cally optimum trajectories require thrust modulation, but this paper is virtually the first to point out how small are the time savings due to thrust modulation.

The example which the author treats in detail is the minimum-time transfer from one position of rest to another position of rest in field-free space. This may be looked upon as a limiting case of the practically important problem of transfer between two coplanar circular orbits when the transfer time is very short compared to the orbital periods. For this limiting case the paper shows that the optimum thrust program produces a 4 per cent saving in transfer time when compared to constant acceleration. It can be shown that for the other limiting case, when the transfer time is much longer than the orbital periods, the optimum thrust program is constant acceleration. Although the time saving might not vary monotonically from 4 per cent in one limiting case to zero in the other, it seems reasonable to assume that it does. It follows that constant-acceleration programs may well yield near-optimum performance for all transfer times. Since it has been shown in the literature that the differences between constant-thrust programs and constant-acceleration programs are usually small, it can be concluded that simple, constant-thrust systems may also yield near-optimum performance.

This paper provides a complete and elegant treatment of an interesting optimization problem. The maximum obtainable system performance is derived and compared with the performance previously calculated for a simpler, nonoptimum thrust program. This determines the maximum performance to be expected from low-thrust propulsion as well as the performance penalties of a simplified mode of operation.

The Effect of a Longitudinal Gravitational Field on the Supercavitating Flow Over a Wedge[1]

B. R. PARKIN.[2] The author has given an elegant solution to a difficult problem of practical importance. It is worth noting that within the framework of the linearized free-streamline theory he has been able to account exactly for the influence of gravity. This is the first exact solution which has been obtained under such general conditions (we are excluding certain well-known but very special solutions from the nonlinear theory of cavity flows).

In his introductory remarks the author indicates the existence of some work by the writer in which the effect of a gravity field normal to the direction of a cavity flow has been given a simplified treatment. Although this simplified treatment is not capable of describing cavity flows with large buoyant effects, the resultant approximation is consistent with the scheme of linearization and should give results valid for that Froude-number range in which the effects of gravity, like the effects of the body, are of first order in smallness. In this linearized theory the solution is obtained by summing a number of fundamental singularities in much the same manner as has been illustrated in the present paper. Therefore the writer will resist the temptation to present any of these details. However, since his findings tend to complement those given already by the author, a brief discussion of them might be useful.

In reference [6][3] of the paper, the problem considered was that of a fully cavitated flat-plate hydrofoil in a flow of unlimited extent. The closed-cavity linearized theory was used to consider the effect of gravity acting in the downward direction. The flow configuration is shown in Fig. 1 of this discussion. This figure shows some calculated cavity shapes which indicate the effect of gravity upon cavity length. Four Froude numbers are shown: $F^2 = 1$, 16, 30, and ∞. The theory does not predict a tilting up of the cavity as a result of buoyancy, and two-dimensional experiments at $F = 2.8$ and 4.5 with ventilated cavities by Dawson[4] have shown no apparent tilting of the cavity.

The effect of gravity upon the lift coefficient is shown in Fig. 2. These results are given for an attack angle of 8 deg, which is rather large for the linearized theory, in order to emphasize the effect of gravity upon the profile lift. These results show that as the cavitation number approaches zero and the cavity length becomes infinite, the present approximate theory breaks down causing the gravitational effect to be exaggerated. Of greater importance than this pathological point is the fact that the lift is reduced as the effect of gravity becomes more important. This result is opposite to the behavior one would expect to obtain from a flow with gravity if he were to correct for its effect by simply adding a term to the gravity-free result to account for buoyancy on the hydrofoil surface. The present theory does take account of buoyancy in this manner. However, the detailed results show that the direct buoyant effect is more than compensated for by the effect upon the flow field of the difference in

[1] By A. J. Acosta, published in the June, 1961, issue of the Journal of Applied Mechanics, vol. 28, Trans. ASME, vol. 83, Series E, pp. 188–192.

[2] The RAND Corporation, Santa Monica, Calif.

[3] B. R. Parkin, "A Note on the Cavity Flow Past a Hydrofoil in a Liquid With Gravity," California Institute of Technology, Engineering Division Report No. 47-9, 1957.

[4] T. E. Dawson, "An Experimental Investigation of a Fully Cavitating Two Dimensional Flat Plate Hydrofoil Near a Free Surface," Thesis, California Institute of Technology, Pasadena, Calif., 1949.

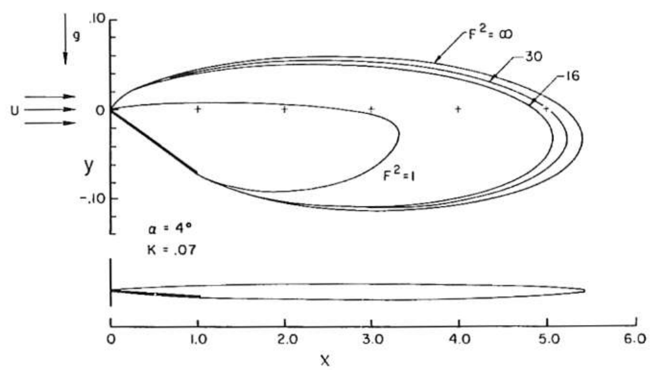

Fig. 1 Effect of gravity on calculated cavity shapes

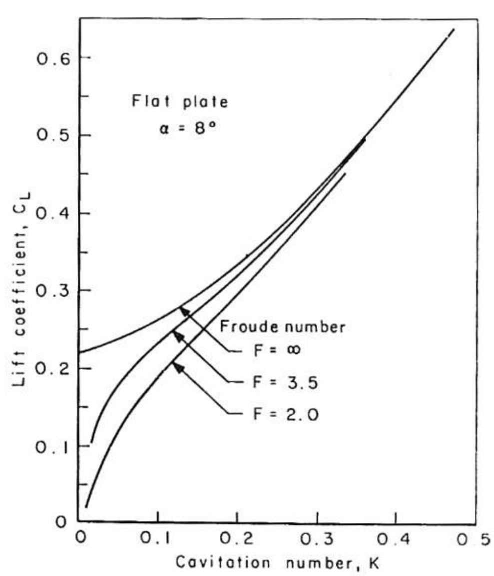

Fig. 2 The effect of gravity on lift coefficient

velocity between the upper and lower surfaces of the cavity. In addition, it is found for the present flow configuration at least that if the Froude number based upon the hydrofoil chord exceeds 6, the effects of gravity can be neglected.

Author's Closure

It is a pleasure to acknowledge the contribution by Dr. Parkin to the relatively few numbers of free streamline flows in which the effect of gravity has been taken into account. Although not mentioned by him, it should be clear that the longitudinal gravity field is considerably easier to treat than the one normal to the flow as discussed above. For this reason the author hopes that Dr. Parkin will consider making the details of his solution more generally available. The two problems are not, of course, strictly comparable; one is for a lifting hydrofoil with a normal gravitational force, and the other for a pure drag body with the force of gravity directed parallel to the flow. The effects of gravity on the cavity proportions are strikingly different in the two cases and are probably typical of the two orientations of gravity rather than the details of the hydrofoil system. However, it would be of great interest to have the solution for a cavitating wedge for the situation treated by Dr. Parkin. The somewhat surprising result that the cavity is not inclined upward under the action of gravity has been further confirmed in recent experiments on cavitation behind two-dimensional bluff bodies at The California Institute of Technology, and it is hoped to present these in the near future.

On Classical Plate Theory and Wave Propagation[1]

NORMAN DAVIDS.[2] The author has indeed shown that the "classical theory" of plates may be used under certain conditions for predicting the response of plates of large radius to transverse disturbances. As implied in the paper, equation (1) represents only the lowest mode of transverse vibration of the plate and gives rise to infinitely long flexural waves, hence the lack of a distinct wave-front. Second-order corrections were investigated by Mindlin[3] but their use in impact problems does lead to excessive mathematical complexities. Under most conditions of collision the equation is reasonably valid, as the author's measurements also confirm.

Somewhat similar measurements have been made by Press and Oliver [4,5] by exciting flexural waves in a $^1/_{32}$-in. plate of 24S-T aluminum by means of an impulsive spark source. The resulting curves of displacement show the characteristic aperiodic transverse oscillations steadily increasing in amplitude, similar to Figs. 1 and 2 of the paper. These authors also cite other references to vibrations of thin plates which have bearing on the present problem.

Experimental studies of flexural waves of infinite plates (ice sheets 1.5–2 ft thick) have also been made by Project Michigan, because of their interest in ice prospecting.[6] Excitation was made by controlled falling weights as impacts. The response was obtained by 3-component geophones placed at intervals along the surface. Again the low-frequency response is similar to that obtained in this paper. However, the theoretical interpretation is considerably more difficult because of the interaction with the water below the ice. The dispersion of the flexural waves was found to be independent of the source-to-detector distance for distances larger than 250 ft.

Starting from the frequency equation for long flexural waves in plates[7]

$$c^2/c_2^2 = (4/3)(2\pi h/\lambda)^2(1 - c_2^2/c_1^2)$$

where

$$c_1^2 = (\lambda + 2\mu)/\rho,\ c_2^2 = \mu/\rho$$

we see that waves of infinitely short wave length ought to travel with infinite (phase) velocity. Actually this does not happen, but a finite velocity is approached because the exact frequency equation based on the wave-propagation theory takes over.

The use of integral transforms, under the conditions described in the paper, is much more satisfactory than the classical method of solution of equation (1), which is based on expansions in infinite series of eigenfunctions, and which are generally unsatisfactory for calculating the motion of the plate because of convergence difficulties.

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[1] By M. A. Medick, published in the June, 1961, issue of the JOURNAL OF APPLIED MECHANICS, vol. 28, TRANS. ASME, vol. 83, Series E, pp. 223–228.

[2] Professor, Engineering Mechanics, The Pennsylvania State University, University Park, Pa.

[3] R. D. Mindlin, "Influence of Rotatory Inertia and Shear on Flexural Motion of Isotropic Elastic Plates," JOURNAL OF APPLIED MECHANICS, vol. 18, TRANS. ASME, vol. 73, 1951, p. 31.

[4] F. Press and J. Oliver, "Model Study of Air-Coupled Surface Waves," *Journal of the Acoustical Society of America*, vol. 27, 1955, p. 43.

[5] W. M. Ewing, W. S. Jardetzky, and F. Press, "Elastic Waves in Layered Media," McGraw-Hill Book Company, Inc., New York, N. Y., 1957, p. 287.

[6] D. H. Clements, D. E. Willis, and J. T. Wilson, "Waves in Lake Ice From Impacts," Report No. 2144-265-T, Project Michigan, Willow Run Laboratories, University of Michigan, Ann Arbor, Mich., August, 1958.

[7] For example, footnote 5, p. 285.