The $\Sigma$-$\Lambda$ Relative Parity and the $K$-$N$ Reaction

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Possible resonances in pion-hyperon scattering at low energies are qualitatively examined and some noticeable differences are found between two cases in which the $\Sigma$-$\Lambda$ relative parity is even and odd. Then, supposing that, in the process $K+N \rightarrow \pi + Y$, the strong resonances of pion-hyperon scattering in the final state will dominate over the relatively weak $K$-meson interaction, we discuss the possibility to conjecture the relative parities of $K$-meson and baryons from the experiment of this process.

§ 1. Introduction

As was discussed by Gell-Mann\(^1\) the coupling of $K$-meson to baryons seems to be somewhat weaker than that of pion. Hence in the process $K + N \rightarrow \pi + Y^*$, the pion-hyperon scattering is expected to play an important role as a final state interaction. If the pion-baryon interaction has universal form and strength, strong resonances similar to the $(3/2, 3/2)$-pion-nucleon scattering will occur in the final pion-hyperon scattering\(^2\) and enhance relevant amplitudes of the $K$-absorption. It was further noticed that the condition of universal strength should be somewhat relaxed. Namely, as far as $2 \gtrsim (f_\Sigma/f_\Lambda)^2 \gtrsim 1/2$, the qualitative character of resonances will not be radically altered\(^3\), $f_\Sigma$ and $f_\Lambda$ being respectively the renormalized $(\Sigma\Sigma\pi)$- and $(\Sigma\Lambda\pi)$-coupling constants. In this paper\(^4\) we mean by universal pion-baryon interaction that the $\Sigma$-$\Lambda$ relative parity $P_{\Sigma\Lambda}$ is even and that $f_\Sigma^2$ and $f_\Lambda^2$ are of the same order of magnitude with $f_\Sigma^2 \approx 0.08$.

The possibility of experimental test of the universal pion-baryon interaction by the $K$-absorption has been discussed by several authors\(^5\). It should be noticed, however, that in order to know what are characteristic of the universal pion-baryon interaction or of even $P_{\Sigma\Lambda}$, consequences of other types of interaction must be examined at the same time. Indeed, strong resonances of the pion-hyperon scattering are, as we shall see, not necessarily characteristic of the universal pion-baryon interaction but will occur, in some states, also by another type of pion-hyperon in-

\(^{*}\) Here and henceforth $Y$ and the word hyperon will mean either $\Sigma$- or $\Lambda$-hyperon. This process will simply be called $K$-absorption.

\(^{**}\) The spins of hyperons are all assumed to be $1/2$. It is, of course, assumed that the isospin, strangeness and parity are conserved in the strong interactions and that the members of a charge multiplet have same parity relative to others.
teraction ($P_{\Sigma A}$ is odd). Therefore, some processes that have been considered to serve as experimental tests of the universal pion-baryon interaction will turn out to be useless.

In Sec. 2 we shall examine the pion-hyperon scattering at low energies in the case of odd $P_{\Sigma A}$. It will be pointed out that in some states resonance will be found commonly in both cases of even and odd $P_{\Sigma A}$, while in some states resonance will occur only in one of the two cases. Then implications of these resonances in the $K$-absorption will be discussed in Sec. 3 in relation to the possibility to determine the relative parities of $K$-meson and baryons. In Sec. 4 some comments will be given on previous investigations on this topics.

§ 2. Pion-hyperon scattering

If the $K$-meson interaction is sufficiently weak the amplitudes of the $K$-absorption can be approximately expressed by the pion-hyperon scattering amplitudes. The $S$-wave pion-interaction will, however, be of the same order of strength with that of $K$-meson, and perhaps cannot be clearly disentangled in the $K$-absorption. Whereas strong $P$-wave resonances of the pion-hyperon scattering, if exist, will be confirmable by marked enhancement of relevant amplitudes of the $K$-absorption.

We do not intend to make detailed analysis but confine our attention only to strong resonances of the $P$-wave scattering at low energies. Throughout this section $P_{\Sigma A}$ is taken to be odd, unless otherwise mentioned. We use the static approximation\(^6\), reducing the pseudoscalar ($\Sigma\Sigma\pi$)-coupling to the familiar $P$-wave vertex and the scalar ($\Sigma A\pi$)-coupling to the $S$-wave vertex. The $\Sigma\cdot A$ mass difference and the virtual effect of $K$-meson will be ignored.

There are three types of scattering,

\[
\begin{align*}
(\Sigma' \rightarrow \Sigma) & : \pi + \Sigma \rightarrow \pi + \Sigma, \\
(\Sigma' \rightarrow A) & : \pi + \Sigma \rightarrow \pi + A, \\
(A \rightarrow A) & : \pi + A \rightarrow \pi + A.
\end{align*}
\]

In $(A \rightarrow A)$, since $P$-wave is not scattered in the Born approximation, the $P$-wave resonance will not occur. On the contrary, strong resonance will occur in $(A \rightarrow A)$ if $P_{\Sigma A}$ is even\(^*\). This is the first difference to be noticed. One may conceive here that, if the $(\Sigma A\pi)$-coupling constant is considerably large, a resonance may take place in $S$-wave scattering in $(A \rightarrow A)$. It will not be the case, however. It is shown that $S$-wave scattering in $(A \rightarrow A)$ does not occur in the Born approximation,

\[^*\] Remember that if the pion-baryon interaction is strictly universal the pion-hyperon scattering amplitudes are related to those of pion-nucleon scattering, $T_{1/2}$ and $T_{3/2}$, as follows,

\[
\begin{align*}
T(\Sigma \rightarrow \Sigma) & = T_{1/2}, & T(\Sigma \rightarrow \Sigma) & = (2T_{1/2} + T_{3/2})/3, \\
T(\Sigma \rightarrow \Sigma) & = T_{3/2} & T(A \rightarrow A) & = (T_{1/2} + 2T_{3/2})/3,
\end{align*}
\]

where the subscripts indicate the isospin states.
if the $\Sigma$-$A$ mass difference is ignored. Though if the mass difference is taken into account the matrix element of the scattering is nonvanishing, its sign indicates that the scattering potential is repulsive.

In $(\Sigma \leftrightarrow A)$, pion’s angular momentum changes as $S \leftrightarrow P$; repeating of $(\Sigma \leftrightarrow A)$ contributes to $(\Sigma \leftrightarrow \Sigma)$. We write the $T$-matrix element of $(\Sigma \rightarrow \Sigma)$ with $P$-wave and that of $(\Sigma \rightarrow A)$ with $(P \rightarrow S)$ wave, respectively, as follows.

$$(\Sigma \rightarrow \Sigma) : -v_{\pi} \mathbf{p}_{\pi} \cdot 2\pi (\omega_{\pi} \mathbf{p}_{\pi})^{-1/2} \mathcal{P}_{J} \mathcal{h}'(\omega_{\pi}),$$

$$(\Sigma \rightarrow A) : -v_{\pi} \mathbf{p}_{\pi} \cdot 2\pi (\omega_{\pi} \mathbf{p}_{\pi})^{-1/2} (\mathbf{\sigma} \cdot \mathbf{q}) \mathcal{h}'(\omega_{\pi}).$$

Here $q$ and $p$ are respectively the initial and the final momenta of pion, $v_{\pi}$ is a cut-off factor, $\omega_{\pi} = (1 + p^{2})^{1/2}$, the unit $c = \hbar = m_{\pi} = 1$ is used. $P_{J}$ is a projection operator for the $\pi + \Sigma$ state of $I = 0, 1$ or $2$. $P_{J=1/2} = (\mathbf{\sigma} \cdot \mathbf{p}),(\mathbf{\sigma} \cdot \mathbf{q})$, $P_{J=3/2} = 3pq - (\mathbf{\sigma} \cdot \mathbf{p})(\mathbf{\sigma} \cdot \mathbf{q})$. $\mathbf{\sigma}$ is the spin vector of hyperon. The $(\Sigma \rightarrow A)$ belongs to $(I, J) = (1, 1/2)$. The Low equation for $h_{IJ}$ is, in the one meson approximation,

$$h_{IJ}(\omega) = \frac{\lambda_{IJ}}{\omega} + \frac{1}{\pi} \int d\omega' \mathbf{p}_{\pi}^{2} \left[ \frac{\mathbf{p}^{2} h_{IJ}^{2}(\omega_{\pi})^{2} + \delta_{IJ,12} \delta_{\sigma,12} |\mathcal{h}'(\omega_{\pi})|^{2}}{\omega_{\pi} - \omega - i\epsilon} \right] + \sum_{I', J'} C_{I'J'} \mathbf{p}^{2} |h_{I'J'}(\omega_{\pi})|^{2} + \delta_{IJ} \delta_{\sigma,12} |\mathcal{h}'(\omega_{\pi})|^{2} \right] \omega_{\pi} + \omega.$$ (2.3)

The coefficient of the Born term is (see Appendix)

$$\lambda_{IJ} = \frac{f_{\Sigma}^{2}}{3} \times \begin{pmatrix} 2 \\ -4 \\ -7 \\ 2 \\ -1 \\ 2 \end{pmatrix} \quad \text{for} \quad (I, J) = \begin{pmatrix} 0, 1/2 \\ 0, 3/2 \\ 1, 1/2 \\ 1, 3/2 \\ 2, 1/2 \\ 2, 3/2 \end{pmatrix}$$ (2.4)

$h'$ in the Born approximation is given by

$$h'(\omega)_{\text{Born}} = \frac{\lambda'}{\omega}, \quad \lambda' = -2\sqrt{2} f_{\Sigma} f_{A}.$$ (2.5)

$f_{\Sigma}$ and $f_{A}$ are the renormalized and unrationialized coupling constants of the $(\Sigma + \pi)$-interaction, respectively. The crossing matrix is

$$C_{I'J',I,J} = C_{1I'} \times C_{J'J},$$

$$C_{I'I} = \frac{1}{6} \begin{pmatrix} 2 & -6 & 10 \\ 2 & 3 & 1 \end{pmatrix}, \quad C_{J'J} = \frac{1}{3} \begin{pmatrix} -1 & 4 \\ -2 & 3 & 5 \end{pmatrix}.$$ (2.6)

In the case of even $P_{\Sigma A}$, resonances occur in the states with $I = 1, 2, J = 3/2$. Whereas in the case of odd $P_{\Sigma A}$ the sign of $\lambda_{IJ}$ of eq. (2.4) suggests that, in addition to these states with $I = 1, 2, J = 3/2$, resonance possibly occurs also when
\(I=0\) and \(J=1/2\). \(\lambda_{1/2}>0\) means that the scattering potential is attractive. For even \(P_{\Sigma A}\), \(\lambda_{0,1/2}=2/3 \cdot (f^2_\Sigma - 5f^2_\Lambda)^3\). Therefore, the sign of \(\lambda_{0,1/2}\) changes with \(P_{\Sigma A}\). Indeed, if \(f^2_\Sigma\) is very small \((5f^2_\Lambda \ll f^2_\Sigma)\) when \(P_{\Sigma A}\) is even, the two cases of even and odd \(P_{\Sigma A}\) will be qualitatively similar to each other, as far as resonance behaviors in the pion-hyperon scattering are concerned. According to Ferrari and Fonda's analysis of the \(A-N\) force, however, the case of \(f^2_\Sigma=0\) is ruled out irrespectively of \(K\)-meson's parity. We think their conclusion on this point is reliable, hence we do not consider the case of very small \(f^2_\Sigma\).

Following Chew and Low, we rewrite eq. (2.3) in the form,

\[
h_a(\omega) = \frac{\lambda_a}{\omega} \left[ 1 - \frac{\omega}{\pi} \int d\omega_p \frac{p \nu p^2}{\omega_p^2} \right]^{-1} \left\{ \frac{X_a(\omega_p)}{\omega_p - \omega - i\epsilon} + \frac{Y_a(\omega_p)}{\omega_p + \omega} \right\},
\]

\(X_a(\omega_p) = p^2 + \delta_a |h'(\omega_p)|^2/|h_a(\omega_p)|^2,\)

\(Y_a(\omega_p) = \sum \limits_{3} \Gamma_{3} |p|^2 h_3(\omega_p)|^2 + \delta_{3} |h'(\omega_p)|^2/|\sum \limits_{3} \Gamma_{3} h_3(\omega_p)|^2,\)

where \(\omega \equiv (I, J)\) and \(\delta_{a} \equiv \delta_{I_{1/2},I_{1/2}}\). An approximate solution will be given by substitution for \(h_a\) and \(h'\) in \(X_a\) and \(Y_a\) by the corresponding Born terms, \(\lambda_a/\omega_p\) and \(\lambda'/\omega_p\). Namely,

\[
h_a(\omega) = \frac{\lambda_a}{\omega} \left[ 1 - \frac{\omega}{\pi} \left\{ \lambda_a F_-(\omega) + \delta_a \frac{\lambda'^2}{\lambda_a} F'_-(\omega) + A_a F_-(\omega) + A'_a F'_-(\omega) \right\} \right]^{-1},
\]

where

\[
A_a = \sum \limits_{3} \Gamma_{3} \frac{\lambda'^2}{\lambda_a} = \frac{f^2_\Sigma}{3} \times \left\{ \begin{array}{ccc}
13 \\
7 \\
8 \\
8 \\
-2 \\
10 \\
\end{array} \right\} = \frac{f^2_\Sigma}{17},
\]

\[
A'_a = \sum \limits_{3} \Gamma_{3} \delta_3 \frac{\lambda'^2}{\lambda_a} = 4f^2_\Sigma \times \left\{ \begin{array}{ccc}
1 \\
1 \\
1 \\
1 \\
\end{array} \right\},
\]

and

\[
F_\pm(\omega) = \int d\omega_p p^3 \nu p^2/\omega_p^2 (\omega_p \pm \omega - i\epsilon),
\]

\[
F'_\pm(\omega) = \int d\omega_p p^3 \nu p^2/\omega_p^2 (\omega_p \pm \omega - i\epsilon).
\]

In the \((I, J) \neq (1, 1/2)\) states, the phase shift \(\delta_a\) is given by \(\text{Re} \ h_a^{-1}(\omega_p) = p^3 \nu p^2 \cot \delta_a(p)\). In the \((I, J) = (1, 1/2)\) state, the phase shift is not real because of the
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coeexistence of two channels, ($\Sigma \to \Sigma$) and ($\Sigma \to \Lambda$).

As for $F_+^\pm (\omega)$ and $F_-^\pm (\omega)$, we simply replace them by their values at $\omega=1$, ignoring their $\omega$-dependence. Using a cut-off energy $\omega_{\text{max}}=6$, we have for $\omega=1$

$$F_+=6.0, \quad F_-^+ = 1.2, \quad \text{Re } F_- = 3.0, \quad \text{Re } F_-^+' = 0.4. \quad (2.12)$$

Then eq. (2.9) becomes

$$(\lambda_2/\omega) \text{Re } h_2^{-1}(\omega) \approx 1 - r_{2\omega} \quad (2.13)$$

where

$$r_{2\omega} \approx f_2^2 + f_2^2 \approx 0.51 \begin{pmatrix} 5.4 \\ -0.3 \\ -1.9 \\ 3.8 \\ -1.3 \\ 4.5 \end{pmatrix} \approx 0.51 \begin{pmatrix} -1.24 \\ 0.51 \\ 0.51 \\ 0.51 \end{pmatrix} \quad (2.14)$$

If $f_2^2 \approx 0.08$ and, for example, $(f_\Lambda/f_\Sigma)^2 \approx 5$ are assumed, we have

$$r_{2\omega} \approx \begin{pmatrix} 0.43+0.21 \\ -0.03+0.21 \\ -0.15-0.50 \\ 0.31+0.21 \\ -0.10+0.21 \\ 0.36+0.21 \end{pmatrix} \approx \begin{pmatrix} 0.64 \\ 0.18 \\ -0.65 \\ 0.52 \\ 0.11 \\ 0.57 \end{pmatrix} \quad (2.15)$$

Here the first numbers show the value when $f_\Lambda^2=0$. The resonance energy is given by $1/r_{2\omega}$. Therefore it may be concluded that, if $f_2^2 \approx f_2^2$, and $(f_\Lambda/f_\Sigma)^2 \approx 5$, resonances will occur at about $\omega \approx 2 \sim 3$ in the states with $(I, J) = (0, 1/2), (1, 3/2)$ and $(2, 3/2)^*$. If $(f_\Lambda/f_\Sigma)^2 \gg 5$, resonances may occur also in other states, but for the present we tentatively assume that $f_\Lambda^2$ is not so large. Significant results of this section are summarized in Table I. Contrasts between two cases of even and odd $P_{\Sigma\Lambda}$ are found only in ($\Sigma \to \Sigma$) of $I=0$ and in ($\Lambda \to \Lambda$).

Table I. $\bigcirc (\times)$ means presence (absence) of resonance.

<table>
<thead>
<tr>
<th>$P_{\Sigma\Lambda}$</th>
<th>$(0, 1/2)$</th>
<th>$(0, 3/2)$</th>
<th>$(\Sigma \to \Sigma)$</th>
<th>$(\Lambda \to \Lambda)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>even</td>
<td>$\times$</td>
<td>$\times$</td>
<td>$\bigcirc$</td>
<td>$\times$</td>
</tr>
<tr>
<td>odd</td>
<td>$\bigcirc$</td>
<td>$\times$</td>
<td>$\times$</td>
<td>$\bigcirc$</td>
</tr>
</tbody>
</table>

* The present approximation applied to the pion-nucleon scattering gives the effective range $r \approx 16 f_\Sigma^2/\pi = 0.41$ to the $(3/2, 3/2)$ state.
§ 3. The \( \bar{K} \)-absorption

First, we present some kinematical considerations. The interaction range \( R \) of the \( K \)-meson-nucleon reactions would not be much greater than the Compton wave length of the \( K \)-meson, \( 1/m_K \). As a result of this we may suppose that a \( K \)-meson with angular momentum greater than

\[
l = R/l
\]

will not play an important role. Here \( l = 1/p_K \) is the de Broglie wave length of the \( K \)-meson with momentum \( p_K \). To illustrate this, in Fig. 1, we plot \( l \) as determined from eq. (3·1) with two choices of \( R = 1/m_K \) and \( 2/m_K \). The same quantity in case of the pion-nucleon system with \( R = 1/m_\pi \) is shown by the dotted line for convenience of comparison. It is seen that the limitation on \( l \) is more stringent for the \( K \)-meson-nucleon case than for the pion-nucleon case. At very low energies, say, below 50 Mev (\( K \)-meson’s kinetic energy in the laboratory system), an \( S \)-wave \( K \)-meson will dominantly interacts with a nucleon.

Now we discuss the effects of resonances found in the preceding section on the \( \bar{K} \)-absorption, in relation to possible variety of parities of the \( K \)-meson and the hyperons. Since the \( Q \)-values for the \( \Sigma \) - and \( \Lambda \)-production are respectively about 100 Mev and 180 Mev, the resonances may be placed at very low energies near the threshold. We assign parity + for \( N \) and \( \Lambda \). Then four possibilities are \( \Sigma^+, K^+, \Sigma^0, K^- \), \( \Sigma^-, K^0 \), and \( \Sigma^+ K^0 \), where the subscript indicates parity relative to \( N - \Lambda \).

If the initial state is \( S_{1/2} \), resonance can be involved only in \( \Sigma^- K^- \). In the \( \Sigma^- K^- \) case the amplitude of the \( \Sigma \)-production related to the resonance \((I=0, P_{1/2})\) will be predominant, hence the branching ratio will be \( \Sigma^+ : \Sigma^0 : \Sigma^- \approx 1 : 1 : 1 \). The superscript indicates the charge. While the \( \Lambda \)-production will be relatively scarce because the final state of the \( \Lambda \)-production is \( S_{1/2} \) and is not concerned with resonance. If this is the case, the production ratio of \( \Sigma \) and \( \Lambda \) will rather sensitively depend on energy. The enhancement of the \( \Sigma \)-production alone is especially favorable for the interpretation of the experimental result which shows infrequent \( \Lambda \)-production.

Namely, Alvaz et al.\(^9\) reported that, in flight plus a rest events yield \( \Sigma^+ : \Sigma^0 : \Sigma^- : \Lambda \)
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$=4:4:8:1$. Eisenberg et al.'s analysis$^7$ of events in flight showed the rise of the $A$-production with increasing energy. In cases other than $\Sigma_-$,$K_-$, this $\Sigma$-$A$ ratio could not be understood without going into detailed mechanism of pion and $K$-meson interactions.

If the incident $K$-meson is $P$-wave, $P_{1/2}$ resonance will occur not only in the case of $\Sigma_-$,$K_-$ but also in the case of $\Sigma_+K_+$, and that $I=1$ in both cases$. If we observe only the $\Sigma$-production, these two cases could not be distinguished. There will, however, be remarkable difference in the $\Sigma$-$A$ ratio between two cases. As has been discussed by Ross$^8$ the $I=1$, $P_{3/2}$ resonance in the $\Sigma_+K_-$ case will result $\Sigma: A \approx 1:2$ (more precisely $\Sigma^+: \Sigma^0: \Sigma^-: A \approx 1:0:1:4$) while in the $\Sigma_+K_+$ case $\Sigma$ will dominate over $A$, because only the $\Sigma$-production will be enhanced by the final state resonance. In case of $\Sigma_-$,$K_-$, one more resonance is involved ($I=0$, $P_{1/2}$). If we could obtain information on the pure $I=0$ state ($K^- + p \rightarrow \pi^0 + \Sigma^0$), the $P_{1/2}$ resonance could be a clue to distinguish the $\Sigma_-$,$K_-$ case from the $\Sigma_+K_+$ case. The above results are summarized in Table II.

Table II. $\Sigma(1,3/2)$, for example, means that resonance is involved in the $\Sigma$-production of $(I,J)=(1,3/2)$. The $\times$ means absence of resonance.

<table>
<thead>
<tr>
<th>initial state</th>
<th>case</th>
<th>$\Sigma_+K_+$</th>
<th>$\Sigma_+K_-$</th>
<th>$\Sigma_+K_+$</th>
<th>$\Sigma_+K_-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$</td>
<td>$\times$</td>
<td>$\times$</td>
<td>$\times$</td>
<td>$\Sigma(0, 1/2)$</td>
<td></td>
</tr>
<tr>
<td>$P$</td>
<td>$\times$</td>
<td>$\Sigma(1, 3/2)$</td>
<td>$\Sigma(0, 1/2)$</td>
<td>$\Sigma(1, 3/2)$</td>
<td>$\times$</td>
</tr>
</tbody>
</table>

In actual phenomena, the initial state will contain both of $S$- and $P$-waves, so the analysis will be rather involved. At sufficiently low energies, however, the $S$-wave will be dominant in the initial state, and the $P_{1/2}$ resonance in the final state will provide useful clues to determine the parities of the $K$-meson and the hyperons.

§ 4. Discussion

Though the $\bar{K}$-absorption will be an efficient tool to determine the relative parities of the $K$-meson and the baryons, this alone will not suffice to be decisive. If $P_{\Sigma \Lambda}$ is once determined, the dispersion relations applied to the $K$-$N$ and $\bar{K}$-$N$ scatterings will be effective for the determination of the $K$-meson's parity relative to the baryons$^9$. As for $P_{\Sigma \Lambda}$ itself, however, the dispersion relations seem to offer rather poor information$^9$. Therefore, it is desirable to find clues on $P_{\Sigma \Lambda}$ in other

* These cases could be distinguished from the above mentioned $I=0$, $P_{1/2}$ resonance in $\Sigma_-$,$K_-$ case by that (i) in $I=0$ state the production ratio $\Sigma^+: \Sigma^0: \Sigma^-$ is $1:1:1$, while it is $1:0:1$ in the $I=1$ state and (ii) the angular distribution in $P_{3/2}$ state contain $\cos^2 \theta$ term while that in $P_{1/2}$ state does not.
processes and/or methods.

Here, confining ourselves to processes wherein the resonances of pion-hyperon scattering at low energies will play important roles, we give some comments on previous investigations. It has been suggested by Ito, Minami and Tanaka\textsuperscript{10} that the processes

\begin{align}
(\text{a}) & \ K^- + D \rightarrow \pi^- + p + A, \\
(\text{b}) & \ \pi^+ + p \rightarrow \pi^+ + K^0 + \Sigma^+, \\
(\text{c}) & \ \pi^+ + p \rightarrow \pi^+ + K^+ + A,
\end{align}

will serve to the test of the universal pion-baryon interaction or of the $P_{\Sigma A}$. Their discussion on (a) and (c) holds good because the $\pi^++A$ system involves resonance if $P^A$ is even and does not if $P^A$ is odd. While the process (b) will be useless because the $\pi^++\Sigma^+$ system ($I=2$) involves resonance commonly in both cases of even and odd $P_{\Sigma A}$.

Ross\textsuperscript{3} suggested that the $\pi^+ + A$ resonance will be found in the processes

\begin{align}
\Sigma^- + p \rightarrow \begin{cases} 
\pi^- + p + A, \\
\pi^0 + n + A.
\end{cases}
\end{align}

These will be effective as tests by the same reason with the above mentioned processes (a) and (c).

The author wishes to thank Drs. A. Komatsuzawa and R. Sugano for helpful discussions, and Prof. Z. Shirogane for his encouragement.

Appendix

In this appendix we obtain the coefficient $\lambda_\alpha$, eq. (2.4). In order to proceed in parallel with the case of pion-nucleon scattering, it is convenient to introduce isospin operator $\rho_\alpha$ ($\alpha = 1, 2, 3$) for $\Sigma$-hyperon and write the Hamiltonian of the $(\Sigma \pi \Sigma)$-interaction in the form $\sum_{\alpha=1,2,3} N_{\alpha} N_{\pi_\alpha}$ analogous to the $(NN\pi)$-interaction, $\sum_{\alpha=1,2,3} N_{\alpha} N_{\pi_\alpha}$. Then $\rho_\alpha$ satisfies

$$\rho_1 \rho_2 - \rho_2 \rho_1 = i \rho_3, \quad \text{(cyclic)} \quad \rho_3 = \sum_{\alpha=1,2,3} \rho_\alpha^2 = 1(1+1) = 2.$$  \hspace{1cm} (A.1)

An explicit representation is

$$\rho_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \rho_2 = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}, \quad \rho_3 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$  \hspace{1cm} (A.2)

with $\Sigma = (\Sigma_\alpha, \Sigma_\beta, \Sigma_\gamma)$ and $\pi = (\pi_\alpha, \pi_\beta, \pi_\gamma)$.

The method used by Wick\textsuperscript{11} in pion-nucleon scattering can be applied to this case. Let pion's isospin operator, which has properties similar to $\rho$, be $t$ and the total isospin operator of the $\pi + \Sigma$ system be $I$. Then
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\[ I^2 = (t + \rho)^2 = t^2 + \rho^2 + 2t\rho. \]  
(A·3)

Remembering that $I^2 = I(I+1)$ and $t^2 = \rho^2 = 2$, we see

\[ t\rho = I(I+1)/2 - 2. \]  
(A·4)

Next, introduce matrices $Q$ and $Q'$ with elements

\[ Q_{\lambda\lambda} = \rho_{\lambda}\rho_{\lambda} \quad \text{and} \quad Q'_{\lambda\lambda} = \rho_{\lambda}\rho_{\lambda}, \]  
(A·5)

which are analogues of Wick's $Q$ and $Q'$. When the index $\lambda$ ($\mu$) indicates the isospin component of the initial (final) pion, $Q$ and $Q'$ correspond respectively to the uncrossed and the crossed types of scattering diagrams in the Born approximation. The eigenvalues of $Q$ and $Q'$ are obtained by the relations

\[ Q = (1 - t\rho)(2 + t\rho) \quad \text{and} \quad Q' = 2 - (t\rho)^2. \]  
(A·6)

$\lambda_\alpha$ is obtained by using the result shown in Table A instead of Table II in Wick's paper and replacing $f_N$ by $f_2$.

The projection operator $P_i$ in eq. (2·2) can be expressed in terms of $Q$ and $Q'$ as follows.

\[ P_0 = (1 - Q')/3, \quad P_1 = Q'/2, \quad P_2 = (2 - 3Q/2 + Q')/3. \]  
(A·7)

The crossing matrix $C$ in eq. (2·5) is also obtained by a fashion similar to that explained in Sec. 6 of Wick's paper.

References

8) See, e. g., D. Amati and B. Vitale, Lectures on strong interactions of strange particles (1958), (unpublished). I thank these authors for sending the print of this lecture.
11) G. C. Wick, Rev. Mod. Phys. 27 (1955), 339, Sec. 4.