Note on the Low Energy Electron-Hydrogen Scattering

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In our previous paper, the low energy electron-hydrogen scattering was discussed on the effective range approximation. In this note, we shall examine the question of energy range of the incident electron to which this method can be applied. An error in the normalization of the ground state wave function of the negative hydrogen ion is also corrected here.

The effective range and the scattering length for the singlet (symmetric) state

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are given by the formulae (12) and (15) of ref. 1). Namely, the effective range is calculated by

\[ \rho = \frac{1}{\gamma} - \frac{1}{4\pi} \int_0^\infty \mathcal{F} \, dr_1 \, dr_2 \]  

(1)

where \( \phi(r_1, r_2) \) is the wave function of H\(^-\) ion, normalized as

\[ \psi(r_1, r_2) = \phi_g(r_1) \exp(-\gamma r_2) / r_2 \text{ for } r_2 \to \infty \]
\[ - \phi_g(r_2) \exp(-\gamma r_1) / r_1 \text{ for } r_1 \to \infty \]

and \( \phi_g(r) = e^{-r}/\sqrt{\pi} \) is that of the hydrogen atom in the ground state, \( \gamma^2/2 \) being the affinity of H\(^-\) ion. The phase shift \( \delta_+ \) for the singlet \( s \) wave scattering is discussed by the expansion,

\[ k \cot \delta_+ = -\gamma + (\rho/2) (\gamma^2 + k^2) \]

\[ + O((\gamma^2 + k^2)^2), \]

and the scattering length \( a_+ \) is related to \( \gamma \) and \( \rho \) through the relation,

\[ -1/a_+ = (k \cot \delta_+)_{\kappa=0} = -\gamma + \rho \gamma^2/2. \]  

(3)

Now, in ref. 1) we used the function of H\(^-\) calculated by Hart and Herzberg. The definition of the normalization of these authors is the same as that given by the formula (9) of Chandrasekhar and Herzberg, in which, however, the factor 2 is unnecessary, so that our previous value for \( (1/4\pi) \int \phi^2 \, dr_1 \, dr_2 \) is to be doubled. With this correction and \( \gamma^2/2 = 0.02764 \) \( (\gamma = 0.2351) \) obtained by Hart and Herzberg, we find

\[ \rho = 2.44 \sim 2.60 \]

and \( a_+ = 6.04 \) for \( \rho = 2.52 \), if the effective range approximation is a good one. To investigate the range of energy for which this is the case, however, the 3rd term in \( k \cot \delta \) must be estimated. This is not an easy task. Here, we shall compare the results obtained by the variational method with the prediction by the effective range theory. In Fig. 1, \( k \cot \delta \) from the \( s \) wave phase shifts adopted by Bransden et al. are plotted. The good agreement would indicate the validity of the effective range approximation to fairly high energy \( (k^2 \leq 0.75, \text{ elastic region}). \)

1) T. Ohmura, Y. Hara and T. Yamanouchi, Prog. Theor. Phys. 20 (1958), 82
2) J. F. Hart and G. Herzberg, Phys. Rev. 106 (1957), 79
3) S. Chandrasekhar and G. Herzberg, Phys. Rev. 98 (1955), 1060
4) private information from Dr. G. Herzberg
5) The definition of the normalization (3·2) of Kinoshita\(^6\) is consistent with (9) of
ref. 3), so that a factor 1/2 must be added to (3·2), or the numerical values of L, M, N of the Table 1 of ref. 6) must be doubled. This was confirmed by the correspondence with Prof. Kinoshita. (We were also informed that the doubling was not necessary in the case of 10 parameters.)

6) T. Kinoshita, Phys. Rev. 105 (1957), 1490