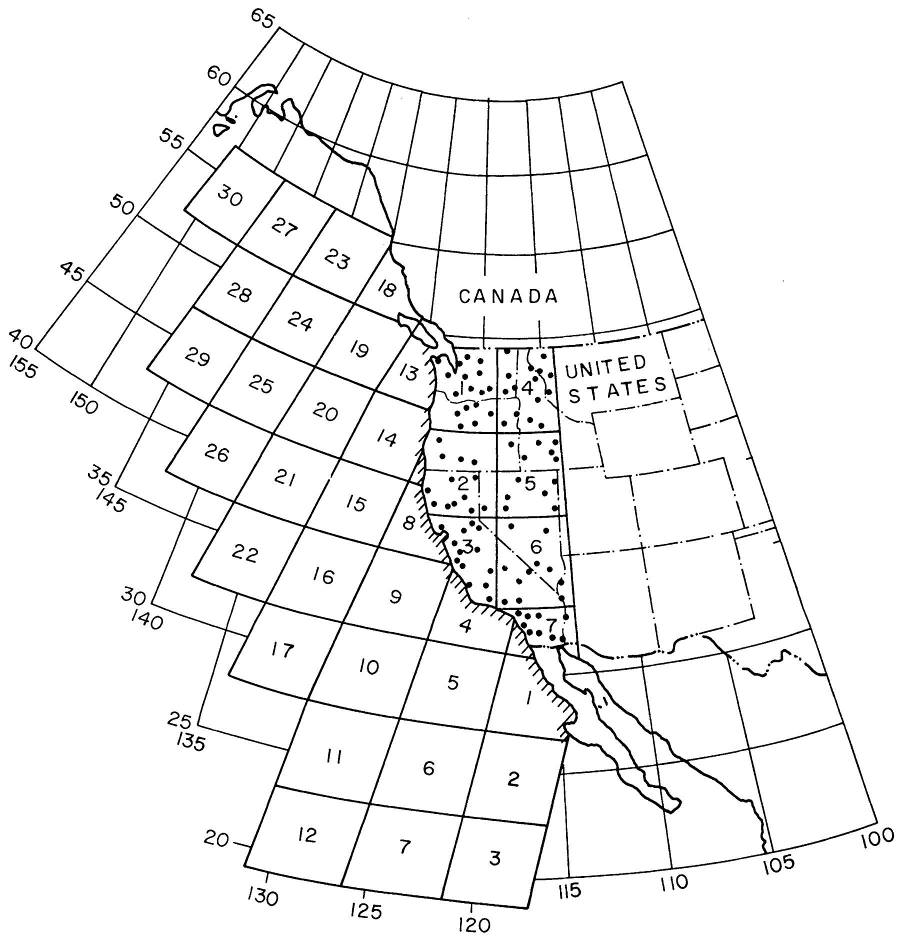


Fig. 13.
Positions of coastal and ocean areas for the investigation of relations of continental monthly precipitation to the monthly temperature of Pacific Ocean surface.

STOCHASTICITY IN OCEAN TEMPERATURE

The results given in this paper on the dependent and independent stochastic components in the surface ocean temperature time series are from a study conducted at Colorado State University on the long range hydrologic forecast

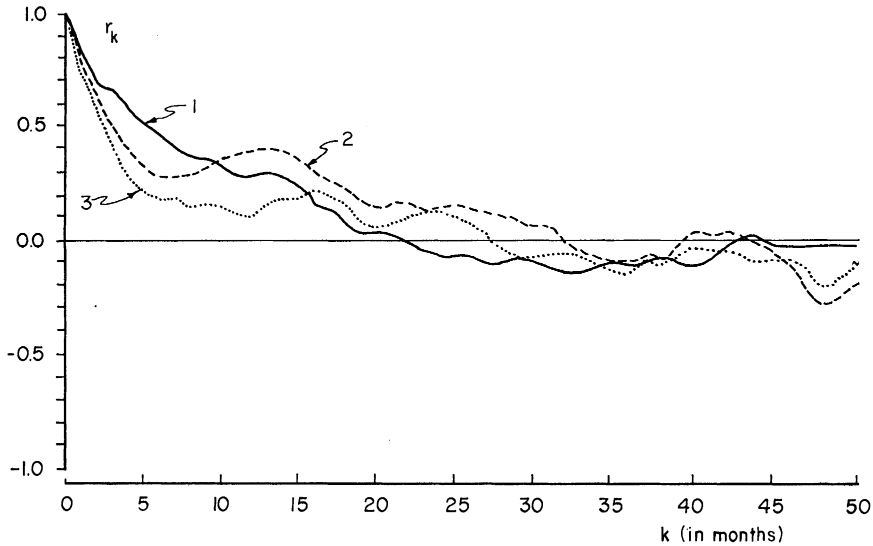


Fig. 14.
Correlograms of the dependent stochastic components of the ocean surface temperature of monthly series, for the areas of Pacific Ocean Nos. 10, 20, and 25 of Fig. 13.

as a part of a study on large continental droughts *. It was attempted to relate the average monthly precipitation of areas close to the Pacific Ocean in the Western United States and the average monthly ocean surface temperature of areas adjacent to the continental areas. Figure 13 presents the areas both of the Western United States (areas designated 1–7) and the adjacent surface of the Pacific Ocean (areas designated 1–30). The dots on the continental areas represent the precipitation station positions that were used to average the monthly precipitation over that area.

The average monthly temperatures of the ocean surface were available for 14 years, so that the series of 168 monthly values represented the periodic-stochastic process of the ocean surface temperature, averaged both over the area and over the month. By using the mean monthly temperature for each of the 12 months of the year and the corresponding monthly standard deviations (or m_τ and s_τ) and Eq. 1, the dependent stochastic components are computed. Figure 14 gives three correlograms of these dependent stochastic components, $\varepsilon_{p,\tau}$, and particularly for the ocean areas 10, 20, and 25.

* Sponsored by U.S. National Science Foundation, Grant GK-11564.

Figure 14 shows a large stochastic dependence in the $\varepsilon_{p,\tau}$-series; similar results are obtained for all other 27 areas. The carry-over of the accumulated heat is on the order of 20–30 months, or for approximately 2–3 years. The autoregressive dependence models may be indicated. The first autocorrelation coefficient for the dependent stochastic components of the average monthly ocean surface temperature ranged for the 30 areas between 0.639 and 0.828, representing a very large time-dependence for monthly series. It is evident, therefore, that the nearly independent (or dependent with very short memory) stochastic component of the incoming solar radiation to the ocean surface is transformed by the heat accumulation in the upper layers of sea water into a highly dependent stochastic component of time series of ocean surface temperature. The oceans and seas, as the earth's environment of significant influence on the stochasticity of geophysical processes, are basically the attenuators of the stochasticity in the energy variables, created by the atmosphere.

Assuming that the first-order linear autoregressive scheme is a sufficient first-order gross approximation for the dependence model of the $\varepsilon_{p,\tau}$-series for the ocean temperature, Eq. 2 then produces the independent stochastic component, $\xi_{p,\tau}$. This "whitening" of the dependent stochastic component presents the real noise in the ocean temperature. Figure 15 gives three correlograms of the $\xi_{p,\tau}$-series, and particularly for the ocean areas 1, 10, and 20. The first autocorrelation coefficients are now for these three example cases –0.083, –0.087,

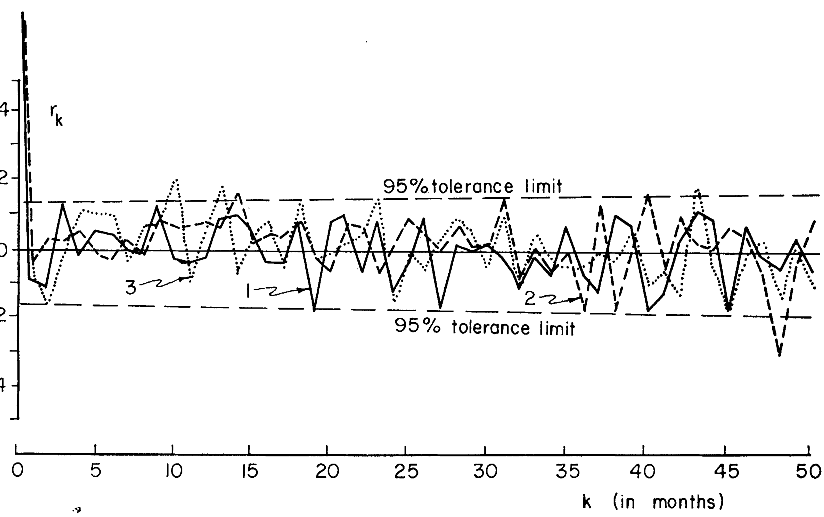


Fig. 15.
Correlograms of the independent stochastic components of Eq. 2 for the monthly series of the ocean surface temperature for the areas 1, 10, and 20 of Fig. 13.

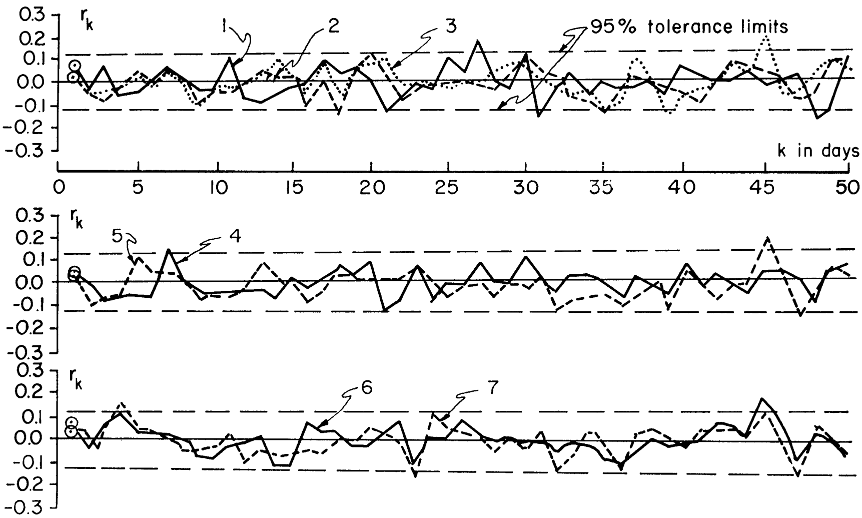


Fig. 16.
Correlograms of the stochastic components of Eq. 1 of the monthly precipitation series of seven areas of Western United States as shown in Fig. 13.

and –0.030. The second-order or higher-order linear autoregressive schemes may be more indicated than the first-order model. However, the tolerance levels on the 95 percent level indicate in all three cases that the correlograms of their $\xi_{p,\tau}$-series cannot be distinguished from the independent stochastic components (white noise), for all practical purposes. It is clear that the ocean surface temperature is only an indicator, and most likely a poor one, of the accumulated heat in the upper layers of oceans and seas. However, in the absence of better data for measuring the stored heat, it gives a general picture of the effects of heat storage on the stochasticity in geophysical processes.

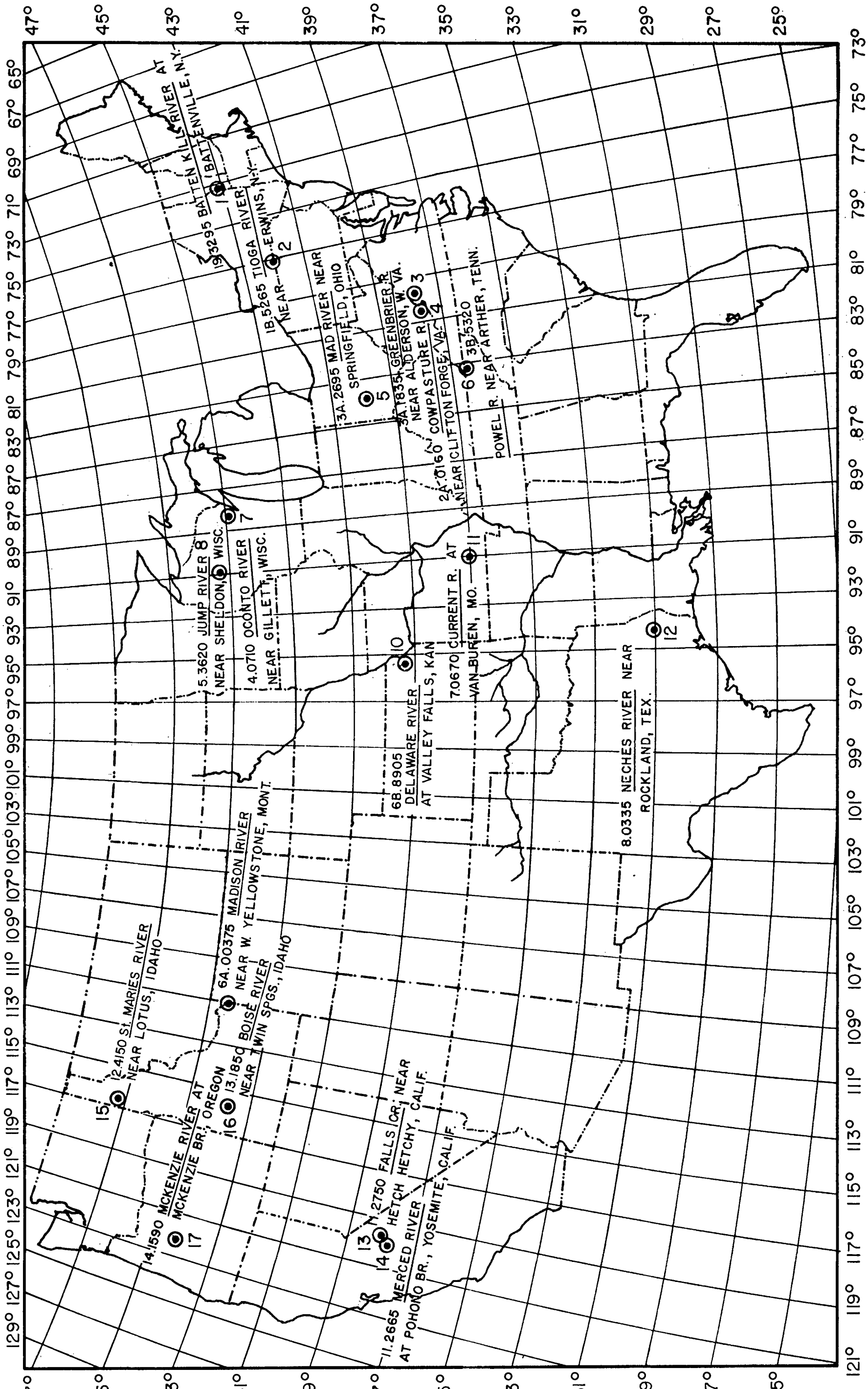
STOCHASTICITY IN MONTHLY PRECIPITATION

The monthly precipitation values of stations represented by dots in Fig. 13 are averaged in order to obtain the monthly precipitation values for the seven areas of Fig. 13. The mean monthly values m_τ and the corresponding monthly standard deviations s_τ, with $\tau = 1, 2, \ldots, 12$, are computed, and then by using Eq. 1, replacing m_τ and s_τ with μ_τ and σ_τ, the $\varepsilon_{p,\tau}$-series are obtained. Their correlograms are given in Fig. 16 for all seven areas (1 to 7). All seven first autocorrelation coefficients are somewhat greater than zero, and particularly somewhat greater than the expected sample mean values of r_1 for independent series. However, these positive values are so small that their squares, as the explained variance of $\varepsilon_{p,\tau}$-series by the first-order linear autoregressive scheme, may be considered negligible, for all practical purposes. These seven correlograms, with the tolerance limits on the 95 percent probability level, are practically confined inside these limits, except for several values which are outside these limits and which are expected to be outside them by the definition of tolerance limits. For all practical purposes, the stochastic components $\varepsilon_{p,\tau}$ of monthly precipitation series are independent components.

Taking into account that the stored heat in the uppermost layers of oceans and seas is the major energy necessary for the evaporation of water, then it becomes clear that the atmosphere with its dynamic stochastic processes is a major stochasticity source, and in this case of the relation of coastal monthly precipitation to the ocean surface temperature, the atmosphere is a disruptor of effects of a large stochastic dependence of ocean heat storage. The evaporation from ocean surfaces depends roughly on the difference between vapor pressure of adjacent layers of water and air, but primarily on the diffusion process which is dependent in turn on the turbulence of the air. The air is then the major controlling factor of evaporation from ocean surfaces.

STOCHASTICITY IN RIVER RUNOFF

The river runoff is represented in this paper by the daily river flows. Figure 17 gives the positions of 17 river gauging stations, and their numbers (1 to 17) and names. The river flow time series are all 40 years in length, and have the least amount of man-made changes or natural accidental disruptions. The daily time series are periodic-stochastic processes and are analyzed in the same way as the other series. Figures 18 and 19 give the estimated 365 mean daily flows and the daily standard deviations m_τ and s_τ as well as the fitted periodic functions μ_τ and σ_τ, with $\tau = 1, 2, \ldots, 365$, for the Tioga River (station no. 2 in Fig. 17). The deviations $\mu_\tau - m_\tau$ and $\sigma_\tau - s_\tau$ may be explained mainly by the sampling variations, though the estimated parameters of the significant harmonics in the periodic functions also contain sampling errors. As expected, these deviations are greater for s_τ than for m_τ because of the use of the second and the first statistical moments in their estimation, respectively.



1B.3295 BATTEN KILL RIVER AT BATTENVILLE, N.Y.
1
1B.5265 TIOGA RIVER NEAR ERWINS, N.Y.
2
3A.2695 MAD RIVER NEAR SPRINGFIELD, OHIO
5
3A.1835 GREENBRIER R. NEAR ALDERSON, W. VA.
3
2A.0160 COWPASTURE R. NEAR CLIFTON FORGE, VA.
4
3B.5320 POWEL R. NEAR ARTHER, TENN.
6
7
5.3620 JUMP RIVER NEAR SHELDON, WISC.
8
4.0710 OCONTO RIVER NEAR GILLETT, WISC.
7.0670 CURRENT R. AT VAN BUREN, MO.
11
6B.8905 DELAWARE RIVER AT VALLEY FALLS, KAN
10
8.0335 NECHES RIVER NEAR ROCKLAND, TEX.
12
6A.00375 MADISON RIVER NEAR W. YELLOWSTONE, MONT.
12.4150 ST. MARIES RIVER NEAR LOTUS, IDAHO
15
13.1850 BOISE RIVER NEAR TWIN SPGS., IDAHO
16
14.1590 MCKENZIE RIVER AT MCKENZIE BR., OREGON
17
11.2750 FALLS CR. NEAR HETCH HETCHY, CALIF.
13
11.2665 MERCED RIVER AT POHONO BR., YOSEMITE, CALIF.
14
47° 45° 43° 41° 39° 37° 35° 33° 31° 29° 27° 25°
65° 67° 69° 71° 73° 75° 77° 79° 81° 83° 87° 87° 89° 91° 93° 95° 97° 99° 101° 103° 105° 107° 109° 111° 113° 115° 117° 119° 121° 123° 125° 127° 129°
73° 75° 77° 79° 81° 83° 85° 87° 89° 91° 93° 95° 97° 99° 101° 103° 105° 107° 109° 111° 113° 115° 117° 119° 121°

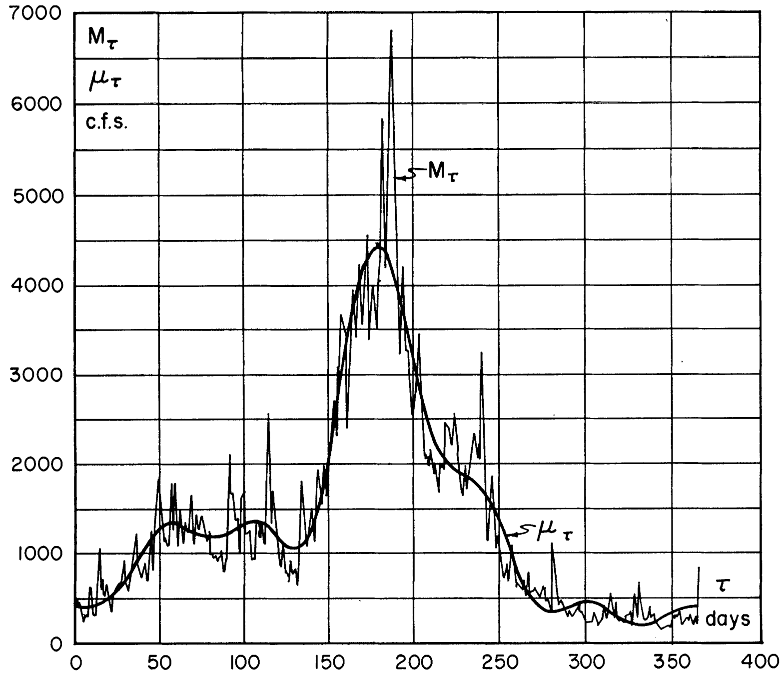


Fig. 18.
The estimated mean daily flows of 365 days, m_τ, and the fitted periodic function to them, μ_τ, for the daily river flow series of Tioga River (no. 2 in Fig. 17).

Figure 20 gives the 365 first autocorrelation coefficients $(r_1)_\tau$, with $\tau = 1, 2, \ldots, 365$, of the $\varepsilon_{p,\tau}$-series of the daily flows of the Tioga River, with the $\varepsilon_{p,\tau}$-series obtained by using Eq. 1 and the μ_τ and σ_τ functions of Figs. 16 and 19. The Fourier analysis of this series of 365 values of $(r_1)_\tau$ shows no significant harmonic, so that the autocorrelation function may be considered as non-periodic in this case. This was not the case for all 17 stations studied. Some of them showed clearly the periodicities in the autocorrelation coefficients. This was particularly the case when both the rainfall and snow melt contributed

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Fig. 17.
Positions, numbers, and names of 17 river gauging stations in the United States, for which stations the daily flow series are analyzed.

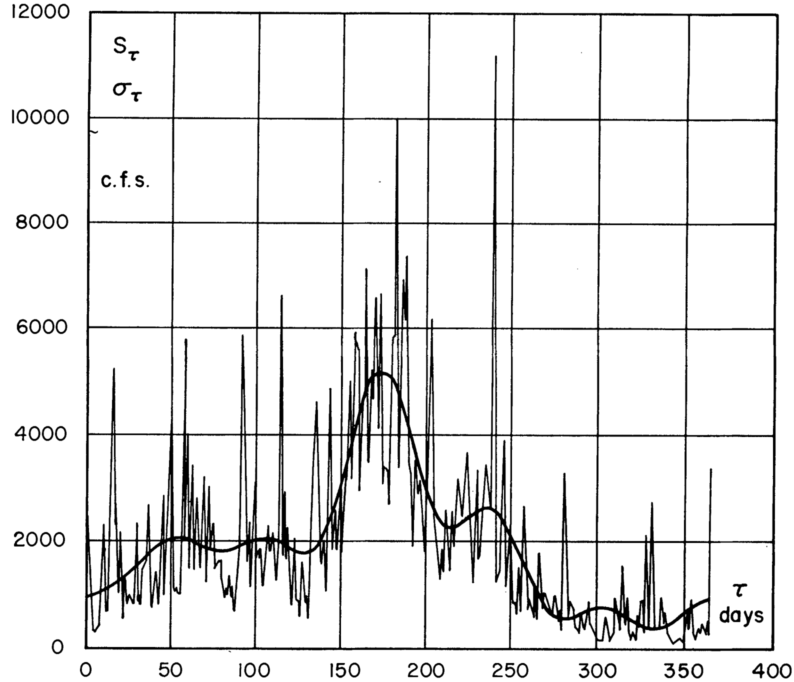

Fig. 19.

The estimated standard deviations of daily river flow series for 365 days, s_τ, and the fitted periodic function to them, σ_τ, for the Tioga River (No. 2 in Fig. 17).

equally to the runoff. The mean autocorrelation coefficient is about $(\varrho_1)_\tau$ = 0.74, which is a somewhat greater value than the overall first autocorrelation coefficient of the entire $\varepsilon_{p,\tau}$-series. The autoregressive model (first, second, or third, whichever came out to be the most appropriate by a simple test) of the type of Eq. 2 was used to compute the independent second-order stationary stochastic component, $\xi_{p,\tau}$.

Figure 21 presents the empirical frequency density distributions for the 6 rivers in the Eastern United States (nos. 1–6). They are nearly identical frequency density curves as it concerns the higher-order moments (third, fourth, ...). This fact points out that the standardized and independent stochastic component (white noise) is nearly identical for that geographical area though the mean and variance depend on the river basin size, climate, and other factors.

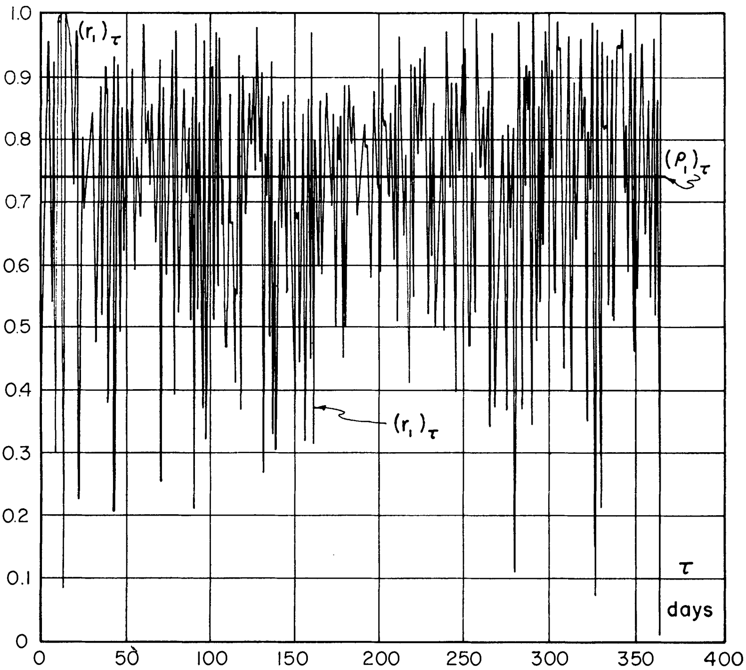

Fig. 20.

The estimated first autocorrelation coefficients, $(r_1)_\tau$, for 365 days, of the stochastic component of Eq. 1 for the daily river flow series of the Tioga River (No. 2 in Fig. 17), with the mean $(\varrho_1)_\tau$.

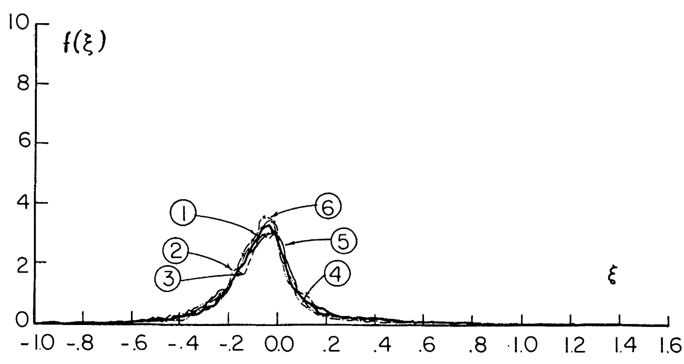

Fig. 21.

The empirical frequency density curves of the independent stochastic component of six daily river flow series in the Eastern United States.

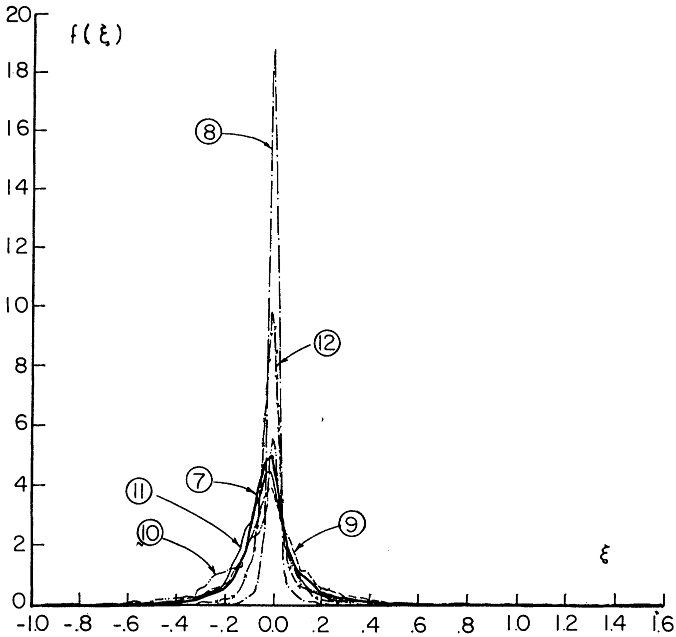

Fig. 22.

The empirical frequency density curves of the independent stochastic component of six daily river flow series in the Midwestern United States.

Figure 22 gives 6 other empirical frequency density curves (nos. 7–12) mainly in the Middlewest United States. The differences in this independent second-order stationary stochastic component are much greater for this area than for the eastern part. Similarly, Fig. 23 gives the empirical frequency density curves for 5 rivers in the Western United States (nos. 13–17) also showing a large amount of consistency in these distributions. The variances for all frequency density curves are equal.

Although the rainfall and snowfall have relatively small time-dependence in the stochastic component of the periodic-stochastic processes of these random variables, the large storage capacities for the retention of water in surface and underground spaces, or in the delays of the water outflow from the river basins, create a large time dependence in the stochastic component of the periodic-stochastic process of river runoff. Therefore, the earth's continental surfaces and the groundwater accumulations act as the attenuators of the highly stochastic input of precipitation. It is easy to demonstrate that the time-dependence of the stochastic component of river runoff is primarily dependent on the retention and time delaying capacity of river basins for the effective precipitation (the part of precipitation which becomes the river runoff).

It can be shown that the potential evaporation from a river basin is a highly dependent process, as it concerns the stochastic component of the periodic-stochastic process of evaporation. The real evaporation has a much less dependent stochastic component than the potential evaporation. Again, the atmosphere acts as a disruptor of the time dependence of stochastic components when passing from a process on the continental areas to a connecting process of the atmosphere.

CONCLUSIONS

The following conclusions may be drawn from the preceding discussions:

1. The atmosphere is the major source of stochasticity in all those geophysical time processes which are connected to the incoming solar radiation, though the other environments (oceans and seas, earth's surface, earth's crust) also produce the stochasticity.

2. The opacity of atmosphere for the incoming and outgoing solar energy is the first and the main source of stochasticity, with very short memory in the observed incoming solar radiation on the earth's surface, thus creating the time

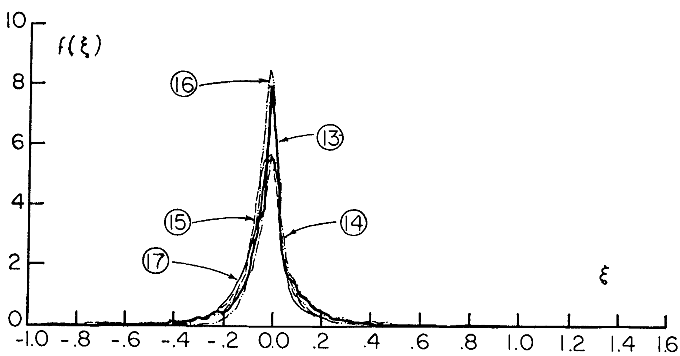

Fig. 23.
The empirical frequency density curves of the independent stochastic component of five daily river flow series in the Western United States.

and space randomness in the distribution of energy in earth's environments.
3. The randomness of the atmospheric opacity of periodic-stochastic type for the radiation transforms a deterministic-periodic process of solar radiation into a periodic-stochastic process of the measured radiation on the earth's surface, with a large degree of stochasticity.
4. The oceans and seas attenuate the stochasticity produced by the atmosphere, because the water is a very conservative fluid for the storage of heat and kinetic energy.
5. The continental surfaces and underground water and heat storage attenuate the high stochasticity produced by the various inputs from the atmosphere to the surface and the underground of continents.
6. The atmosphere usually disrupts high dependences of stochastic components of inputs from the other earth environments into the atmosphere.
7. The most stochasticity-producing environment has been studied for decades mainly by the classical methods of deterministic laws of fluid mechanics and thermodynamics. What is needed at present is stochastic meteorology, as a counterweight to the classical deterministic meteorology. Without stochastic meteorology a good understanding and description of many geophysical stochastic processes will be difficult, particularly in hydrology.

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