AUTHOR'S CLOSURE

The observation of Dr. Plunkett is an interesting one and the author concurs with his opinion that it may have value for estimating the buckling load in the manner that he describes.

The author also wishes to correct a numerical value occurring in the paper. Under Discussion of Results the stated increase of strain of 49 per cent caused by an axial load which is one half of the Euler critical load should read 17 per cent.

Biaxial Tension-Tension Fatigue Strengths of Metals[1]

F. B. SCHNEIDER.[2] The author refers to the accumulator A, Fig. 3, to maintain pressure, and to the gages H and L to measure the maximum and minimum pressures. While this will work at static conditions and up to a certain minimum number of fluctuations, it is not pointed out that the accumulator and the gages tend to respond to the average pressure at 300 fluctuations per min instead of to the maximum or minimum pressure. From the description given it would seem possible that the pressures could exceed the static test limits without noticeable indication on the gages, and the amount of liquid could decrease without corrective action from the accumulator as long as the average pressure remained within these limits. Perhaps the remark of the author that it was necessary to apply the low rate of 300 fluctuations per min to eliminate possible errors also refers to these conditions.

From Fig. 2 it appears that the connection to the pressure gages is in the piping leading to the test specimen. This assumes that the remainder of the piping to the test specimen must be in perfect order, since any leaks or air bubbles in this part of the piping would not register. Was there no possibility of air being trapped at the inlet connection to the test specimen even though the vent at the top of the specimen indicated no air? Why were the gages not connected to the specimen directly?

Between the gage connection and the gages is a tee with a pipe leading upward to the valve M. The vent on top of the test specimen cannot show whether the pipe to valve M contains trapped air. While this condition will not prevent the operation of the valve M, it will impair the maximum and minimum pressure readings of the gages at higher fluctuations. In other words, the test specimen will not be subjected to the extreme pressures as adjusted at static conditions, but will be exposed to pressures closer to the average pressure.

Considering Fig. 5 of the paper, with its peculiar crossing of curves, it could be that at the test of $R = 2.0$, air bubbles were present. Has the author an explanation for the crossing of these curves? Is it possible that at $R = 1.0$, a resonance condition shortens the life of the test specimen? The axial load is measured at a point far removed from the test specimen. While this is correct at static conditions, it is certain that the elastic system of the lever K will cause a time delay in the application of the load at the test specimen.

Since calculations have been made to ascertain the influence upon the value of the applied load of the error introduced by neglecting the inertia forces of the lever K, it would be interesting to know if similar calculations were made to check the influence of the elastic system of the lever K upon synchronism of the two principal loads. Synchronism has been adjusted statically only.

Since the presence of air bubbles together with the elasticity of the piping in the hydraulic system and the elasticity of the lever K in the mechanical system tend to destroy the desired synchronism in the test specimen, it would be interesting to learn whether it has been checked at the test specimen by electric gages. As a matter of record, Equation [1] of the paper should read

$$\sigma_1' = \frac{P'}{\pi d t} + \frac{p'd}{4t}$$

In the next paragraph of the paper, the ratios of the minimum to the maximum stresses, σ_1'''/σ_1' and σ_2'''/σ_2', were kept at about 0.10 to 0.30. The letters G_2, G_1, E_2, and C are not shown in Figs. 2 and 3. The eccentric E_1 is not shown in Fig. 3, but in Fig. 2.

The results of such tests are valuable to the designing engineer. They furnish him with the required data properly to design hydraulic apparatus with simultaneous fluctuating axial loads and hydraulic pressures. They caution him especially to strengthen the designs of apparatus where the maximum longitudinal stress is equal or one half the maximum circumferential stress.

AUTHOR'S CLOSURE

The author appreciates the comments made by Mr. Schneider. There are possible errors in the value of stresses calculated and in the synchronism of the stresses as indicated by Mr. Schneider. It has been assumed that with the rates of fluctuation used these errors are small. However, in a current project using the same testing machine, the biaxial dynamic strains will be measured on the specimen. In this way, the actual values of the stresses and the synchronism of these stresses will be determined.

Determination of the Buckling Load for Columns of Variable Stiffness[1]

G. SONNEMANN.[2] The author points out a very good method by which problems of columns with variable stiffness can be solved. It is rather unfortunate that he did not go into greater detail to point out some very enlightening facts.

If one rewrites Equation [7a] of the paper in the manner suggested by the author, one would obtain

$$\mu_1 = \frac{\int_0^L (F_1'')^2 EI dx}{\int_0^L (F_1')^2 dx} \qquad [1]$$

This is the standard form of the Rayleigh quotient to give the critical buckling load for any column, as long as the shear deformation is negligible.

Similarly, if one rewrites Equation [7b] as suggested by the author, one obtains

$$\mu_2 = \frac{\int_0^L (F_1')^2 dx}{\int_0^L [(F_1)^2/EI] dx} \qquad [2]$$

This is the standard form suggested by Timoshenko to give a closer approximation of the critical load for any column with negligible shear deformation.

[1] By Joseph Marin, published in the December, 1949, issue of the JOURNAL OF APPLIED MECHANICS, Trans. ASME, vol. 71, pp. 383–388.

[2] Locomotive Engineering Division, General Electric Company, Erie, Pa. Mem. ASME.

[1] By C. C. Miesse, published in the December, 1949, issue of the JOURNAL OF APPLIED MECHANICS, Trans. ASME, vol. 71, pp. 406–410.

[2] Assistant Professor, Department of Mechanical Engineering, Drexel Institute of Technology, Philadelphia, Pa. Jun. ASME.