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An optimal design of water distribution networks with hydraulic-connectivity

Hyun-Gon Shin and Heekyung Park

ABSTRACT

In designing water distribution networks (WDN) against emergency situations, efforts must be made to achieve two objectives: maximizing reliability and minimizing cost. For such optimization, it is necessary first to quantify the reliability of WDN using a surrogate measure. Hydraulic-connectivity is selected as a surrogate measure in this study, which indicates a probability that every demand node in a network is connected to at least one supply source with required flow at adequate pressure. An optimization model using hydraulic-connectivity and the genetic algorithm (GA) is formulated. For illustration, the optimization model is applied to the New York City water supply tunnel. This tunnel cannot satisfy the minimum pressure with existing diameters. The new optimal design with required flow at adequate pressure is found to be lower than the existing tunnel cost by US$25.252 million when the hydraulic-connectivity is 0.9778. It is also noted that this approach allows the exploration of the trade-off between cost and reliability directly. This permits designers to design WDN based on more quantitative information regarding cost and reliability.

Key words | hydraulic-connectivity, GA, optimization, reliability, WDN

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INTRODUCTION

Many algorithms to find a minimum cost design for water distribution networks (WDN) have been developed during the last decades, using linear, non-linear and dynamic programming. Lai and Schaake (1969) were the first to present a type of linear program model which could determine pipe diameter at minimum cost; Alperovits and Shamir (1977) introduced an optimum design method called linear-programming-gradient (LPG); and Quindry *et al.* (1979, 1981) introduced a variation of the LPG model. Fujiwara and Khang (1990) suggested a two-phase decomposition method for optimal design of looped water distribution networks. Its main feature is that it generates a sequence of improving local optimal solutions. As the genetic algorithm (GA) developed, Murphy and Simpson (1992) first applied GA to the optimization of WDN and then Savic and Walter (1997) developed a ganet model using GA. The model uses pipe diameter as decision variable.

At the same time, many research projects have been carried out to quantify reliability and adopt it into the optimization process. Researchers tried to simulate the characteristics of emergency situations by defining systems failure conditions. Some researchers developed models to estimate system reliability with frequency, duration, and quantity of system failure. Generally speaking,

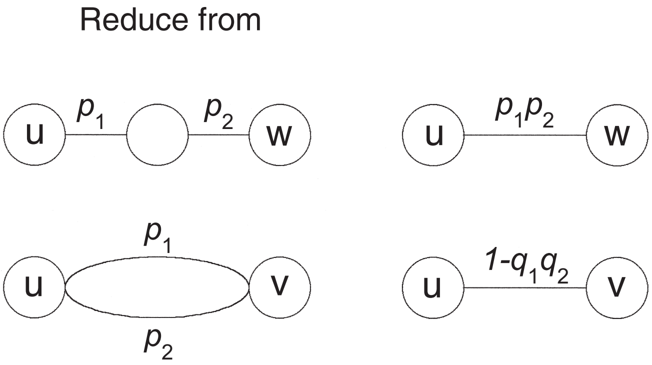

Figure 1 | Series and parallel reduction.

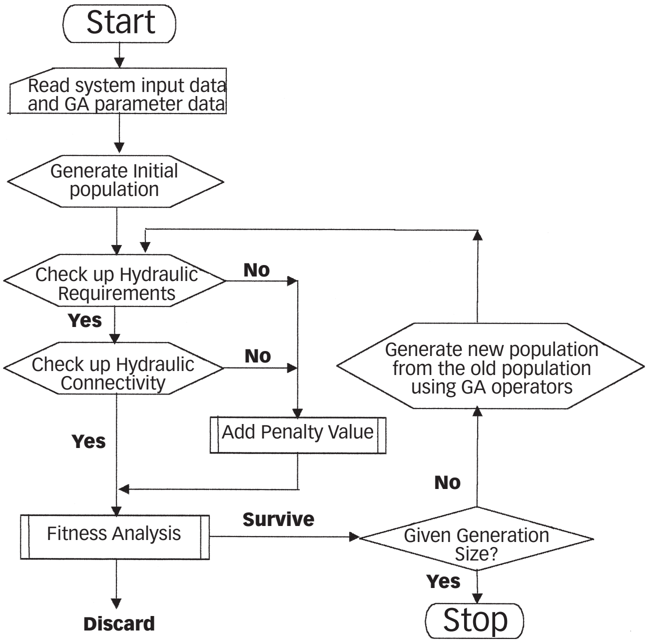

Figure 2 | Flowchart of the main program.

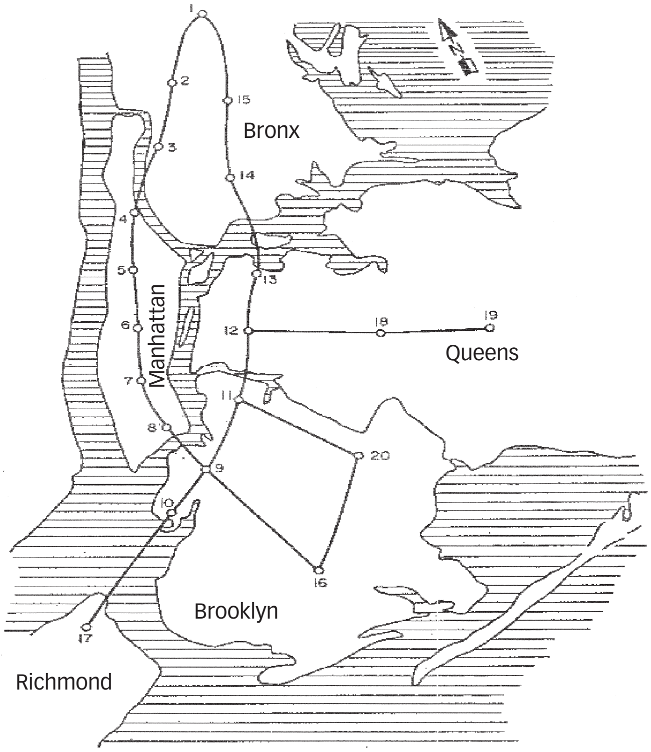

Figure 3 | New York City water supply tunnels network.

however, no universally accepted measure of reliability for WDN has yet been developed. Hobbs and Beim (1988) classified reliability into two categories using the bulk supply and the distribution network. The former measures reliability from the point of view of total supply and demand, while the latter measures reliability of the distribution network against random failure of components.

Shamir (1981) presented a reliability theory for WDN, defining reliability in terms of the total volume and the rate of shortage. A volume reliability factor and a discharge reliability factor were derived according to these two definitions. System reliability was then expressed with an overall reliability factor of an average of the two. Su *et al.* (1987) introduced a heuristic method using the minimum cutset method and the generalized reduced-gradient model (GRG2). Wagner *et al.* (1986) introduced analytical methods to find two probabilistic reliability measures, 'reachability' and 'connectivity'. These measures are calculated with series and parallel reduction techniques. Reachability of demand node is the probability that a node is connected to at least one source. Connectivity of the network is defined as the probability that every demand node in the network is connected to at least one source. Heekyung Park (1990) and Heekyung Park and Liebman (1993) presented expected shortage as a reliability measure and developed multi-objective optimization models which minimize cost while constraining expected shortage.

In this paper, connectivity is introduced as a surrogate measure of reliability. The mechanical connection between source and node does not guarantee reliability to WDN unless the required flow at adequate pressure is provided at the node. But, if it could guarantee such quantity and pressure requirements, connectivity can be a good measure of WDN reliability. This paper develops ways of providing such conditions while connectivity is used as a reliability measure against emergency situations. In addition, an optimization method has been developed to produce a minimum cost design satisfying given reliability constraints expressed in terms of connectivity. For optimization, GA has been used; the connectivity in the model is termed 'hydraulic-connectivity'.

Table 1 | Node data for New York City water supply tunnels network

Node	Min. head (m)	Demand ($m^3 s^{-1}$)	Node	Min. head (m)	Demand ($m^3 s^{-1}$)	Node	Min. head (m)	Demand ($m^3 s^{-1}$)
1	91.44	– 57.1292	8	77.724	2.497546	15	77.724	2.616477
2	77.724	2.616477	9	77.724	4.813864	16	79.248	4.813864
3	77.724	2.616477	10	77.724	0.028317	17	83.14944	1.628219
4	77.724	2.497546	11	77.724	4.813864	18	77.724	3.315903
5	77.724	2.497546	12	77.724	3.315903	19	77.724	3.315903
6	77.724	2.497546	13	77.724	3.315903	20	77.724	4.813864
7	77.724	2.497546	14	77.724	2.616477			

HYDRAULIC-CONNECTIVITY AS A SURROGATE OF RELIABILITY

Hydraulic-connectivity is selected as a surrogate measure that indicates a probability that every demand node in the network is connected to at least one supply source with required flow at adequate pressure. When an emergency situation breaks out, the most important necessary condition for water supply is that the demand nodes must remain connected at some positive pressure to one of the sources. In this sense, therefore, maximizing the hydraulic-connectivity can be a good way of increasing reliability against emergency situations.

For the calculation of this parameter, water distribution systems are often modelled as networks of supply and demand nodes, connected by links. The nodes are modelled as being perfectly reliable. Each link i is said to have a probability p_i of functioning at any point in time and probability q_i $(=1-p_i)$ of failing. At any point in time, the system can assume one of a large number of configurations, with some links functioning and other links failed. The probability of any one configuration occurring can be calculated as the product of the p_i values for the operative links multiplied by the product of the q_i values of the failed links. For connectivity calculation, each configuration corresponds to either a connected system, where every demand node is connected via functioning links to some source, or a disconnected system.

In order to calculate connectivity of a series-parallel network, a reduction method has been used in this paper. As shown in Figure 1, a series reduction can be performed by replacing two links (u–v and v–w), incident to the same node of degree 2 (node v) by one link (u–w). If the probabilities of operation of original links are p_1 and p_2, respectively, the probability of operation of the new link will be p_1p_2. A parallel reduction can be performed by replacing two links connecting the same two nodes by one link. If the probability of operation of the original links is p_1 and p_2, respectively, the probability of operation of the new link will be $1-q_1q_2$. The overall probability that the network will remain connected is not affected by these reductions.

For simplicity, this study considers only pipes, even if a network consists of many components such as pipes, tanks, pumps and valves. To simulate the emergency situation, each pipe i is assigned a probability of failure, q_i, which is a function of diameter. By using the probability and diameters of individual pipes and the network configuration, connectivity is calculated.

Many studies to identify the characteristics of failure probability of pipes have been carried out. O'Day (1982)

Table 2 | Pipe data for New York City water supply tunnels network (Hazen-Williams coefficient for all tunnels = 100)

Pipe #	From node #	To node #	Length (m)	Pipe #	From node #	To node #	Length (m)
1	1	2	3535.68	12	13	12	3718.56
2	2	3	6035.04	13	14	13	7345.68
3	3	4	2225.04	14	15	14	6431.28
4	4	5	2529.84	15	1	15	4724.40
5	5	6	2621.28	16	10	17	8046.72
6	6	7	5821.68	17	12	18	9509.76
7	7	8	2926.08	18	18	19	7315.20
8	8	9	3810.00	19	11	20	4389.12
9	9	10	2926.08	20	20	16	11704.32
10	11	9	3413.76	21	9	16	8046.72
11	12	11	4419.60				

and Ciottoni (1983) analysed the New York City and Philadelphia water system, and identified the factors responsible for main breaks. Both Ciottoni and O'Day concluded that higher failure rates are experienced in smaller-diameter pipes. Sullivan (1982) also noted increased failure rates for smaller-diameter pipes in Boston. Kettler and Goulter (1985) presented the equation of linear regression analysis between the failure probability and pipe diameter: $Y = 2.002 - 0.0064\ X$ (Y = failure ($\mathrm{km}^{-1}\ \mathrm{yr}^{-1}$), X = diameter (mm), range 0–300 mm). Wagner *et al.* (1986) presented the failure probability: $q_i = 1.557 \times 10^{-6}\ L$ (L = pipe length (ft)). Note that some of the findings discussed above were adopted for the calculation of failure probabilities of pipes in this study.

HYDRAULIC-CONNECTIVITY CONSTRAINED OPTIMAL DESIGN MODEL: HYCCODEM

Objective

The objective of optimal design is to find pipe diameters minimizing cost, while satisfying a given reliability expressed in hydraulic-connectivity. The method for minimizing cost is to ascertain fitness in GA. The fitness necessary for the GA is formulated as follows:

$$\text{Fitness} = \text{Total Cost}_{\max} - \text{Total Cost} \tag{1}$$

Where, $\text{Total Cost}_{\max}$ = a given large value and Total Cost = total cost of each string from the objective function.

As the total cost is low, the fitness is high and the string will have high probability of survival for the next generation. In this mechanism, the GA will find a minimum cost solution.

On the other hand, each string in GA consists of pipe diameters for all links. The cost function is as follows.

$$\text{Cost} = 1.136\ d_{ij}^{1.24}\ l_{ij}, \tag{2}$$

where, Cost = dollars, d_{ij} = diameter of pipe between node i and node j (cm) and l_{ij} = length of pipe between node i and node j (m).

The total cost of each string is calculated as follows.

Table 3 | Discrete pipe diameters (cm)

91.44	121.92	152.4
182.88	213.36	243.84
274.32	304.80	335.28
365.76	396.24	426.72
457.20	487.68	518.16
548.64	579.12	609.60
640.08	670.56	701.40
731.52	762.00	

$$\text{Total Cost} = \Sigma\ \text{cost} + pc, \qquad (3)$$

where, pc = penalty cost for the violation of the hydraulic requirements.

Check up connectivity

The pipe failure probability (q_{ij}) is expressed as a function of length (l_{ij}) and diameter (d_{ij}) only for the purpose of optimization. The pipe failure probability is given as follows.

$$q_{ij} = 8.14124 \times 10^{-6} \cdot \frac{l_{ij}}{d_{ij}^{1/2}} \qquad (4)$$

Where, l_{ij} = length of pipe (m), d_{ij} = pipe diameter (cm).

The connectivity, which is related to the operational probability (p_{ij}) and the failure probability (q_{ij}) of each pipe, is expressed as follows:

$$\text{Connectivity}_{ij} = \text{Function}\ (p_{ij}, q_{ij}) \geqslant \text{Giv. Connectivity} \qquad (5)$$

where, p_{ij} = operational probability of link from node i to node j, q_{ij} = failure probability of link from node i to node j and $p_{ij} + q_{ij} = 1$.

Check up hydraulic connectivity

Each string is solved by a network solver. The Hazen-Williams equation is used to calculate the hydraulic

Table 5 | The comparison between existent and optimal results: pipe diameters and cost (US$ millions)

Pipe #	Existing diameter (cm)	New diameter (cm)	Pipe #	Existing diameter (cm)	New diameter (cm)
1	457.2	579.12	12	518.16	579.12
2	457.2	335.28	13	518.16	609.6
3	457.2	365.76	14	518.16	609.6
4	457.2	243.84	15	518.16	579.12
5	457.2	243.84	16	182.88	91.44
6	457.2	182.88	17	182.88	121.92
7	335.28	152.4	18	152.4	91.44
8	335.28	152.4	19	152.4	213.36
9	457.2	152.4	20	152.4	91.44
10	518.16	457.2	21	182.88	243.84
11	518.16	579.12	Cost	180.0	154.748

Table 4 | Genetic algorithm parameters

Population size	Generation size	Probability of crossover (Pc)	Probability of mutation (Pm)	Probability of recombination (Pr)
80	120	0.7	0.001	0.01

Table 6 | Results of head for New York City network problem using GA

Node	Min. head (m)	Head (m)	Node	Min. head (m)	Head (m)	Node	Min. head (m)	Head (m)
1	91.44	91.44	8	77.724	78.6272	15	77.724	89.5364
2	77.724	91.2177	9	77.724	83.8127	16	79.248	79.8847
3	77.724	87.3746	10	77.724	83.8032	17	83.14944	83.7779
4	77.724	86.7693	11	77.724	84.3596	18	77.724	84.5622
5	77.724	83.8403	12	77.724	84.9640	19	77.724	84.4765
6	77.724	82.3902	13	77.724	85.9453	20	77.724	80.0714
7	77.724	78.6373	14	77.724	87.7455			

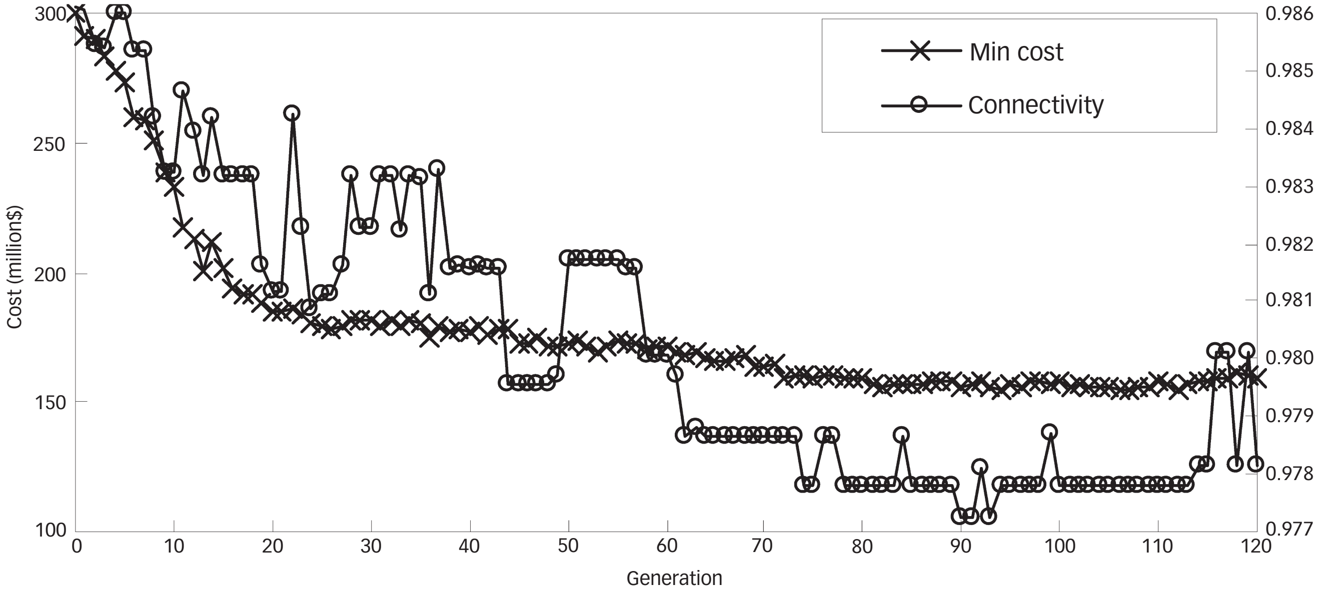

Figure 4 | Optimal cost and connectivity with varying generations using GA

requirements. The obtained residual heads are checked with the given minimum head, H_{min} as follows:

$$H_{min} \leqslant H_i. \tag{6}$$

If this requirement is violated at any node, a penalty is added to the objective function, as shown above, which will make the cost high and the fitness low. That is, the string will have lower probability of survival for the next generation. In this way, those strings that satisfy hydraulic requirements survive and the hydraulic connectivity is established in the model.

Optimization procedure

The optimization model with two modules is driven by the GA: the hydraulic module conducting hydraulic analysis

Table 7 | The optimization results: pipe diameters (cm) and costs (US$ millions)

	Connectivity				
Pipe #	**0.9778**	**0.978**	**0.979**	**0.980**	**0.981**
1	579.12	579.12	609.6	579.12	579.12
2	335.28	365.76	335.28	365.76	365.76
3	365.76	426.72	335.28	426.72	426.72
4	243.84	243.84	335.28	365.76	304.8
5	243.84	304.8	304.8	243.84	274.32
6	182.88	182.88	182.88	182.88	182.88
7	152.4	152.4	182.88	91.44	182.88
8	152.4	182.88	152.4	152.4	152.4
9	152.4	91.44	91.44	182.88	91.44
10	457.2	365.76	396.24	457.2	518.16
11	579.12	579.12	640.08	579.12	609.6
12	579.12	640.08	579.12	518.16	518.16
13	609.6	609.6	579.12	609.6	579.12
14	609.6	609.6	609.6	609.6	609.6
15	579.12	609.6	579.12	579.12	609.6
16	91.44	91.44	121.92	91.44	182.88
17	121.92	121.92	121.92	243.84	121.92
18	91.44	121.92	152.4	121.92	213.36
19	213.36	213.36	243.84	213.36	213.36
20	91.44	91.44	91.44	91.44	91.44
21	243.84	243.84	243.84	243.84	243.84
Cost	154.748	158.765	159.623	163.149	165.249

and the reliability module, calculating connectivity. These modules work in series, as shown in Figure 2. The optimization procedure is as follows:

Table 8 | Pressure heads for critical nodes (m)

		Connectivity				
Node	**Min. head**	**0.9778**	**0.978**	**0.979**	**0.980**	**0.981**
16	79.248	79.8847	79.7373	79.5599	79.3382	79.6617
17	83.14944	83.7779	83.5051	83.2065	83.2356	83.6098
19	77.724	84.4765	85.2678	84.1193	83.8925	83.8833

1. A population with strings that consist of pipe diameters for all links is generated.
2. Hydraulic requirements of each string are checked with the hydraulic module.
3. Hydraulic-connectivity of each string is checked with the reliability module.
4. Fitness of each string is checked.
5. Only those strings with higher fitness survive and go through the genetic procedures.
6. The process repeats to a certain number of generations.
7. The optimal solution is determined among the strings with the highest fitness value of each generation.

MODEL APPLICATION

For application of HYCCODEM, the New York City water supply tunnel system shown in Figure 3 was used and physical data are shown in Tables 1 and 2. This system cannot satisfy the minimum pressure with existing diameters. So, we developed an optimization technique to determine the most economically effective design with the new tunnels to enable the system to meet minimum acceptable pressures. For optimization with GA, each pipe diameter is designed in accordance with Table 3; other GA parameter values are shown in Table 4.

RESULTS

HYCCODEM optimized the New York network, not including connectivity constraints. A minimum cost of

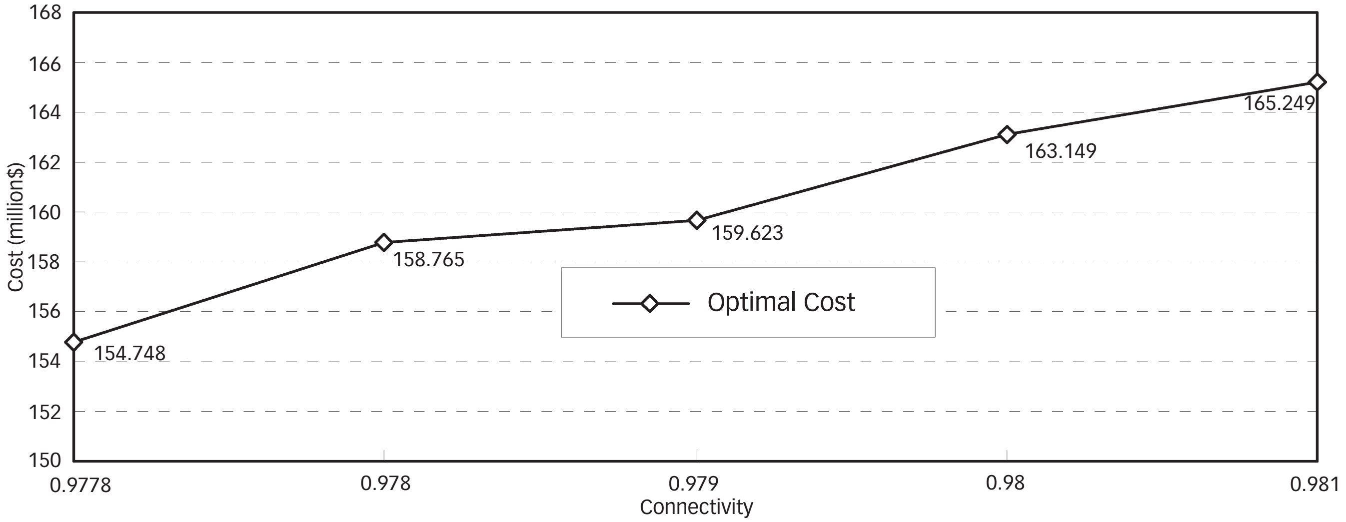

Figure 5 | Optimal cost with varying hydraulic-connectivity.

US$154.748 million was obtained, which is lower than the existing tunnel results (US$180 million) as shown in Table 5. This shows that the optimization with GA can be applied to WDN. The optimization results with HYCCODEM are shown in Tables 5 and 6 and Figure 4. The connectivity was also confirmed at 0.9778. To demonstrate HYCCODEM's capability of handling connectivity, a number of optimizations were conducted with varying connectivities. As shown in Tables 7 and 8 the connectivity increased from 0.9778 to 0.981. The optimization results with those connectivities are shown in Figure 5. As expected, the cost increases as the connectivity increases.

SUMMARY

The conclusions of this study are as follows:

1. Hydraulic-connectivity can be used as a surrogate of reliability against emergency situations. It can differentiate the design of WDN according to a given emergency situation.
2. Since HYCCODEM can provide valuable information regarding trade-offs between cost and connectivity, planners and other decision-makers can make their decisions based on more quantitative results regarding cost and reliability.
3. If any emergency situation can be expressed in the pipe failure probability function with design variables, the approach with HYCCODEM can be applied for finding an optimal design of WDN against the emergency situation.
4. This study result proves again that GA can be a more effective tool for the optimization of WDN since it results in a lower cost solution with required flow at adequate pressure than the existing tunnel.
5. A sensitivity analysis with GA parameters is necessary to further investigate GA application for the optimization of WDN.

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