Plastic Biaxial Stress-Strain Relations for Alcoa 24S-T Subjected to Variable-Stress Ratios[1]

D. C. Drucker.[2] As the authors state, the data available on stress-strain relations under combined stress are meager indeed. Therefore it is important to analyze all new results carefully and obtain as much basic information of theoretical and of practical value as possible. Also, interpretation in terms of preconceived notions should not be permitted to obscure the real meaning of the data.

A point of great interest at present is how close an approximation simple deformation theory provides in practical problems. Deformation theory, in general, postulates that the existing state of stress determines the state of strain. The simplest form is based upon octahedral shearing stress τ_0

$$\epsilon_1 = \frac{1}{C(\tau_0)}\left[\sigma_1 - \nu'(\sigma_2 + \sigma_3)\right]$$

$$\epsilon_2 = \frac{1}{C(\tau_0)}\left[\sigma_2 - \nu'(\sigma_3 + \sigma_1)\right]$$

where the usual notation is followed, and $C(\tau_0)$ is a plastic Young's modulus, and ν' a plastic Poisson's ratio. In particular, therefore, the theory supposes that if σ_1 and σ_2 are given, and σ_3 is zero, ϵ_1 and ϵ_2 are determined regardless of the path of loading.

What do the authors' data prove? Their Figs. 5 and 11 are especially significant. Fig. 5 shows that when $\sigma_1/\sigma_2 = 1$ at all times $\epsilon_1 = 0.011 = \epsilon_2$ (approx.) at $\sigma_1 = \sigma_2 = 51{,}000$ psi. On the other hand, Fig. 11, which has a path of 5000-psi internal pressure followed by axial tension at constant pressure, gives $\epsilon_1 = 0.007$, and $\epsilon_2 = 0.016$ at $\sigma_1 = \sigma_2 = 51{,}000$. As the elastic-strain components are over 0.003, it is clear that the plastic-strain components of 0.004 and 0.013 are predicted very badly at this value of stress. Instead of being equal as deformation theory postulates, they are in the ratio of more than 3 to 1.

As another example, take the loading path of 3000 psi internal pressure followed by tension. Fig. 11 gives $\epsilon_1 = +0.0025$, $\epsilon_2 = -0.002$ at $\sigma_1 = \sigma_2 = 32{,}000$ psi, which means plastic strains of about 0.0005 and -0.004, respectively, which are again far from equal.

Obviously, the authors' statement that deformation theory is adequate does not hold for small but appreciable plastic strains when the path of loading changes abruptly. Clearly, it will not be adequate either for large deformations if the loading path is frequently varied appreciably. It is, however, a sufficiently good engineering approximation when the major part of the plastic deformation occurs over the portion of the loading path which is fairly close to a path with constant value of σ_1/σ_2.

In conclusion, it should be pointed out that if octahedral shearing stress or any isotropic function of shearing stress is the governing criterion,[3] the proper significant strain variable to plot against is proportional to $\int \sqrt{(d\epsilon_1^p)^2 + (d\epsilon_2^p)^2 + (d\epsilon_3^p)^2}$ where the $d\epsilon^p$ are the principal plastic-strain increments. Again, if the major part of the plastic deformation occurs when the loading path is not too far from $\sigma_1/\sigma_2 = $ const, the integral is closely a constant times the octahedral shearing strain.

[1] By Joseph Marin and B. J. Kotalik, published in the December, 1950, issue of the Journal of Applied Mechanics, Trans. ASME, vol. 72, pp. 372–376.

[2] Professor of Engineering, Brown University, Providence, R. I. Mem. ASME.

[3] "Effective Stress and Effective Strain in Relation to Stress Theories of Plasticity," by G. N. White, Jr., and D. C. Drucker, *Journal of Applied Physics*, vol. 21, 1950, pp. 1013–1021.

Authors' Closure

The authors' statement that the deformation theory is adequate was intended to apply for the specific material tested, for large plastic strains for the stress condition used. They agree in general with the statements made by Mr. Drucker regarding the limitations of the deformation theory.

The Uniform Distribution of a Fluid Flowing Through a Perforated Pipe[1]

J. D. Keller.[2] This paper presents a worth-while extension of the theory of pipe burners, although the taking into account of the variation of the pipe friction coefficient with velocity which forms its chief claim to novelty has not the great importance seemingly attributed to it by the author (somewhat reminiscent of Maupertuis and the polar flattening of the earth).

The author wrongly assumes that previous investigators did not recognize this variation. Actually, it was well known but was judged to be of only minor importance; in the writer's opinion, the author has failed to show that this judgment was incorrect for the great majority of commercially usable pipe burners, although beyond question it was so in some special cases. Furthermore, except in the laminar-flow ranges, it is by no means certain that in pipe burners the friction coefficient decreases appreciably with increasing Reynolds number, as required by the author's Equation [8], since it is well known that in very rough pipes the coefficient remains almost constant; see, for example, the curves representing the experimental results of Nikuradse and of Fromm.[3] It seems very probable that the disturbance of flow produced by the diversion of fluid into the successive outlet ports along the pipe length is equivalent in effect to great roughness of the pipe wall.

Again, the author's assumption that the flow changes abruptly from turbulent to laminar at some point in the pipe length is directly contrary to experience. Long ago, Osborne Reynolds showed conclusively that turbulence once started tends to persist, and that a quieting section of considerable length is required to cause it to die out. This is also shown by the difference between the higher and lower critical velocities.

Of the four photographs cited by the author as proving his point, Figs. 3(*b*) and 6(*c*), which according to the author's Table 1 were obtained with burners having a ratio of total port area to pipe cross-sectional area greater than 1, evidently correspond to operation in the laminar-flow range. Figs. 5(*a* and *b*) were obtained with a burner having an area ratio less than 1 (actually, about 0.72 according to Table 1) and therefore are not pertinent to the argument; in these, again, the flow evidently was in the laminar and not in the turbulent range. Hence any conclusions drawn from these four photographs could apply only in the laminar-flow range.

In the writer's opinion, where fairly uniform gas distribution and flame heights are obtained along the length of pipe burners having port area/pipe area ratios considerably in excess of 1, this result is not due (or at least, not chiefly due) to variation of the friction coefficient, but to a factor hitherto overlooked by investigators, including Keller as well as Dow, namely, the increase of pressure below the flame, due to the expansion of volume caused by combustion and the acceleration of the gas par-

[1] By W. M. Dow, published in the December, 1950, issue of the Journal of Applied Mechanics, Trans. ASME, vol. 72, pp. 431–438.

[2] Associated Engineer, Pittsburgh, Pa. Mem. ASME.

[3] "Industrial Furnaces," by W. Trinks, third edition, John Wiley & Sons, Inc., New York, N. Y., 1934, Fig. 310.