DISCUSSION

K. A. Blenkarn[4]

As the authors make quite clear, there are other possible bucking modes of drill strings beside the two-dimensional modes studied in this paper. Thinking in terms of the straight but inclined holes, one envisions primarily buckling wherein the drill string remains in contact with the hole. For such modes the string would presumably take the shape of a sine wave wrapped around inside the hole, ultimately approaching a helix. The authors promise to tell us about these three-dimensional modes in a forthcoming paper.

In the present paper, it is demonstrated that lack of straightness promotes buckling into two-dimensional modes. It is perhaps not premature to consider cases in which two-dimensional buckling might play a governing role. The most obvious such case involves the so-called "nose-diving technique" for decreasing hole inclination. In utilization of this technique, the first step is to give the hole a sharp dog-leg back toward vertical. This is done, for instance, by slowly drilling a few feet with exceedingly low weight on the bit. After the initial kick is made, a very limber pipe is placed immediately above the bit. The explanation given for the action of the limber pipe in further decreasing hole inclination is as follows: The dog-leg near the bottom of the hole causes the limber pipe to buckle as described in the present paper. The buckled pipe contacts the "high" side of the hole, and application of greater weight on the bit causes the pipe to push very hard against this side of the hole. The resulting reaction at the bit tends to push the bit into the "low" side of the hole making the hole drill back toward vertical.

The technique just described is at present largely a matter of guesswork. Possibly the ideas presented in this paper could bring some refinements into such procedures. For example, it might be useful to specify just how much of an initial dog-leg is needed to cause the limber pipe to buckle in the desired mode.

D. W. Dareing[5]

The authors present an interesting analysis of a difficult problem. Even though the final solution is approximate, it makes a worthwhile contribution to the knowledge of drill string behavior, and will no doubt be of interest to others working in this area.

The question of accuracy always arises when an approximate method is used, especially when an assumption is made on the deflection function (or buckled configuration). Realizing it is difficult to make a comparison when $0 < \alpha < \pi/2$, since apparently no exact solution has been made for this case, I wonder if the authors compared the deflection function of their two-term approximation with the functions of the already known solutions for the cases when $\alpha = 0$ and $\alpha = \pi/2$. When $C = 1/2$, the two-term approximation appears to be reasonable for drill collar analysis, but the authors might reassure the reader if the above comparisons were made.

It seemed worthwhile, from a practical standpoint, to use the final solution in this paper, equation (17), to determine the "critical angle of inclination," for a given bit weight, for which the drill collars do not buckle but lie along the bottom side of the well bore. If we let L be the length of the drill collars in compression, then

$$L = \frac{W}{\gamma \cos \alpha}$$

where W is the component of the bit weight along the line of inclination and γ and α are the same as given in this paper. For small angles (α) this is essentially the magnitude of the bit weight. The only value of C that is of interest is 1/2 since only this value satisfies the boundary condition of zero slope at $x = l$. The values of γ and a_1^* were determined for different weights, W, and angles of inclination, α, by using the same size drill collars as those of the authors. The maximum transverse displacement (say Δ), located at $x = (1/3)l$, can be shown to be equal to $1.1299a_1^*$. The resulting data are given in Fig. 9.

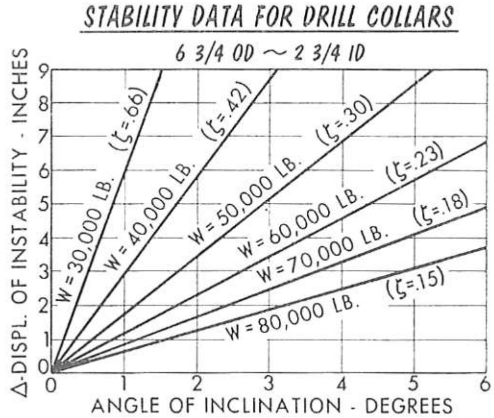

Fig. 9

For the small angles, α, given in Fig. 9, γ was constant for a given W. By determining the permissible movement, Δ, of the collars in the well bore, the critical angle of inclination can be determined according to the authors' theory.

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Authors' Closure

The authors wish to express their thanks to the discussers for their thought and effort in clarifying our work.

The results of our first investigation of the three-dimensional modes mentioned by Dr. Blenkarn, "The Stability of a Circular Rod Laterally Constrained to be in Contact With an Inclined Circular Cylinder," have been accepted for presentation at the Applied Mechanics Summer Conference, June 9–11, 1964, Boulder, Colo.

Dr. Blenkarn's explanation of hole straightening based on the type buckling considered here is believed by the authors to have merit. Although precise predictions of the effectiveness of such a procedure are probably too much to hope for, we are currently trying to establish predictions which may serve as guidelines for such a procedure.

The question raised by Dr. Dareing concerning the accuracy of the solution is, of course, pertinent. For the relatively simple case of the equations (90 deg) a beam theory solution for short spans may be deduced from an equilibrium approach based on the following equations

$$EIf''''(x) + Pf''(x) + \rho g = 0$$

$$f\left(\frac{L}{2}\right) = 0, \quad f\left(-\frac{L}{2}\right) = 0, \quad f''\left(\frac{L}{2}\right) = 0, \quad f''\left(-\frac{L}{2}\right) = 0$$

to obtain

$$\frac{f(x)}{L} = \frac{1}{2}\frac{\rho g L}{P}\left(\frac{1}{4} - \frac{x^2}{L^2}\right) + \frac{\rho g L}{P}\frac{EI}{PL^2}\left(1 - \frac{\cos\left(\sqrt{\frac{PL^2}{EI}}\cdot\frac{x}{L}\right)}{\cos\left(\sqrt{\frac{PL^2}{EI}}\cdot\frac{1}{2}\right)}\right)$$

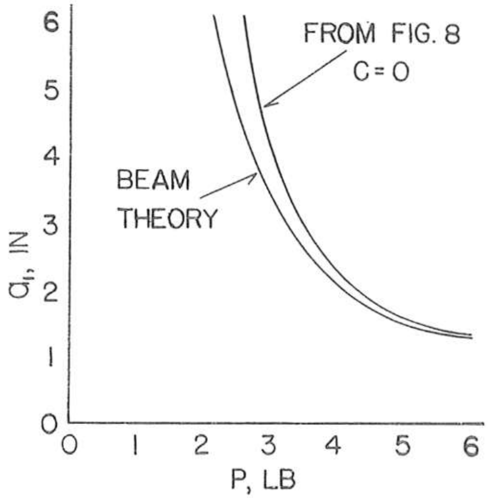

Fig. 10 Comparison of result given in the paper with result from beam theory equilibrium solution

At x equals zero $f(x)$ equals α_1 for this case so that

$$\frac{\alpha_1}{L} = \frac{1}{8}\frac{\rho g L}{P} + \frac{\rho g L}{P}\frac{EI}{PL^2}\left(1 - \frac{1}{\cos\left(\sqrt{\frac{PL^2}{EI}}\cdot\frac{1}{2}\right)}\right)$$

A comparison of the relationship between P and α_1 based on this result and the one given in Fig. 8 of the paper is of interest. This comparison, shown in Fig. 10, indicates that the results are, for engineering usage, in good agreement. Since the shape of the critical configuration as obtained by the equilibrium method depends on the load P, it is not so simple to compare shapes from each procedure; however, it is generally accepted that when an energy method approach is used the shape need only be approximately correct in order to obtain a fairly accurate prediction of the critical amplitude.

Fig. 9, from Dr. Dareing's discussion, is an interesting and informative means of presenting the results of the analysis.