as soon as shock-wave behavior begins.[5] All these factors undoubtedly help to cause differences between observed and predicted behavior.

It must also be emphasized that the original paper was concerned only with the steady-state behavior during impact. As has been pointed out elsewhere,[6] this analysis does not describe the initial stages of the impact process (before the steady state is attained). Moreover, as was shown by the authors in connection with tension impact,[7] it is very dangerous to assume that a specimen is instantaneously frozen at the instant the impact ends; a good many things can happen to a specimen after the impact force disappears. Correct comparison between theory and experiment requires either the making of instantaneous measurements during the impact, or the extension of the theory to predict the final state after impact. The latter has been done for low impact velocities but not for high.

Professor Clark mentions the effect of the l/d ratio of compression specimens, pointing out that the stress-strain relation found experimentally will depend on this ratio. In their analysis, the authors were concerned with the behavior of very long specimens, for which l/d would be infinite. The authors' Fig. 1 (stress-strain curve for copper in compression) was obtained by calculation from a tensile stress-strain diagram on the assumptions that the volume of material remains constant during plastic flow, and that strain-hardening depends on the natural strain ($\log_e 1 + \epsilon$). The curve obtained is not considered to be particularly exact, but was adequate for the authors' purpose, namely, illustration.

The authors are grateful for the remarks of Dr. E. H. Lee who was responsible for a large part of the important contributions to the problem of propagation of plasticity that were made in England during the war. The authors hope that the English work will soon be published.

Gyroscopic Effects on the Critical Speeds of Flexible Rotors[1]

A. M. G. Moody.[2] While this paper is not of much direct value to the designer, its indirect value should not be overlooked. The effect of so-called gyroscopic stiffening can be of controlling importance. Unfortunately, it is difficult to analyze. The author, by using a number of ideal assumptions, is able to carry out some analysis, but to apply this to an actual machine is quite difficult.

Take, for example, the simple case of a single overhung wheel, such as a supercharger impeller, having a spline mounting on the shaft. When the wheel is running at full speed, the bore is larger than the shaft, and we can no longer assume that the plane of the wheel is normal to the axis of the shaft. We cannot, however, assume that it is completely unrestricted, since the increase in diameter of the bore may be slight at one point on the axis and larger elsewhere. The axial variation of stress along the hub of a rotating centrifugal impeller is something which cannot ordinarily be determined analytically to any reasonable degree of precision.

Thus, as just mentioned, the author's work is of little direct value in this case. But it has considerable indirect value. It permits the determination of a limiting value of critical speed. If, by making one set of assumptions, we can find a higher limit for critical speed, and by making another set, a lower limit, we shall be considerably better off than if we know only one of these.

Referring to the simple case just discussed, a test made by Mr. A. W. McClure and the writer should be of interest. A calculation of critical speed, assuming full gyroscopic effect, i.e., wheel rigidly mounted on shaft, gave a value some 40 per cent higher than that obtained by neglecting gyroscopic stiffening. A test showed a 25 per cent increase. This indicates that the departure between the ideal analysis and the actual situation is considerable, but it also indicates that a knowledge of the upper limit is of real value.

Stephen H. Crandall.[3] The detailed solutions for the critical frequencies which the author has given for the several special cases are interesting and will undoubtedly be useful. It should be noted that this same class of problems has recently been discussed by J. L. Bogdonoff[4] who also solves the cantilever disk problem in detail and gives a general method which is equivalent to that employed by Green, namely, the use of the auxiliary diagrams, e.g., Figs. 2, 5, and 7 to construct the critical speed diagrams, e.g., Fig. 4, 6, and 8.

It would have been interesting to investigate the mode shapes for the more complicated systems. The nomenclature of Fig. 3 is not clear. The words positive and negative seem to refer to the direction of the resultant moment on the disk rather than to the direction of precession. The highest and lowest curves of Fig. 4 have the mode shape (disposition of δ and φ) as sketched in Fig. 3(c) while the inner two curves have the mode shape sketched in Figs. 3(a) and 3(b).

Author's Closure

A. M. G. Moody adds considerably to the design value of the paper by practically evaluating the assumption of rigid disk mounting. The numerical values are of particular interest.

Bogdanoff[4] presented a very general mathematical analysis of this class of problems and outlined a method of solution. He carried through one solution, namely, for the simple cantilever system, and presented that solution directly on dimensionless coordinates proportional to those of Fig. 4 of the paper, plotting a family of curves for varying D_1. His general solution was thus given there instead of on a diagram such as Fig. 2, which he did not use.

Professor Crandall properly interprets Fig. 3 of the paper, in which the words positive and negative refer strictly to the quantity $(2h - 1)$ rather than to h, the ratio of rotation velocity to whirl velocity. In Fig. 2, Fig. 3(a) refers to those curves below the ordinate 1, Fig. 3(b) to those between 1 and 4, and Fig. 3(c) to those above 4.

The mode shapes for common whirl in a simply supported system with two disks at quarter points (Figs. 9 and 11 of the paper), are as follows, referring to Fig. 1 of this closure:

(a) Negative precession: dotted lines Fig. 9 of paper; lines two and six from the top, Fig. 11.

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[5] "On the Impact Behavior of a Material With a Yield Point," by M. P. White, Journal of Applied Mechanics, Trans. ASME, vol. 71, 1949, p. 39.

[6] "The Force Produced by Impact of a Cylindrical Body," by M. P. White, NDRC Report, A-157.

[7] "The Permanent Strain in a Uniform Bar Due to Longitudinal Impact," by M. P. White and LeVan Griffis, Journal of Applied Mechanics, Trans. ASME, vol. 69, 1947, p. A-337.

[1] By R. B. Green, published in the December, 1948, issue of the Journal of Applied Mechanics, Trans. ASME, vol. 70, pp. 369–376.

[2] Elliott Company, Jeannette, Pa.

[3] Assistant Professor of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, Mass.

[4] "A Method for Simplifying the Calculations of the Natural Frequencies for a System Consisting of n Rigid Disks Mounted on an Elastic Shaft," *Journal of the Aeronautical Sciences*, January, 1947, vol. 14, no. 1, pp. 5–18.

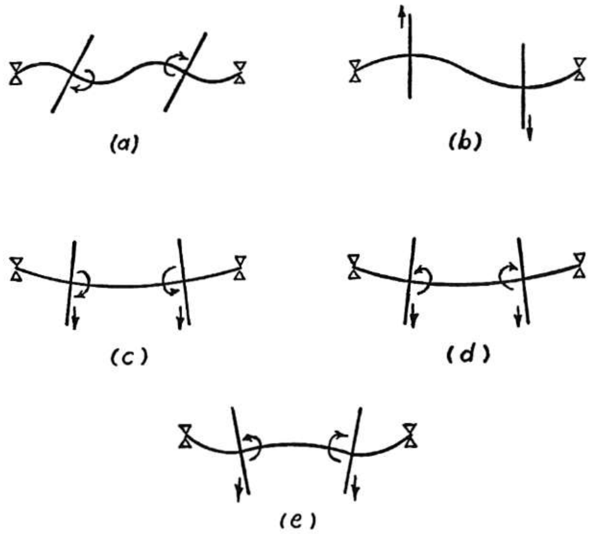

FIG. 1

(*b*) All precessions. K_3^2 equals 8, Fig. 9; lines three and eight, Fig. 11.

(*c*) Lower negative precession. K_3^2 below 1, Fig. 9; line five, Fig. 11.

(*d*) Positive precession. K_3^2 between 1 and 4, Fig. 9; line four, Fig. 11.

(*e*) Higher negative precession. K_3^2 above 4, Fig. 9; lines one and seven, Fig. 11.

The mode shapes of the other systems are not too difficult to visualize, once those of the two systems described are understood.

Investigations of the Flow in Curved Ducts at Large Reynolds Numbers[1]

W. E. TRUMPLER.[2] The paper is very instructive in conveying the exact nature of flow in curved passages. Such presentations are of great help to engineers in hydraulic machines, where flow losses generally are lumped together in a percentage and little is known of its real nature. It would be of further advantage if such an investigation could be extended to curved channels in rotating elements, such as centrifugal-pump or compressor wheels.

[1] By J. R. Weske, published in the December, 1948, issue of the JOURNAL OF APPLIED MECHANICS, Trans. ASME, vol. 70, pp. 344–348.

[2] Chief Engineer, Centrifugal Compressors, Clark Bros. Company, Inc., Olean, N. Y. Mem. ASME.

AUTHOR'S CLOSURE

Certain aspects of secondary flows in rotor wheels of turbomachines have been studied by the author and a paper, "Secondary Flows in Rotating Passages at High Reynolds Numbers" presented at the VII Congress of Applied Mechanics in London, England, in September, 1948, which will be published in the Proceedings of that Congress.

Theory of the Damped Dynamic Vibration Absorber for Inertial Disturbances[1]

C. F. GARLAND[2] AND F. M. SAUER.[3] The results of a similar analysis of the dynamical vibration absorber, together with experimental data, are included in a paper by the writers.[4] A comparison of the writers' analysis with that of the author indicates a discrepancy in the expressions for the optimum damping in the "Lancaster-type" absorber. It is believed that the author's Equation [34*b*] is incorrect and that the optimum damping for this case should be

$$h_{\text{opt}}^2 = \frac{1}{2(2 + \mu)}$$

The author's Equation [34*b*] leads to a corresponding distortion of curve 3, Fig. 6 of the paper. It is noted that curve 1 in Fig. 6, is not in agreement with Equation [34].

AUTHOR'S CLOSURE

Professor Garland and Mr. Sauer give the correct form of Equation [34*b*] and note that curves 1 and 3 of Fig. 6 are not consistent with the correct formulas [34] and [34*b*] which they are supposed to represent. I am grateful to them for correcting the record.

Also the author would like to point out that the expression $\pi c\omega\xi^*$ appearing in Equation [45] of the paper and two lines earlier should read $\pi c\omega\xi^{*2}$. This oversight does not affect later results.

[1] By J. E. Brock, published in the March, 1949, issue of the JOURNAL OF APPLIED MECHANICS, Trans. ASME, vol. 71, pp. 86–92.

[2] Associate Professor, Department of Mechanical Engineering, University of California, Berkeley, Calif. Mem. ASME.

[3] Instructor, Department of Mechanical Engineering, University of California, Berkeley, Calif. Jun. ASME.

[4] "Performance of the Viscously Damped Vibration Absorber Applied to Systems Having Frequency-Squared Excitation," by C. F. Garland and F. M. Sauer, published in this issue of the JOURNAL OF APPLIED MECHANICS, pp. 109–116.