Why did Einstein’s Programme supersede Lorentz’s? (II)

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In section 1 I showed that Lorentz’s classical programme was progressive until after 1905—the year in which Einstein published his Theory of Special Relativity (hereafter referred to as S.R.T.). In the next two sections I shall try to deal with the following three questions. First what were Einstein’s reasons for objecting to the classical programme and hence for starting his own? (I have already shown in section 1 that these reasons could not have been of an empirical kind.) My second question is this. Once Einstein’s programme was launched, why did other scientists like Planck, Lewis and Tolman work on Einstein’s programme rather than on Lorentz’s? \(^1\) Thirdly I shall try to answer the question, at what stage, if any, did the relativity programme empirically supersede Lorentz’s.

2.1 Einstein’s Appraisal of Classical Physics.

Why did Einstein object to Classical Physics? Let me immediately say that the answer to this question will not be a psychologistic answer; I shall not for example be indulging in speculations about Einstein’s childhood. What

\(^1\) My answer will also show that Kuhn’s theory of paradigm-change is not applicable to the Einsteinian Revolution. (Cf. below, pp. 237–8.)
I shall try to show is that certain (unfalsifiable) metaphysical beliefs—at first sight rather vague and empty—which Einstein held, correspond to heuristic prescriptions which, when skilfully applied to particular cases, become specific and powerful tools for the invention of scientific theories. Thus metaphysics can play an important role in starting a new programme, especially when the existing one is empirically successful. Of course, the triumph of a programme can be achieved only by empirical means. However interesting its metaphysics, the programme will ultimately be judged by its ability to anticipate facts. I should like to formulate, as clearly as I can, two devices which formed part of Einstein’s heuristics. To these devices correspond metaphysical beliefs which Einstein articulated in his later years.

(I) Theories have to fulfil the so-called internal requirement of coherence.1 Science should present us with a coherent, unified, harmonious, simple, organically compact picture of the world. The mathematics used in the theory should reflect the degree of internal perfection of the world. ‘The aim of science is, on the one hand, a comprehension as complete as possible of the connection between the sense experiences in their totality, and on the other hand, the accomplishment of this aim by the use of a minimum of primary concepts and relations. (Seeking as far as possible, logical unity in the world picture, i.e. paucity in logical elements.)’2

Einstein went as far as asserting that reality, although independent of the mind, was nonetheless knowable a priori. His so-called aestheticism was not meant in any subjective sense but was linked to a definite metaphysical position. Because Nature is simple, scientific hypotheses ought to be organically compact. Simplicity or coherence are not aimed at because they please our minds or because they effect economy of thought, but because they are an index of verisimilitude.

If it is true that the axiomatic foundations of theoretical physics cannot be derived from experience but have to be freely invented, can we at all hope to find the right way? Or worse still: does this ‘right way’ exist only as an illusion . . . To this I answer with complete confidence that this right way exists and that we are capable of finding it. In view of our experience so far we are justified in feeling that Nature is the realisation of what is mathematically simplest . . . It is my conviction that we are able, through pure mathematical construction, to find

1 There is also an external requirement on theories, namely that they be consistent with empirical results. Thus Einstein writes: ‘The first point of view is obvious: the theory must not contradict empirical facts’ (cf. Einstein [1949], p. 21). Also: ‘The great attraction of the theory [General Relativity] is its logical consistency. If any deduction from it should prove untenable, it must be given up. A modification of it seems impossible without destruction of the whole.’ (Einstein [1950], p. 110. For Einstein, ‘logical consistency’ meant ‘coherence’ or ‘organic compactness’.) Fortunately Einstein did not follow this rule.

those concepts and the law-like connections between them, which yield the key to the understanding of natural phenomena . . . The really creative principle is in mathematics. In a certain sense I consider it therefore to be true—as was the dream of the Ancients—that pure thought is capable of grasping reality.¹

I shall illustrate the importance of prescription (I) in 2.2.

(II) The second heuristic device is more difficult to formulate. Its metaphysical underpinning is the claim that since God is no deceiver, there can be no accidents in Nature. All observationally revealed symmetries signify fundamental symmetries at the ontological level. Hence the heuristic rule: replace any theory which does not explain symmetrical observational situations as the manifestations of deeper symmetries—whether or not descriptions of all known facts can be deduced from the theory. This will become much clearer with two examples.

(a) The Induction Experiment.

\[ F = e \left( \vec{D} + \frac{\vec{v}}{c} \right) \wedge \vec{H}, \text{ where } \vec{D} = 0 \text{ and } \vec{v} = 0 \]

the electron will experience a force which generates a current in the conductor.

We now keep the magnet fixed and move the conductor with velocity \( \vec{v} \) with respect to the medium. No electric field is created because \( H \) is static, i.e. independent of the time. The situation is very different from the previous one, so we might expect the current in the loop either not to arise at all or at any rate to be different from what it was in the first case. However, in view of \[ F = e \left( \vec{D} + \frac{\vec{v}}{c} \right) \wedge \vec{H}, \text{ where now } \vec{D} = 0 \text{ but } \vec{v} \neq 0, \] a current does arise, and, if the relative motion between the conductor and the magnet is the same as in the previous case, the current also turns out to be the same. This result is wholly explained by Maxwell’s theory; in other words, if we assume the existence of a preferred frame and accept Maxwell’s equations,

¹ Cf. Einstein [1934], p. 116; my translation.
we can infer that the outcome of the experiment depends solely on the relative motion of the magnet and the conductor, and not on their absolute motion with respect to the ether. Hence, this time without the aid of any auxiliary hypothesis, an ether theory yields the undetectability of the ether.

Thus in classical electromagnetism there is a basic ontological difference between a situation in which a magnet moves in the ether (presence both of a magnetic and of an electric field) and one in which the same magnet is stationary (presence of a magnetic field alone). However, when we apply Maxwell's equations to compute the current due to the motion of a conductor in the field created by the magnet, the result depends only on the relative motion between the magnet and the conductor. Thus there exists at the 'observational' level, a symmetry between the following two situations: (a) magnet moving towards the conductor, and (b) conductor moving towards the magnet. This conflicts with the asymmetry obtaining at a higher level. Special Relativity eliminates the asymmetry: equations of exactly the same form apply, whether we choose the magnet or the conductor as our frame of reference. There are no separate electric and magnetic fields but one anti-symmetric tensor which transforms globally.1

(b) Equality of gravitational and inertial masses.

In Newtonian theory the inertial mass $m_i$ of a body represents its laziness, i.e. its capacity for resisting acceleration. Inertia is a primary irreducible property of matter which appears in the fundamental laws of motion. The gravitational mass $m_g$ is a measure of the body's receptiveness to the gravitational field. According to Newton gravity is not a primary quality to be treated on par with inertia or impenetrability. Hence inertia and gravity ought to be independent properties. One should for instance be able to alter the gravitational 'charge' $m_g$ without affecting the inertia of the body, in the same way that one can alter the electric charge $e$ while keeping the

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1 Exactly similar considerations as apply to the classical explanation of the induction experiment apply to Lorentz's explanation of the Michelson result (cf. Part I, p. 115). Once we accept the existence of an ether as the carrier of the electromagnetic field, we are led to look upon the latter as a state of the substratum. Molecular forces are transmitted by the same medium, so they also form part of its state; we thus have a good reason for supposing that molecular and electromagnetic forces are similar, i.e. for accepting the M.F.H. (Molecular Forces Hypothesis). From this assumption follow the L.F.C. (Lorentz-Fitzgerald Contraction Hypothesis) and Michelson's null result. There is something paradoxical in that, through postulating the ether as a universal medium, we are driven to the conclusion that it must be undetectable. Was it not dissatisfaction with this paradox so closely connected with the crucial experiment, which caused Einstein to look for another explanation? The answer is that Einstein had become aware of the paradox independently of Michelson, as is indicated in the first paragraph of his [1905] where the induction experiment is mentioned. What from Einstein's point of view, was an unsatisfactory feature of classical physics is already evinced by Maxwell's account of the induction experiment and is, in this sense, completely independent of Michelson.
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inertial mass \(m_t\) constant. However, this is not the case: a doubling of \(m_t\) is instantaneously matched by a doubling of \(m_g\). Newton postulated that the two masses, \(m_t\) and \(m_g\), are equal, but did not explain why. In other words, there is a symmetry between doubling \(m_t\) and doubling \(m_g\), which is at odds with the disparity between the two properties of inertia and gravity. Let us note that the observational 'cash-value' of \(m_t = m_g\) is the proposition that all bodies fall with the same acceleration in a given gravitational field.

The problem can be put a little differently as follows. If a moving train suddenly decelerates, the passengers, being thrown forward, imagine that they are subject to a field of force to which they respond proportionately to their inertial masses. The Newtonian physicist will tell them that this field is a fictitious inertial field due to an inappropriate choice of coordinate system (the train); in fact, by virtue of their inertia, the passengers are still moving uniformly in Absolute Space (that is, if we neglect the attraction of the earth).

According to the Newtonians this fictitious inertial field differs fundamentally from the 'real' gravitational field created by the earth; it is only by accident, namely because \(m_t = m_g\), that all objects respond to the two fields in exactly the same way.

Einstein eliminates this asymmetry between gravity and inertia by proposing that all gravitational fields are inertial; i.e. that all gravitational fields are created by a (local) acceleration of the frame of reference. To put it crudely: being thrown forward in a moving train and being attracted by the earth are basically one and the same phenomenon.\(^1\) It is no wonder that all bodies fall with the same acceleration, since it is the common frame which is accelerating under their feet.

These prescriptions may be susceptible of a more precise formulation, but I leave this question open.\(^2\) Whatever the case may be, the lack of a more accurate rendering in no way entails that the propositions in question must be given a subjective (or psychological) interpretation. Einstein's metaphysical statements are admittedly vague, yet they may still correspond to real properties of an external world independent of the scientist's mind, of his private feelings about harmony, perfection and the like. My main object will now consist in examining the role the above prescriptions played in the genesis of S.R.T. In this specific context it turns out that these otherwise vague rules and propositions assume a very precise form, leaving no doubt as to their intended objective meaning.

\(^1\) This is not strictly speaking true. In the case of the train the field is globally eliminable, whereas in the case of the earth the field is irreducible.

\(^2\) Einstein himself thought 'that a sharper formulation would be possible. In any case it turns out that among the augurs there usually is agreement in judging the inner perfection of the theories and even more so the degree of external confirmation' (Einstein [1949], p. 23)

Let us look at the more general features of Einstein’s objections to Classical Physics. According to Einstein, one of Maxwell’s and Faraday’s greatest contributions to science was the introduction of the field as a constituent of physical reality to be treated on a par with other constituents such as corpuscles and electric charge. Lorentz’s electromagnetic theory confronts us with a dualism to whose removal Einstein was to devote much of his life: on the one hand there are discrete charged particles whose motions are governed by Newton’s laws, and on the other hand a continuous field obeying Maxwell’s equations. It is true that the charged corpuscles and their motions generate the field; but, once started, an electromagnetic disturbance propagates itself with velocity \( c \) independently of its source; the field may act back on the particles, thereby modifying their motion. Fields and particles are therefore ontologically on a par. One way of resolving this dualism is to explain the behaviour of the field in terms of the mechanical properties of an all-pervading medium. Lorentz clearly recognised that all such attempts throughout the nineteenth century had failed; he was about to try a solution in the opposite direction, and in particular to explain inertial mass in electromagnetic terms.

Einstein was clearly dissatisfied with this dualism, as is apparent from the following passage:

If one views this phase of the development of the theory critically, one is struck by the dualism which lies in the fact that the material point in Newton’s sense and the field as continuum are used as elementary concepts side by side. Kinetic energy and field-energy appear as essentially different things. This appears all the more unsatisfactory inasmuch as, in accordance with Maxwell’s theory, the magnetic field of a moving electric charge represents inertia. Why not then total inertia? Then only field-energy would be left and the particle would be merely an area of special density of field-energy. In that case one could hope to deduce the concept of the mass-point together with the equations of the motions of the particles from the field-equations,—the disturbing dualism would have been removed.

This lack of unity in the physical foundations, which violates prescription (II), was reflected in the mathematical formulation of the theory. Einstein explains:

The weakness of the theory lies in the fact that it tried to determine the phenomena by a combination of partial differential equations (Maxwell’s field equations for empty space) and total differential equations (equations of motion of point masses), which procedure was obviously unnatural.

The dualism was made far worse by Newton’s classical Principle of

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1 Einstein [1934], p. 160.  
2 Einstein [1949], p. 36.  
3 Einstein [1950], p. 75.
Relativity which applies to mechanics but apparently not to electrodynamics.\(^1\) In view of the Galilean transformation which physicists took for granted, Maxwell’s equations seem to presuppose the existence of an ether, or at any rate of a unique frame of reference in which they would hold good. Assessing Lorentz’s work, Einstein wrote:

For him [i.e., Lorentz], Maxwell’s equations concerning empty space applied only to a given system of co-ordinates, which, on account of its state of rest, appeared excellent in comparison to all other existing systems of co-ordinates. This was a truly paradoxical situation since the theory appeared to restrict the inertial systems more than classical mechanics.\(^3\)

The Absolute Space Hypothesis, *i.e.*, the assumption that among all inertial frames there exists a privileged one, is an idle metaphysical component of Classical Mechanics. That its elimination does not reduce the empirical content of Classical Dynamics was clearly recognised by Newton who wrote: ‘The motion of bodies included in a given space are the same among themselves, whether that space is at rest or moves uniformly forward in a right line without any circular motion.’\(^3\) One could further maintain that the Absolute Space Hypothesis was scientifically useless in that one could not even in principle define the Absolute Frame as that in which Newton’s laws of motion hold good; for if these laws are true in one of the inertial frames, they are automatically true in all.\(^4\)

With the advent of the wave theory of light, of Fresnel’s and Lorentz’s postulation of a stationary ether,\(^6\) the situation changed dramatically. One could now define the Absolute or Ether Frame as that in which Maxwell’s equations are true. Given the old Kinematics and in particular the Galilean transformation, this definition singles out a unique frame in which, because Maxwell’s equations hold in it, light propagates itself in all directions with the same speed \(c\). The ether frame was taken to be inertial, so that in all other frames, whether inertial or accelerated, light would not have a constant velocity. This implied the possibility of devising experiments which might detect the ‘absolute’ motion of ponderable bodies. The experiment would be such that its outcome tells us whether the body in question was in motion or at rest in the ether. In this connection Michelson’s experiment is typical: a null outcome would tell us that the earth is at rest in the ether, and from a shift of the fringes it would be concluded that the earth moves. In this particular case however we know that the earth changes its velocity with respect to the inertial frame determined by the

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\(^1\) Newton [1686], p. 20.  
\(^2\) De Haas-Lorentz [1957], p. 7.  
\(^3\) Newton [1686], p. 20.  
\(^4\) In the ‘Science of Mechanics’, which Einstein carefully read, Mach attacked the concept of Absolute Space and went as far as proposing that even the distinction between inertial and non-inertial frames ought to be abolished (Mach [1883], chapter 2, vi).  
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stars, so the earth must at one point of its trajectory be moving in the ether. Hence we can predict that the experimental result must be positive.

Why did Einstein find such developments in the evolution of physics 'paradoxical'? We have seen that Einstein disliked the dualism of particles and fields. The fact that the laws of mechanics (which govern the motions of particles) obey the Principle of Relativity, while Maxwell's equations (which govern the behaviour of the field) do not, makes the dualism much worse. Given the problem-situation, there were to my mind two courses of action open to a unificationist like Einstein: he could maintain either that the Relativity Principle applies \textit{neither} to mechanics \textit{nor} to electrodynamics or else that it applies to \textit{both} at the same time. In the first case he could have modified mechanics in such a way that it only holds in the ether frame; in the second case he would have to extend the Relativity Principle to electrodynamics. In its Galilean form, the Relativity Principle is inapplicable to electromagnetic theory. At this point however, Einstein's critique of the induction experiment proved crucial in that it tipped the balance in favour of extending Relativity to electrodynamics and thereby modifying classical kinematics.\footnote{I do not of course mean that the experiment was 'crucial' in the traditional sense of refuting one theory while confirming another.}

To repeat, in the induction experiment there is complete symmetry between the two experimental results, which is at odds with the asymmetry introduced by the 'theoretical' explanation. To put it more pedantically, the observational statements describing the behaviour of the currents in the conductor are identical in the two cases, but the high-level explanations in terms of the accepted theory differ widely. There would be nothing intrinsically wrong in this state of affairs, had the asymmetry not been introduced through considerations of absolute motion which the Relativity Principle forbids. Seen from that angle however, the experiment suggests that an extension of the Relativity Principle to include electrodynamic phenomena might abolish the 'theoretical' asymmetry; it promises to make the symmetry between the two experimental outcomes appear, not as a fortuitous result, but as a direct manifestation of a general principle, the principle of Lorentz-covariance. In this he was following Prescription II.

In his [1905] Einstein concluded that:

\textit{examples [like the induction experiment] together with the unsuccessful attempts to discover any motion of the earth relatively to the light medium, suggest that the phenomena of electrodynamics as well as of mechanics possess no properties corresponding to the idea of absolute rest. They suggest rather, as has already been shown to the first order of quantities, that the same laws of electrodynamics and optics will be valid for all frames of references for which the equations of mechanics hold good. We will raise this conjecture (the purport of}
which will hereafter be called the Principle of Relativity) to the status of a postulate and also introduce another postulate, which is only apparently irreconcilable with the former, namely that light is always propagated in empty space with a definite velocity $c$ which is independent of the state of motion of the emitting body.$^1$

Note the two references made to mechanics, underlining the important part which Classical Relativity played in Einstein's thinking prior to 1905.$^2$

In this passage Einstein alludes to the absence of any first-order effects of absolute motion, which Lorentz had explained in the Versuch through an early version of the theory of corresponding states. This first-order equivalence between observers, which runs counter to the preference given to a unique frame, must have increased Einstein's suspicion that the Relativity Principle applies to electrodynamics as well as to mechanics; under the new theory the absence of first-order effects, instead of being a stray fact, would directly reveal the presence of a universal principle.

What commended the Relativity Principle was therefore its universality, its unifying role in subsuming mechanics and electrodynamics under the same law and in providing a unified explanation for various features of phenomena such as the symmetry in the induction experiment and the absence of first-order effects due to the earth's motion.

The phrase:

'... together with the unsuccessful attempts to discover any motion of the earth relatively to the light medium'

has given historians and philosophers of science some problems.$^3$ It also seems inconsistent with the thesis of Section 1 that the Michelson experiment played a negligible role in the genesis of S.R.T.

Einstein might be referring in the quoted phrase to Michelson's experiment, which must have been in the back of his mind, if only through Lorentz's [1895].$^4$ This is perfectly compatible with his assertion that the experiment came to his attention only after 1905. To my mind the above phrase is no more than a casual allusion to a number of results which he had registered without any surprise, for they anyway followed from his own

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$^1$ Einstein and others [1923], p. 38.

$^2$ This was later confirmed in his more philosophical writings. (See De Haas-Lorentz [1957], quoted above, p. 229. Also cf. Einstein [1930], p. 55.)

$^3$ Grünbaum, for example, says: 'Unless they provide some other consistent explanation for the presence of the latter statement in Einstein's text of 1905, it is surely incumbent upon all those historians of Relativity Theory who deny the inspirational role of the Michelson-Morley experiment to tell us specifically what other "unsuccessful attempts to discover any motion of the earth relatively to the light medium" Einstein had in mind here.' (Cf. Pearce Williams [1968], p. 114.)

$^4$ He admitted to Shankland that 'he had also been conscious of Michelson's result before 1905, partly through his readings of the papers of Lorentz and more because he had simply assumed this result of Michelson to be true'. (Holton [1969], p. 154.)
conjectures. On this point I agree with Holton; for otherwise Einstein would certainly have cited Michelson's result in support of his second postulate, the Light Principle $P_2$. This postulate presents us with a new difficulty.

Unlike his first postulate (the Relativity Principle) $P_1$, the Light Principle $P_2$ is thrown out with no justification whatever. Moreover, on the face of it, $P_2$ runs counter to Einstein's prescription ($I$): there seems to be no connection at all between the fundamental properties of space-time and those of light. Why should purely kinematical considerations involve $c$? The light principle is quite a low-level statement which is not as yet integrated into a more general system. It is precisely for this reason that philosophers and scientists supposed that Einstein was obeying the dictate of experience, basing his second postulate on Michelson's result. Later on in his [1905], Einstein does say that the light principle is in agreement with experience: he had after all heard of various experiments trying to detect the earth's absolute motion. Nowhere, however, does he assert that experience had suggested the second postulate, or even made it look plausible to him.

I think the problem can be solved simply by examining more carefully Einstein's later writings—in particular his Autobiography—and then comparing them with his [1905]. In his [1934] Einstein writes:

Then came the Special Theory of Relativity with its recognition of the physical equivalence of all inertial systems. In conjunction with Electrodynamics or the law of propagation of light, it implied the inseparability of space and time.

Perhaps the most illuminating passage occurs in Einstein's [1950]:

The second principle on which the Special Relativity theory rests is that of the constancy of the velocity of light in the vacuum. Light in a vacuum has a definite and constant velocity, independent of the velocity of its source. Scientists owe their confidence in this proposition to the Maxwell-Lorentz theory of electrodynamics.

Also, in his [1949], Einstein tells us about a thought-experiment in which, at about the age of sixteen, he imagined himself to be following a ray of light at speed $c$:

If I pursue a beam of light with a velocity $c$ (velocity of light in a vacuum), I should observe such a beam of light as a spatially oscillatory electromagnetic...
field at rest. However, there seems to be no such thing, whether on the basis of experience or according to Maxwell's equations. From the very beginning it appeared to me intuitively clear that, judged from the standpoint of such an observer, everything would have to happen according to the same laws as for an observer who, relatively to the earth, was at rest. For how, otherwise, should the first observer know, i.e. be able to determine, that he is in a state of fast uniform motion.¹

What is most striking about this passage is the conclusion which Einstein draws from his thoughtexperiment. He does not restrict himself to what seems warranted by the experiment, namely that c is an unattainable speed or that the addition law of velocities must break down. He immediately jumps to a general conclusion, or rather puts forward the sweeping conjecture: the laws of physics—more specifically those of electromagnetism—would have to be the same for the moving and for the stationary observers. Both historically and epistemologically speaking, Einstein's second starting point—the first one being the Relativity Postulate—is not the Light Principle but the proposition:

$$\text{(P3) Maxwell's equations express a law of nature;}$$

in virtue of $Pr$, they must therefore assume the same form in all inertial frames. Maxwell's equations imply that, within each co-ordinate system in which they hold, electrodynamic disturbances propagate themselves with velocity c, which velocity must therefore be an invariant. Thus $Pr$ and $P3$ imply $P2$.

In the electrodynamical part of the 1905 paper, Einstein does in fact suppose that Maxwell's equations are Lorentz-covariant and then deduces the transformation laws for $E$ and $H$. He does not try to infer $P3$ from $Pr$ and $P2$; so the electrodynamic part, by exhibiting a transformation which makes Maxwell's equations covariant, simply established that the latter are compatible with the Relativity Postulate and the light-principle. It is a consistency proof. Although $P3$ is a stronger statement than $P2$, it is more plausible and incidentally less counter-intuitive. In accordance with (I), $P3$ derives its plausibility from being a unified, well-knit theory in which the primitive concepts (electric field, magnetic field, charge density) are all closely connected; also it had been tested for a whole generation prior to 1905. Thus the logical order is reversed through *a priori* heuristic considerations: $P3$ is more plausible, though stronger, than $P2$.

Another piece of evidence which confirms the view that Einstein approached the problem of Relativity through Maxwell's equations and their covariance is to be found in Lorentz's [1895]. In a part of this work which is completely independent of Michelson's experiment and of the

¹ Einstein [1949], p. 53; my italics.
Contraction Hypothesis Lorentz had proved that no first-order effects of the earth's motion can be detected. Neglecting all terms in \( \left( \frac{v}{c} \right)^2 \), he used a limiting case of the Lorentz transformation; he then found transformation laws for the field \( E, H \), under which the equations take on a form very similar to, and in some cases identical with, the form assumed in the ether frame. It is not far-fetched to suppose that Lorentz's techniques made a strong impression on Einstein; they might well have led him to wonder whether a more general transformation would yield complete covariance together with the result that no effects whatever arise from uniform rectilinear motion. Lorentz himself was to attempt this solution in his [1904], which Einstein did not read before publishing the 'Electrodynamics of Moving Bodies'.

However, Lorentz's programme for a Theory of Corresponding States was outlined in the Versuch and one is struck by the similarity between the methods used by Lorentz and by Einstein. In his [1905] the latter first constructed a transformation law for the coordinates \( x, y, z, t \); then, assuming the co-variance of Maxwell's equation, he deduced the transformation laws for \( \tilde{E}, \tilde{H} \) and \( \rho \). Lorentz's influence on Einstein cannot be overrated; it was not Michelson, the experimentalist, but Lorentz, the theoretician, who played a considerable inspirational role in the genesis of Special Relativity. This is indicated in the all-too-brief second paragraph of Einstein's [1905] by the clause: 'as has already been shown to the first order of small quantities.'

We have seen that Einstein rejected Lorentz's classical approach, but he made use of Lorentz's tremendous technical achievement, albeit under very different kinematical assumptions. I have mentioned that Lorentz's [1892a] already contains the full Lorentz transformation up to a constant factor in the expression of \( t \).\(^1\) Einstein's greatest contribution was to extend Lorentz's methods and give the transformed quantities a realistic interpretation in the 'moving' system.\(^2\)

One might still wonder why Einstein did not start by postulating \( P_1 \) and \( P_3 \) instead of \( P_1 \) and \( P_2 \). He had, I think, at least two good reasons for presenting the new theory in the way he did. On the one hand it is preferable, from the logical point of view, to use the weaker assumption \( P_2 \) which, in conjunction with \( P_1 \), suffices for developing a new kinematics and deriv-

\(^1\) Cf. Part I, p. 112.

\(^2\) It is now finally clear why Einstein could rightly claim that Michelson's experiment had been quite irrelevant to his work and that he could easily have anticipated its null outcome. That almost nothing is cited in support of the Light Principle may be due to the fact that it follows from a well-corroborated hypothesis (Maxwell-Lorentz equations) together with the Relativity Postulate \( P_1 \), for whose acceptance Einstein had already argued. (Cf. Part I, pp. 107–8.)
ing the Lorentz transformation. On the other hand Einstein had come to the conclusion that Maxwell's equations, although true of macroscopic phenomena, did not provide the ultimate foundations for the whole of physics.\(^1\) He might therefore have wanted to make his space-time system independent of electrodynamics.\(^2\)

One question remains unanswered. Einstein faced a problem forced on him by the incompatibility, if taken together, of the following three hypotheses:

\[(P_1)\] The Relativity Postulate
\[(N)\] Newton's second law of motion
\[(P_3)\] The Maxwell-Lorentz equations\(^3\)

His solution consisted in modifying \(N\), or rather in replacing \(N\) by a new theory \(N'\) such that \(P_1\) and \(N'\) and \(P_3\) are consistent. Considering that the (Galilean) Relativity Principle was first shown by Newton to hold for mechanics, it is puzzling that Einstein seems never to have envisaged keeping \(N\) and substituting for \(P_3\) a new set of equations \(P_3'\) covariant under the Galilean transformation. \(P_3'\) would of course have had to yield \(P_3\) as a limiting case.\(^4\)

\(^1\) Einstein did not accept Lorentz's (tentative) assumption that all physical phenomena could be explained in terms of charges and fields governed by Maxwell's equations.

\(^2\) He wrote in his [1955]: 'I knew only of Lorentz's works in 1895—"La Théorie Electromagnétique de Maxwell" [this is in fact Lorentz's [1892a]] and "Versuch einer Theorie der elektrischen und optischen Erscheinungen in bewegten Körpern"—but not Lorentz's later works, nor the consecutive investigations by Poincaré. In this sense my work of 1905 was independent. The new feature of it was the realisation of the fact that the bearing of the Lorentz transformation transcended its connection with Maxwell's equations and was concerned with the nature of space and time in general. A further new result was that Lorentz-invariance is a general condition for any physical theory. This was for me of particular importance because I had already previously found that Maxwell's theory did not account for the micro-structure of radiation and could therefore have no general validity.' Einstein indicates that a connection between the Lorentz transformation and Maxwell's equations clearly existed but was then transcended. The logical picture seems to be as follows:

\[P_1\text{ and } P_3 \Rightarrow P_1\text{ and } P_3\]
\[P_1\text{ and } P_3 \Rightarrow \text{ new kinematics and Lorentz-transformation equations.}\]
\[P_1\text{ and Lorentz-transformation equations } \Rightarrow \text{ requirement of Lorentz-invariance for all physics.}\]

The connection between Maxwell's theory and the Lorentz-transformation is given by:

\[P_1\text{ and } P_3 \Rightarrow \text{ Lorentz equations}\]

This connection is transcended by the result that, from the Relativity Principle \(P_1\) and the Lorentz transformation equations, there follows a new structure of space-time and a condition of Lorentz-invariance which applies not only to Maxwell's equations but to the whole of physics.

\(^3\) Of course there is the underlying and common assumption that the law of inertia should hold in all allowable frames. This is precisely why these frames are called 'inertial'.

\(^4\) W. Ritz adopted this approach. Rather than adjusting the whole of physics to electrodynamics, he tried to alter electrodynamics so as to make it Galileo-covariant. He looked upon the field quantities \(\vec{E}\) and \(\vec{H}\) as intermediate quantities which enable one to compute the Lorentz force \(\vec{F} = e(\vec{E} + \vec{v} \times \vec{H}/c).\) In the last analysis only the particles,
This is all the more intriguing because he realised that Maxwell's equations were not as fundamental or as ultimate as Lorentz had taken them to be. In other words: why did Einstein throw in his lot with Maxwell rather than with Newton? In the end Einstein guessed; it was not however as uninformed a guess as it might at first appear.

Einstein was dissatisfied with the further and fundamental dualism between fields and particles which beset Lorentz's theory. In virtue of his Prescription, it seemed obvious to Einstein that one component of the dual structure ought to be reduced to the other. But which one? He found, as he explained in his Autobiography, that all attempts at a mechanical explanation of the behaviour of the field had failed.

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1 It is also intriguing due to the fact that, according to MacCormmach, Einstein was initially inclined to regard mechanics as the most fundamental branch of physics. (Cf. MacCormmach [1970].)

2 Einstein adduces from Planck's quantum hypothesis a second reason for abandoning mechanics in favour of electrodynamics. Although this reason seems to be a post hoc rationalisation, I shall quote Einstein in full: '[Planck's] form of reasoning does not make obvious the fact that it contradicts the mechanical and electrodynamic basis, upon which the derivation otherwise depends. Actually, however, it presupposes implicitly that energy can be absorbed and emitted by the individual resonator only in quanta of magnitude $h$, i.e. that the energy of a mechanical structure capable of oscillations as well as the energy of radiation can be transferred only in such quanta—in contradiction of the laws of mechanics and electrodynamics. The contradiction with dynamics was here fundamental; whereas the contradiction with electrodynamics could be less fundamental. For the expression for the density of radiation energy, although it is compatible with Maxwell's equations, is not a necessary consequence of these equations.' (Einstein [1949], p. 45; my italics.) This passage indicates that Einstein gave precedence to Maxwell over Newton and took his starting point with electrodynamics rather than with mechanics. Nevertheless, having accepted Planck's quantum hypothesis, Einstein could not regard Maxwell's equations as fundamental. Thus Einstein had enough reservations about electrodynamics to avoid making it into a cornerstone of his kinematics. Although $P_3$ implies and lends plausibility to the Light Principle, the latter is still more fundamental in the sense of applying both to micro- and macroscopic phenomena; Einstein's lucky and unexplained guess was that the invariance of $c$ was a universal principle which transcends its obvious dependence on Maxwell's equations.
If mechanics was to be maintained as the foundation of physics, Maxwell's equations had to be interpreted mechanically. This was zealously but fruitlessly attempted, while the equations were proving themselves increasingly fruitful.\footnote{Einstein [1949], p. 25.}

We have seen how Einstein arrived at his programme and hence at S.R.T. We shall see that this programme finally superseded Lorentz's in the strictly empirical sense in 1915.\footnote{Cf. Part I, p. 116.} But was Einstein's programme objectively superior to Lorentz's in 1905? Did Lorentz's programme, as is generally claimed, really collapse in the face of S.R.T.?\footnote{Cf. Part I, pp. 120-1.}

2.3 The Heuristic Superiority in 1905 of the Relativity Programme: Einstein's Covariance versus Lorentz's Ether. The Power of Einstein's Heuristics: Derivation of a New Relativistic Law of Motion and of $E = mc$.\footnote{For the argument that follows, I do not even need the assumption that $T_e$ and S.R.T. are observationally equivalent. It is enough that: (1) between 1905 and 1908 no 'crucial' experiment between the two rival theories was carried out; and (2) neither hypothesis logically implies the other. (1) is a historical fact; as for (2), Lorentz proposed a 'model' of the electron as a spherical distribution of charge in the ether, while Einstein remained agnostic as to the shape, charge density and mass of the electron; on the other hand Einstein asserted that all physical laws are Lorentz-covariant whereas Lorentz restricted his attention largely to electrodynamics (and did not fully establish the covariance of Maxwell's equations).}

As I have already shown,\footnote{Einstein [1949], p. 25.} Lorentz's theory $T_e$ is observationally equivalent to the S.R.T.; Einstein's transformed coordinates can be interpreted as the measured coordinates in Lorentz's moving frame. In the latter the 'real' coordinates are still the Galilean ones: $x_r = x - vt$, $y_r = y$, $z_r = z$, $t_r = t$; but, due to the contraction of measuring rods, to time-dilation and to the synchronisation of clocks through light signals, the measured coordinates are:

$$x' = \beta(x - vt), \quad y' = y, \quad z' = z, \quad t' = \frac{t - vx}{c^2},$$

where $\beta = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

Thus, as he indicated at the end of his *Theory of Electrons*, Lorentz was in a position so to reformulate his theory that no 'crucial' experiment between his system and Einstein's could have been devised in 1905.\footnote{Cf. below, section 3.}

In view of this situation, why did brilliant mathematicians and physicists like Minkowski and Planck abandon the classical programme in order to work on Special Relativity? Given the lack of any crucial experiment, a Kuhnian account of the 'conversion' of Planck and others may seem plausible. But the idea of a new bandwagon is highly implausible. First, Einstein was a relatively unknown figure while Lorentz was a recognized authority. Secondly, Lorentz's theory was eminently intelligible whereas
Einstein's involved a major revision of our most basic notions of space and time. Thirdly, there was no build-up of unsolved anomalies which Einstein's theory dissolved better than Lorentz's. Moreover, at the time when Planck was converted, that is in 1906, no bandwagon had started. Nor did the leading protagonists of the old paradigm die out unconverted as Kuhn claims is generally the case. Lorentz himself was eventually converted to the new outlook. In the *Theory of Electrons*, first published in 1909, he gives essentially the same account of the theory of corresponding states as in his *Electromagnetic Phenomena* of 1904; however, the footnotes indicate that by 1915 he had already accepted the Relativity Principle.

Kuhnian explanations of the victory of *S.R.T.* do not work. Another explanation is Whittaker's. He tackles the difficulty by considering Lorentz and Poincaré as the real authors of Special Relativity, leaving to Einstein the merit of proposing a new theory of gravitation (i.e. General Relativity). Thus Lorentz's ether programme was not defeated by, but developed into, the Relativity programme. However interesting and plausible this explanation may seem in the light of the foregoing discussion, it is unacceptable. As will be shown, the two programmes possess very different heuristics.

The most commonly held explanation is the third one. According to this, Einstein's theory represented the success of positivism in ridding classical physics of redundant metaphysics.

I shall both develop this positivist claim and present my answer to it by

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Did Lorentz face insuperable difficulties which were known to his contemporaries? We have seen that Lorentz used the 'Molecular Forces Hypothesis' in order to obtain the appropriate laws about rod-contractions and clock-retardations. He had thereby assumed a transformational similarity between electromagnetic and molecular forces. In view of his programme, his next most natural step would have been to give a precise classical law of force for molecular and atomic interactions. We have also seen that, in order to explain the variation of the inertia with the velocity, Lorentz accounted for the mass of an electron in electromagnetic terms (electromagnetic longitudinal and transversal masses). In other words, in producing the revolutionary results which either matched or even anticipated Einstein's, Lorentz had to give a classical account of elementary particles. Lorentz was therefore unlucky in his choice of problems: he was straight away involved in difficulties which were to defeat Einstein himself, but at a much later stage. With hindsight we can see why Lorentz would probably have failed anyway; he was overtaken by the quantum theorists who realised that classical laws and in particular Maxwell's equations, did not explain atomic stability. However, in 1905 there was hardly any indication that Lorentz could go no further in developing his programme and that no satisfactory classical account could be given of the behaviour of elementary particles. Yet, already in 1906, a physicist of Planck's stature and conservatism abandoned the classical approach, knowing very well that the *S.R.T.* might well have been refuted by Kaufmann's experiment. Planck's choice, if rational, must have been guided by considerations different from the ones just given.

A Kuhnian might fall back on individual Gestaltswitches: but if so, the Gestaltswitches would be different for Planck, for Minkowski, for Sommerfeld, for Lorentz!

Cf. Whittaker [1953], Chapters II and V.

Cf. Bridgman [1936], pp. 7–9, von Laue [1952], p. 6, Eddington [1939], pp. 70–5. Also Eddington [1920], pp. 1–16.
comparing the Einsteinian revolution with the Copernican (or rather the Keplerian) one. This comparison will point to a feature which has often been a symptom if not a cause of decline in the heuristic of research programmes. This feature is a divorce between the empirical content and the mathematical formulation of certain scientific hypotheses: these hypotheses contain a large number of (physically) uninterpreted mathematical entities.¹ According to positivists like Mach, the mere elimination of such entities increases simplicity and thereby constitutes progress. My claim is that such eliminations are by-products of new research programmes whose heuristic eliminates certain entities. This may be accompanied by a—contingent—increase in simplicity. Let us take the example of the Copernican Revolution.

The Platonic programme of saving the phenomena by the use of circular and spherical motions was initially successful: to each mathematical entity corresponded a physical one. Each planet was fixed on a physically real crystalline sphere which performed a number of axial rotations. It was however discovered that the distance between the earth and a given planet varied, so the astronomers resorted to eccentrics, epicycles and equants in order to account for the new phenomena. The physical problem was to determine the motion of the heavenly bodies relative to the earth. Since the paths of the planets are non-circular and since their motion is non-uniform, a widening gap appeared between the physical problem and the mathematical methods, which allowed only for circular motions. Although the earth allegedly occupied the centre of the universe, the paths of the planets about the earth were not dealt with directly; epicycles, deferents and equants, all of which had no ‘physical reality’, were introduced in order to predict astronomical data; both the centre of an epicycle and the punctum equans are empty points in space.

Copernicus did not heal this rift between the physical picture and the mathematical description. True, he got rid of the equant; but, although his problem was to determine the motion of the planets with respect to a fixed sun, he interposed between the sun and the planets roughly as many epicycles with as many empty centres as were involved in the Ptolemaic system. It was left to Kepler to investigate the direct relation between the sun and the planets, to abolish epicycles and to find that the planets describe ellipses with one focus at the centre of the sun.

Let us now return to Lorentz. We have seen that the Lorentz-transformation is always carried out in two steps.²

¹ This is so to speak the obverse of the point made earlier about the second heuristic role of mathematics in physics. There we saw how new physical theories can be constructed by interpreting hitherto uninterpreted mathematical entities. (Cf. Part I, pp. 109–11.)

² Cf. Part I, p. 117.
The first step yields the Galilean coordinates:

\[ x_r = x - vt, \quad y_r = y, \quad z_r = z, \quad t_r = t \]

The second one gives us the effective coordinates:

\[ x' = \beta(x - vt), \quad y' = y, \quad z' = z, \quad t' = \beta(t - vx/c^2) \]

The Galilean coordinates are interposed between the absolute coordinates and the effective ones in the same way as various epicycles were placed between the earth or the sun on the one hand and the planets on the other.\(^1\)

In a moving frame only the Galilean coordinates are taken by Lorentz to be ontologically 'real' in the same way that before Kepler only circular motions were considered permissible from a metaphysical point of view. These metaphysical assumptions were naturally reflected in the mathematics: in the Galilean transformation used by Lorentz and in the epicycles used by Ptolemy and Copernicus. The Galilean transformation is a vestige of the original aim of the Classical Programme, namely the aim of giving to the ether frame a privileged status. (Because of the Galilean transformation, Maxwell's equations hold good only in the ether frame.) The assumption of an ether frame no longer has any observational cash-value. Similarly the Ptolemaic epicycles were reminders of a hope which had long vanished, the hope of finding that the motions of the planets are both uniform and circular.

Copernicus was aware that the motions of the planets are neither circular nor uniform and Lorentz later realised that the effective coordinates, and not the Galilean ones, are the measured quantities in the moving frame.

I have drawn a parallel between Copernicus and Lorentz. Kepler and Einstein can be similarly compared. Kepler's greatest contribution to astronomy allegedly consisted in eliminating epicycles and in showing that the 'real' paths of the planets are ellipses with one focus at the sun. Similarly, according for instance to Bridgman and to von Laue, Einstein's chief merit lay in abolishing the Galilean transformation and in identifying the effective or measured coordinates as the only real ones. In equating 'to be' with 'to be perceived or measured' Einstein is supposed to have carried out a positivistic revolution in physics. However, if the merit both of Kepler and of Einstein only consisted in ridding physics of unnecessary

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\(^1\) By drawing a parallel between the Galilean transformation on the one hand and a system of epicycles on the other, I do not want to suggest that, in Lorentz's theory \( T \), the Galilean transformation is physically uninterpreted. In fact, even the epicycles can be interpreted in the following trivial way: God, in contemplating his creation, sees it as a huge system of interlocking circles. Similarly, in Lorentz's case, God would perceive an infinite extended substance, the ether, in which any two events are separated by an absolute time interval. Such interpretations, which do not increase the empirical content of existing theories, could conceivably be made useful by indicating how they are to be heuristically exploited in order to construct new physical theories. Lorentz did not give such an indication in connection with the Galilean transformation. (Cf. below, p. 243.)
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'epicycles', then the importance of these two physicists in the history of science is very much overrated: Copernicus and Lorentz did all the creative work, and Kepler and Einstein only applied Occam's razor in order to demolish the expendable metaphysical scaffolding used by their predecessors. Moreover, Copernicus knew that the paths of the planets were not circular, hence that his epicycles were part of the scaffolding; Lorentz realised that he did not need the Galilean coordinates in order to deduce the null results which he set out to explain. If so, *Kepler and Einstein contributed to the economy of thought and not to the growth of knowledge.*

This is an unacceptable conclusion. Let us start with Kepler. Copernicus's account of the motion of heavenly bodies had been largely Aristotelian in character: because the planets are perfect spheres, their natural motion is both uniform and circular. Through trying to give a dynamical explanation of the motion of heavenly bodies, Kepler provided classical astronomy with its heuristic. He proposed to determine the forces which emanate from the sun and directly act on the planets. He abolished epicycles *because* he wanted nothing but forces to mediate between the sun and the planets. Circles centred on empty points did nothing but conceal the 'true' relation which linked one heavenly body to the other. Kepler proposed a dynamical theory which is now largely forgotten because it was contradicted and supplanted by Newtonian astronomy. But in forgetting Kepler's dynamical theory we should not forget that Kepler created the programme which culminated in the Newtonian system; Kepler's method consisted in trying to discover the law of force responsible for the periodic motion of the planets round the sun. *Getting rid of Copernican epicycles was not an end in itself: it was subordinate to the needs of the new heuristic.*

Einstein, like Kepler, created a programme, not only an isolated theory. We shall see that Einstein's heuristic is based on a general requirement of Lorentz-covariance for all physical laws; we recall that the Lorentz-transformation sends \((x, y, z, t)\) directly into \((x', y', z', t')\) without passing by the Galilean coordinates \(x, y, z, t\). The new heuristic therefore requires the abolition of the Galilean transformation which plays the role of a cumbersome epicycle. The parallel with Kepler is complete.

After these criticisms of the Kuhnian, Whittakerian and positivist 'explanations' of the Einsteinian revolution, let me venture my own. In my view the main difference between Lorentz and Einstein lies in the difference between the heuristics of their respective programmes. *The ether programme did not collapse but was superseded by a programme of greater heuristic power.* This greater heuristic power explains why Planck and others joined Einstein's programme before it became empirically progressive. The difference between the two theories cannot be appreciated by taking an
instantaneous look at Lorentz's and Einstein's systems. One has first to imbed them in their respective programmes. In this way one realises that the two theories are similar because they stand at the intersection of two research programmes which later diverged. It will further be shown that the difference between the two approaches did not emerge with hindsight but guided the deliberate choice of scientists at the beginning of this century.¹

Lorentz, unlike Einstein, did not create the heuristic of his own programme. The heuristic of Lorentz's programme consisted in endowing the ether with such properties as would explain the behaviour both of the electromagnetic field and of as many other physical phenomena as possible. In view of the overwhelming success of Newtonian dynamics it is hardly surprising that the ether was supposed to possess primarily mechanical properties. The ether programme developed rapidly in certain respects, yet towards the end of the nineteenth century its positive heuristic was running out of steam. A succession of mechanical models for the ether were proposed and abandoned. One serious difficulty was the presence in these models of longitudinal as well as transversal waves.² Lorentz faced a daunting problem of a different sort: in order to explain certain electromagnetic phenomena he postulated an ether at rest. He considered a portion of the ether, calculated the resultant \( R \) of the Maxwellian stresses acting on its surface and found that \( R \) is generally non-zero. Hence, if he was to assume that the ether was anything like an ordinary substance, he would have also to suppose that it was in constant motion. But this contradicted his original assumption of an ether at rest. He concluded 'that the ether is undoubtedly widely different from all ordinary matter' and that 'we may make the assumption that this medium, which is the receptacle of electromagnetic energy and the vehicle for many and perhaps for all the forces acting on ponderable matter, is, by its very nature, never put in motion, that it has

¹ I have reached the seemingly paradoxical conclusion that both Einstein (and Planck) on the one hand and Lorentz on the other were perfectly rational in doing what they did, i.e. in doing opposite things. Let me immediately add that they were rational, given their metaphysical positions. The conflict between Lorentz and Einstein is, among other things, the age old conflict between two metaphysical doctrines which, Polanyi notwithstanding, do not belong to the tacit component but can be articulated. Lorentz held that the universe obeys intelligible laws (e.g. wave processes presuppose a medium, there exists an absolute 'now' etc.) and Einstein held that the universe is governed by principles which can be given a mathematically coherent form. (e.g. all laws are covariant.) All major scientific revolutions were accompanied by an increase of mathematical coherence together with a (temporary) loss of intelligibility. (This applies to the Copernican, to the Newtonian, to the Einsteinian and to the quantum-mechanical revolutions.) It can moreover be argued that intelligibility is a time-dependent property, while mathematical coherence is not. We still consider Newtonian astronomy more coherent than Ptolemaic astronomy; but action-at-a-distance was unintelligible before Newton, became perfectly intelligible at the end of the eighteenth century, and again unacceptable after Maxwell.

² Cf. Whittaker [1951], Chapter V.
neither velocity nor acceleration, so that we have no reason to speak of its mass or of forces that are applied to it.\textsuperscript{1} In other words Lorentz had reached a point where the behaviour of the electromagnetic field dictated what properties the ether ought to have, no matter how implausible these properties might be: for example the ether was to be both motionless \textit{and} acted upon by non-zero net forces. The ether was nothing but the carrier of the field. \textit{This involved a reversal of the heuristic of Lorentz’s programme: instead of learning something about the field from a general theory of the ether, he could only get at the ether post hoc by way of the field.} In the case of the \textit{M.F.H.}, for example, Lorentz \textit{first} studied the transformational properties of the electromagnetic field; only \textit{then} did he extend these properties to other molecular forces. Instead of positing \textit{one medium} endowed with certain properties from which all forces inherit some \textit{common} characteristic, we have an electromagnetic field acting as the archetype which determines the respects in which all forces are similar.

I do not claim that the ether programme was beyond redemption. Of course there was no obvious reason why the postulation of some non-mechanical properties of the ether should not account both for electromagnetic phenomena and for molecular interactions. All I claim is that the heuristic, \textit{as it stood}, had petered out and that the ether programme was in need of a ‘creative shift’\textsuperscript{2}—a shift which, as a matter of fact, Lorentz did not provide.

Einstein based his heuristic on the requirement that all physical laws should be Lorentz-covariant; \textit{i.e.} all theories should assume the same form, whether they are expressed in terms of $x, y, z, t$ or in terms of $x’, y’, z’, t’$. But it would be practically impossible to discover new laws simply by looking out for all the equations which are covariant under the Lorentz transformation. A good method is to start from well-tested laws whose past success would anyway have to be explained by any new theory. \textit{Thus the heuristic of Einstein’s programme is based on two distinct requirements: (1) a new law should be Lorentz-covariant and (2) it should yield some classical law as a limiting case.}

We have just seen that Lorentz used the ether in order to extend certain properties of the electromagnetic field to molecular forces. His methods were effective in explaining Michelson’s and other null results. By requiring that all forces and not only the electromagnetic and the molecular forces obey the same transformation laws; by taking Maxwell’s equations and imposing their transformation properties on the whole of physics, Einstein

\textsuperscript{1} Lorentz [1909], p. 30.

\textsuperscript{2} This is a technical term in the methodology of scientific research programmes: cf. Lakatos [1970], p. 137.
both strengthened those Lorentzian methods which had proved effective in particular cases and turned them into a heuristic of general applicability. In this sense Einstein's programme displayed greater heuristic power than Lorentz's.

Let us give a more formal rendering of these two requirements. Let $R(a_1, a_2, \ldots, a_n) = 0$ be an equation which constitutes a physical law in some inertial frame $I$. If $I'$ is any other inertial frame in which the quantities $a_1, a_2, \ldots, a_n$ assume the values $a'_1, a'_2, \ldots, a'_n$, respectively, then by the Relativity Principle:

\[(R(a_1, a_2, \ldots, a_n) = 0) \Rightarrow (R(a'_1, a'_2, \ldots, a'_n) = 0)\]

But as Kretschmann pointed out to Einstein, every empirical law can be given not only a Lorentz-covariant but also a generally covariant expression (of course, general covariance implies Lorentz-covariance). Thus, on the face of it, the most distinctive requirement of Einstein's heuristic is empty. However the requirement is only trivialised if one is allowed complete freedom in reformulating the law. If one is restricted to a given number of entities $a_1, a_2, \ldots, a_n$, then the covariance requirement, far from being empty, becomes a stringent condition. As we shall see, in each particular case in which the heuristic is applied, the entities involved in the covariant law are precisely those involved in the corresponding classical law.*

Now we consider the requirement that a new relativistic law should yield the corresponding classical theory as a limiting case. In the most general case laws will involve the speed of light, the velocities $v_1, \ldots, v_n$ of a finite number of particles or processes and some other quantities $a, b, \ldots$. If $R = 0$ and $K = 0$ are the relativistic and classical laws respectively, we require that:

\[R \to K \text{ as } (v_1/c, v_2/c, \ldots, v_n/c) \to (0, 0, \ldots, 0).\]

There are at least two ways of letting $v_m/c$ tend to zero for $m = 1, 2, \ldots, n$. First we take $c$ to be a constant and let $(v_1, \ldots, v_n)$ approach zero. In this case we put $\bar{w}_m = \bar{v}_m/c$, for all $m = 1, 2, \ldots, n$ and consider both $R$ and $K$ as functions of $c, \bar{w}_1, \ldots, \bar{w}_n, a, b, \ldots$. In other words, we write:

\[R = R(c, \bar{w}_1, \ldots, \bar{w}_n, a, b, \ldots) \text{ and } K = K(c, \bar{w}_1, \ldots, \bar{w}_n, a, b, \ldots)\]

We then make

\[R(c, \bar{w}_1, \ldots, \bar{w}_n, a, b, \ldots) \to K(c, \bar{w}_1, \ldots, \bar{w}_n, a, b, \ldots)\]

* Cf. Kretschmann [1917] and Einstein [1918].

This problem arises also in the case of General Relativity where a different set of restrictions again render the covariance principle non-empty. (Apart from the energy tensor $T_{\mu\nu}$, only the $g_{\mu\nu}$'s and their first and second order derivatives can occur in the field equations.)
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approach zero as \(w_1, \ldots, w_n\) simultaneously tend to zero. It is of course tacitly assumed that \(R\) and \(K\) are continuous functions. Hence

\[
R(c, \bar{w}_1, \ldots, \bar{w}_n, a, b, \ldots) - K(c, \bar{w}_1, \ldots, \bar{w}_n, a, b, \ldots)
\]
approaches

\[
R(c, o, \ldots, o, a, b, \ldots) - K(c, o, \ldots, o, a, b, \ldots)
\]
as \((w_1, \ldots, w_n)\) approaches \((o, \ldots, o)\). Thus the second requirement reduces to the equation:

\[
R(c, o, \ldots, o, a, b, \ldots) = K(c, o, \ldots, o, a, b, \ldots).
\]

In this first case the function \(R\), which is to be determined, will therefore be subjected to the following two conditions:

(1') \([R(c, \bar{w}_1, \ldots, \bar{w}_n, a, b, \ldots) = 0] \iff [R(c, \bar{w}_1, \ldots, \bar{w}_n, a', b', \ldots) = o]\)

(Relativity Principle)

(2) \(R(c, o, \ldots, o, a, b, \ldots) = K(c, o, \ldots, o, a, b, \ldots)\)

(Requirement that the classical law be a limiting case of the new law)

We recall that

\[
R(c, w_1, \ldots, w_n, a, b, \ldots) = R(c, \bar{v}_1/c, \ldots, \bar{v}_n/c, a, b, \ldots) = 0
\]
is the relativistic law which is to replace the classical equation

\[
K(c, w_1, \ldots, w_n, a, b, \ldots) = K(c, \bar{v}_1/c, \ldots, \bar{v}_n/c, a, b, \ldots) = 0.
\]

If the relativistic law holds good in general, it will in particular be true for vanishing velocities: i.e. for \(v_1 = \ldots = v_n = 0\) or equivalently for \(w_1 = \ldots = w_n = 0\). By (2) it follows that:

(3) \(K(c, o, \ldots, o, a, b, \ldots) = o\).

This last equation means that, when \(v_1, \ldots, v_n\) all vanish, the relativistic law collapses into the classical one, which must therefore hold good in this particular case.\(^1\)

There is a second way of making \(v_1/c, \ldots, v_n/c\) tend to zero, namely by treating \(c\) as a variable parameter, fixing the velocities \(\bar{v}_1, \ldots, \bar{v}_n\) and then letting \(c\) tend to infinity.\(^2\) Putting \(c = 1/\gamma\), we can write:

\[
R = R_0(\gamma, \bar{v}_1, \ldots, \bar{v}_n, a, b, \ldots) \quad \text{and} \quad K = K_0(\gamma, \bar{v}_1, \ldots, \bar{v}_n, a, b, \ldots).
\]

We now require that:

\[
[R_0(\gamma, \bar{v}_1, \ldots, \bar{v}_n, a, b, \ldots) - K_0(\gamma, \bar{v}_1, \ldots, \bar{v}_n, a, b, \ldots)] \to 0,
\]
as \(c \to \infty\), i.e. as \(\gamma = 1/c \to 0\).

\(^1\) Both Einstein and Planck assumed that Newton's second law of motion holds good when the velocity vanishes. (Cf. below, p. 247.)

\(^2\) Note that, as \(c \to \infty\), the Lorentz transformation collapses into the Galilean one.
Assuming that $R_0$ and $K_0$ are continuous, we obtain:

$$(3') \quad R_0(o, \tilde{v}_1, \ldots, \tilde{v}_n, a, b, \ldots) = K_0(o, \tilde{v}_1, \ldots, \tilde{v}_n, a, b, \ldots)$$

One last way of meeting the requirement that $R$ should tend to $K$ as $(v_1/c, \ldots, v_n/c)$ tends to zero is to assume that $R$ is a function of certain relativistic quantities and that $K$ is the same function of the corresponding classical quantities. Then, if $R$ and $K$ are continuous and if each relativistic quantity tends to the corresponding classical one, it follows that:

$$R \to K \text{ as } (v_1/c, \ldots, v_n/c) \to (0, \ldots, 0).$$

Having formulated the heuristic of the relativity programme in general terms, let me now give concrete examples illustrating the power of this heuristic.

The first example is concerned with Planck's modification of Newton's second law of motion. Following Einstein, Planck considered a slowly accelerated electron in an inertial frame $I$. By substituting Lorentz's expression for the ponderomotive force in Newton's second law, it is found that the motion of the electron is governed by the equation

$$e \left( \frac{\vec{E} + \frac{\vec{v}}{c} \wedge \vec{H}}{c} \right) - m\ddot{a} = 0$$

where $e$ is the charge of the electron, $\vec{v}$ is its velocity, $\ddot{a}$ its acceleration and $m$ its mass; as usual $\vec{E}$ and $\vec{H}$ are the electric and magnetic fields respectively. It is easily verified that this classical law is not Lorentz-covariant and thus has to be modified. Let us denote $R(\vec{v}/c, \ddot{a}, \vec{E}, \vec{H}) = 0$ be the relativistic equation which is to replace $K(\vec{v}/c, \ddot{a}, \vec{E}, \vec{H}) = 0$.

Consider the electron at the time $t$ when its velocity is $\vec{v}$, and choose an inertial frame $I'$ which moves with the same velocity $\vec{v}$ with respect to $I$. In $I'$ the electron is instantaneously at rest. If we denote by $\vec{v}', \ddot{a}', \vec{E}'$ and $\vec{H}'$ the relativistic law of the conservation of momentum

$$\sum m_i \vec{v}_i / \sqrt{1 - v_i^2/c^2} = 0$$

is a good illustration of equation $(3')$. Letting $(v_1, \ldots, v_n)$ tend to zero serves no purpose in this case; since, if we start from an arbitrary function $f(v_i)$ and consider $\sum f(v_i) \vec{v}_i$, then: as $(v_1, \ldots, v_n) \to (0, \ldots, 0)$, $f(v_i) \vec{v}_i \to 0 = \text{value of } \sum m_i \vec{v}_i$ for $v_1 = v_2 = \ldots = v_n = 0$. This does not help us towards determining $f$.

Denote by $\mu_i$, the relativistic mass $m_i / \sqrt{1 - v_i^2/c^2}$. The relativistic momentum $\sum \mu_i \vec{v}_i$ and the classical momentum $\sum m_i \vec{v}_i$ are the same functions of the masses and of the velocities. Lewis and Tolman (tacitly) assumed that $\mu_i \to m_i$ as $v_i \to 0$. (Cf. Lewis [1908] and Lewis and Tolman [1909].)
the quantities in \( I' \) which correspond to \( \vec{v}, \vec{a}, \vec{E} \) and \( \vec{H} \) respectively, then \( \vec{v}' = 0 \). By the Relativity Principle, \( i.e. \) by the equivalence (1') above:

\[ R(\vec{v}/c, \vec{a}, \vec{E}, \vec{H}) = 0 \iff R(\vec{v}'/c, \vec{a}', \vec{E}', \vec{H}') = 0 \]

We have already explained that, for vanishing velocities, a relativistic equation must coincide with its classical counterpart. (This is a direct consequence of the requirement (2) above that the relativistic law should yield the classical one as a limiting case).

Therefore:

\[ R(0, \vec{a}', \vec{E}', \vec{H}') = 0 \iff K(0, \vec{a}', \vec{E}', \vec{H}') = 0. \]

But:

\[ K(0, \vec{a}', \vec{E}', \vec{H}') = e\left( \vec{E}' + \frac{\vec{a}'}{c} \wedge \vec{H}' \right) - m' \vec{a} = e\vec{E}' - m\vec{a}' \]

By (4), (5) and (6) it follows that:

\[ R(\vec{v}/c, \vec{a}, \vec{E}, \vec{H}) = 0 \iff [e\vec{E}' - m\vec{a}' = 0] \]

Using known transformation equations, we can express \( \vec{E}' \) and \( \vec{a}' \) in terms of \( \vec{E}, \vec{H}, \vec{v} \) and \( \vec{a} \), and thus obtain:

\[ [e\vec{E}' - m\vec{a}' = 0] \iff \left[ \frac{d}{dt} \left( \frac{m\vec{v}}{\sqrt{1 - \vec{v}^2/c^2}} \right) = e\left( \vec{E} + \frac{\vec{v}}{c} \wedge \vec{H} \right) \right] \]

Thus \( \frac{d}{dt} \left( \frac{m\vec{v}}{\sqrt{1 - \vec{v}^2/c^2}} \right) = e\left( \vec{E} + \frac{\vec{v}}{c} \wedge \vec{H} \right) \) is the relativistic equation of motion for an electron moving in an electromagnetic field. Planck took the Lorentz force \( e\left( \vec{E} + \frac{\vec{v}}{c} \wedge \vec{H} \right) \) to be the very paradigm of force and generalised the last equation as follows:

\[ \frac{d}{dt} \left( \frac{mv}{\sqrt{1 - v^2/c^2}} \right) = \text{force} = \vec{f} \]

Equation (9) is the relativistic law which replaced Newton's second law of motion. By using (9), the expression of the relativistic kinetic energy \( k(v) \) can be determined. It is:

\[ k(v) = \int \vec{f} \cdot \vec{v} \, dt = mc^2 \left( \frac{I}{\sqrt{1 - v^2/c^2}} - I \right) \]

Thus, by using Einstein's heuristic together with the simple device of choosing an inertial frame \( I' \) in which the electron is instantaneously at rest, Planck modified the law \( \vec{f} = m\vec{a} \) which had been considered an unshakeable convention of theoretical physics.
Let us now examine how Einstein, by using the same heuristic, arrived at his famous equation relating mass and energy:

\[ E = mc^2/\sqrt{1-v^2/c^2} \]

Einstein assumed that there had to be a relativistic law corresponding to the classical law of conservation of energy. By the Relativity Principle this new conservation law must hold in all inertial frames. Einstein considered an inertial frame \( I \) in which a stationary body \( B \) emits light and thereby loses a certain amount of energy \( Q \). Since energy is conserved:

\[ (ix) \quad E_1 = E_2 + Q \]

where \( E_1 \) is the total energy of \( B \) before radiation and \( E_2 \) its total energy after radiation.

Einstein considers a second inertial frame \( I' \) moving with velocity \(-\vec{v}\) with respect to \( I \). The body \( B \), which is at rest in \( I \), moves with velocity \( \vec{v} \) relatively to \( I' \). By the Relativity Principle, energy must be conserved both in \( I \) and in \( I' \). Hence:

\[ (i2) \quad E'_1 = E'_2 + Q' \]

where \( E'_1, E'_2 \) and \( Q' \) are the quantities in \( I' \) which correspond to \( E_1, E_2 \) and \( Q \) respectively.

Subtracting \((ix)\) from \((i2)\):

\[ (i3) \quad E'_1 - E_1 = (E'_2 - E_2) + (Q' - Q) \]

Einstein interpreted \((E'_1 - E_1)\) as follows. \( E_1 \) is the energy of \( B \) in its rest-frame \( I \) (before light is emitted). \( E'_1 \) is the energy of the same body \( B \) as seen from the moving frame \( I' \). In \( I' \) the body \( B \) moves with velocity \( \vec{v} \) but in \( I \) it is at rest. Hence \((E'_1 - E_1)\) is the energy which accrues to the body \( B \) solely in virtue of its motion; i.e. \((E'_1 - E_1)\) is the kinetic energy of \( B \) to within an additive constant. By \((i0)\)

\[ (i4) \quad E'_1 - E_1 = (Mc^2/\sqrt{1-v^2/c^2}) - Mc^2 + h, \]

where \( h \) is a constant and \( M \) is the rest mass of \( B \) before radiation. Similarly:

\[ (i5) \quad E'_2 - E_2 = (mc^2/\sqrt{1-v^2/c^2}) - mc^2 + h, \]

where \( m \) is the rest mass of \( B \) after radiation.

Note that \( Q \) is the energy lost through radiation in \( I \). Einstein was in possession both of Maxwell's equations and of the transformation laws for the field. From these he calculated that:

\[ (i6) \quad Q' = Q/\sqrt{1-v^2/c^2} \]

Substituting from \((i4)-(i6)\) into \((i3)\):

\[ (i7) \quad (M - m)c^2 = Q; \ i.e. \ c^2\Delta M = Q, \] where \( \Delta M = M - m. \]
Thus the rest mass of $B$ has decreased by the amount $\Delta M = Q/c^2$. A supposedly immutable substance, namely the rest mass $\Delta M$, can vanish and thereby give rise to the equivalent amount of energy $c^2\Delta M$. This revolutionary result is a consequence of the Relativity Principle applied to the law of conservation of energy. True, Lorentz had shown that the electron possesses an electromagnetic inertia which varies with the speed. He had also found that the electromagnetic rest mass is a multiple of the electrostatic energy. But neither in Lorentz's [1904] nor in his [1909] is there any indication that the rest mass is a variable quantity!

The extraordinary power of the Relativity Principle is further displayed by the following fact. Given the Relativity Principle, the law of conservation of energy both implies and is implied by the law of conservation of momentum; where the momentum of a particle of rest mass $m$ and velocity $\vec{v}$ is the vector $m\vec{v}/\sqrt{1-v^2/c^2}$, and the energy of the particle is $mc^2/\sqrt{1-v^2/c^2}$.

These examples show that the revolutionary relativistic laws were not arrived at in a sudden flash of intuition or through some kind of mystical insight. The new laws were mathematically derived from assumptions like the Relativity Principle which seem so 'formal' and innocuous as to be devoid of empirical content.

3 Einstein's Programme Supersedes Lorentz's.

Einstein invented not a theory but a research programme with an immensely powerful heuristic. But research programmes are ultimately judged on their empirical rather than on their heuristic power. No matter how fruitful its heuristic guidelines for the construction of new theories are, the programme will not be successful if these theories are not empirically corroborated. In my view Einstein's relativity programme superseded Lorentz's in the empirical sense in 1915 with its explanation of the precession of Mercury's perihelion. This explanation requires the general theory. There were of course special relativistic results (e.g. $E = mc^2$) which could in principle be tested, but even by 1915 such tests seemed to be only a remote possibility.

My claim that Einstein's programme superseded Lorentz's with the explanation of the perihelion of Mercury raises two difficulties. First, since I wish to claim this as a success for the whole relativistic programme, I have to establish a continuity between the special and the general theories. Secondly, since the behaviour of Mercury was well-known, I shall have to show, in line with my definition of empirical support, that the Mercury

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1 The rest mass of an electron is a function of the charge and of the radius. Lorentz took both the charge and the radius to be constant.

prediction was an unexpected consequence of the general theory. It may seem that this preoccupation with Mercury's perihelion is unnecessary in view of the fact that General Relativity made predictions which were novel in the temporal sense, e.g. the bending of light rays. However the Mercury prediction, in contradistinction to the bending of the light rays, was both in close agreement with observation and also depended on the full field equations.  

3.1 The Continuity between the Special and General Theories of Relativity.

The explanation of gravitation by General Relativity appears to involve a major shift in the methods used by Einstein. But I propose to show that after 1908 Einstein merely strengthened the methods already used in Special Relativity; the only new addition to the programme was in the heuristic function of the Principle of Equivalence (i.e. of the equality of gravitational and inertial masses).

One might think that the General Theory constitutes simply a generalisation to the case of accelerating frames. But this is only a small part of the

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1 Cf. Adler, Bazin and Schiffer [1965], p. 194.
2 My views are in sharp contrast with those of Lanczos. In his [1972] Lanczos distinguishes between a young Einstein who was supposedly a strict empiricist and an older Einstein who indulged in speculation. Lanczos writes: 'Einstein was at that time still a convinced empiricist who would not have dared to argue that perhaps nature is based on rational and universal principles, which cannot be found by experimentation but only by inspired and imaginative speculation;... In fact Einstein in the beginning of his career distrusted mathematics and considered the mathematical formulation of a physical event as the mere form in which a phenomenon is described, which does not touch on its substance. ... To think in experimental terms was Einstein's basic attitude in the prime of his career in marked contrast to his ideas in the last phase of his life when in search for the ultimate unification of nature he often fell victim to mere formalism.' Against this view I maintain that the roots both of General Relativity and of the various unified field theories go back to 1905. Had Einstein really been a strict empiricist and a follower of Mach who saw the task of theoretical physics purely in the more or less accurate description of experimental observations', then why did he insist that the Relativity Principle should hold not only at the observational but also at the highest theoretical level? An empiricist would have been perfectly satisfied with a solution such as Lorentz's which explains why no experiment could detect absolute motion. To insist that the Relativity Principle obtains at the level of laws presupposes a realistic interpretation of these laws beyond their functions as mere tools for the description of experimental observations. For Einstein observational symmetries are nothing but the mere reflection of a deeper symmetry at the ontological level. The empirical success of Einstein's earlier work and the empirical failure of Einstein's later work (unified field theory) cannot be explained by Lanczos's claim that Einstein degenerated from empiricism to speculation. High level speculation paid off handsomely in the case of General Relativity Theory; the programme achieved stunning empirical success in 1915 and in 1919. It is hardly surprising that Einstein 'overestimated' the speculative (or so-called aprioristic) method which enabled him to construct the General Theory and tried to apply the same methods to the construction of unified field theories. He was unlucky, but the lack of—empirical—success cannot be attributed, as Lanczos claims, to a change in methodological attitude. The same attitude helped Einstein in the case of the General Theory, but it failed him later in life. Einstein proposed and Nature disposed.

8 For a more detailed account see Zahar [1973].
answer to the continuity problem. After 1905 Einstein's main problem was to devise a Relativistic Theory of Gravitation. He found it impossible to reconcile his equation:

\[ E = mc^2 = m_0 c^2 \sqrt{1 - v^2/c^2} \]

with the Principle of Equivalence. This Principle implies that, since the gravitational and the inertial masses of any material body are equal, all bodies fall with the same acceleration in the same gravitational field \( \vec{g} \). In classical physics this result follows from Newton's second law

\[ \vec{f} = \frac{d}{dt} (m \vec{v}) = m \vec{a} \]

and from the equality of the inertial and gravitational masses. The masses cancel out on both sides of \( m \vec{a} = m_0 \vec{g} \), leaving \( \vec{a} = \vec{g} \). Einstein assumed that in Special Relativity the corresponding equation would be some relation of the form:

(inertial mass) \( \vec{g} \) = rate of change of momentum;

\[ m_0 \frac{d}{dt} (\vec{v}) = \frac{d}{dt} (m_0 \sqrt{1 - v^2/c^2}) \]

In view of the equation:

\[ E = \text{energy} = m_0 c^2 \sqrt{1 - v^2/c^2} \]

the rest mass \( m_0 \) may vary with the time; in other words we may have \( \frac{dm_0}{dt} \neq 0 \).

Dividing through by \( m_0 \), we obtain on the right-hand side of:

\[ m_0 \frac{\vec{v}}{\sqrt{1 - v^2/c^2}} = \frac{d}{dt} \left( m_0 \frac{\vec{v}}{\sqrt{1 - v^2/c^2}} \right) \]

a term

\[ \frac{I}{m_0} \frac{dm_0}{dt} \frac{\vec{v}}{\sqrt{1 - v^2/c^2}} \]

which may be different from zero.

Thus the motion of a material body under the effect of the gravitational field will generally depend on its rest mass \( m_0 \). The Principle of Equivalence is violated.

Rather than give up the Principle of Equivalence, Einstein gave up the hope of giving a special relativistic theory of gravitation. He changed his tactics and launched a two-pronged attack on the problem of gravitation:

(i) Einstein only now remembered his Machian scruples concerning the

\[ \text{Warning to the reader: In this section } m_0 \text{ denotes the rest mass and } m \text{ the inertial mass i.e. } m = m_0 \sqrt{1 - v^2/c^2}. \]
so-called myth of the inertial frame. If one purges Newtonian Theory of the Absolute Space Hypothesis, one finds that Special Relativity is as 'absolute' as classical mechanics; both theories postulate a set of privileged inertial frames. From a Machian viewpoint this assumption is unacceptable.\(^1\) Einstein decided to treat all coordinate systems on a par and to impose a condition of general covariance on all physical laws. This condition, which is a strengthening of the requirement of Lorentz covariance (General Covariance of course implies Lorentz covariance), is an important element of continuity between the special and the general theories of Relativity.

(2) Einstein decided to go back to his original heuristic, in particular to the heuristic device which consists in scrutinizing known empirical results and in isolating certain features in them which are 'unsatisfactorily' explained by current theories.\(^2\) As I have shown, Einstein analysed the well-known 'fact' that all bodies fall with the same acceleration in the same way that he had analysed the result of the induction experiment.\(^3\) Let me recall that Einstein reached the conclusion that all gravitational fields can be regarded as caused by a local acceleration of the frame of reference. It is thus obvious why the introduction of accelerated frames holds out the promise of solving the problem of gravitation.

The two prongs of the attack can now be seen to converge to the same result. We have here a second element of continuity between the Special and the General Theories: each involves an application of Prescription II. However, Einstein now faced a new difficulty. In Newtonian mechanics there exist so-called inertial fields which arise if one chooses an accelerating frame of reference. These inertial fields are artificial in that they can be transformed away by one global change of coordinates, namely by a change which refers everything back to an inertial frame. This is not the case with 'real' gravitational fields; for example the field at a point near the earth's surface can be transformed away only at the cost of piling it up at the antipode. How could one ever hope to be able to deal with two such dissimilar fields in the same way? By a tremendous stroke of genius Einstein turned this seeming impasse into a powerful heuristic device. Consider an accelerating frame of reference \(S\) in which there exists both a 'real' gravitational field \(\vec{g}\) and an inertial field \(\vec{i}\). Every particle \(P\) is acted upon by a force \(\vec{F} = m\vec{i} + mg\). Since \(m = \text{inertial mass} = \text{gravitational mass} = m\) (say), it follows that \(\vec{F} = mi + mg = m(\vec{i} + g) = m\vec{G}\), where \(\vec{G} = \vec{i} + g\).

\(^1\) This may have been a reason for Mach's rejection of Special Relativity. (Cf. Mach [1913], Preface.)
\(^2\) This is Prescription II; cf. above, p. 225. The heuristic of Special Relativity is only part of Einstein's general heuristic as expressed in Prescriptions I and II.
\(^3\) Cf. above, pp. 225–7.
Thus \( i \) and \( g \) always occur indissolubly fused into one global field \( G \); Einstein typically refuses to consider this 'fact' a mere accident and argues that there exists only one total field \( G \) which a new theory of gravitation would have to explain. In view of \( G = i + g \), \( i \) is now a special case of an Einsteinian gravitational field; \( G \) reduces to \( i \) if all matter in the universe is either annihilated or removed to an infinitely distant point. The field \( i \) can be globally transformed away through a single change of coordinates; in other words \( i \) is a reducible gravitational field. Reducible fields offer the advantage that they can be generated at will through an arbitrary acceleration of the frame of reference. One can heuristically exploit the Principle of Equivalence by generating a reducible field \( i \) through an acceleration of the frame, by studying the properties of \( i \) and finally by extending these properties to non-reducible fields \( G \).

But how is this generalisation to be carried out? This is where the absolute differential calculus proved extremely helpful. The method is as follows: start from an inertial frame \( S(x^0, x^1, x^2, x^3) \) in which Special Relativity applies in its usual form; hence:

\[
ds^2 = (dx^0)^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2 = g_{mn} dx^m dx^n
\]

(we take the velocity of light to be unity) where:

\[
(g_{mn}) = \begin{pmatrix} I & -I & \emptyset \\
-I & -I & \emptyset \\
\emptyset & \emptyset & -I \end{pmatrix}
\]

Accelerate the frame \( \tilde{S} \); in other words carry out a non-linear transformation of coordinates. In the accelerating frame \( S(x^0, x^1, x^2, x^3) \): 

\[
ds^2 = g_{mn} dx^m dx^n, \quad (g_{mn}) \text{ varies from one point to the next. The matrix } (g_{mn}) \text{ is not arbitrary, since it satisfies the following condition which we denote by } K: \text{ through a global transformation of coordinates, namely through the transformation } (x^n) \rightarrow (\tilde{x}^n), \text{ (}g_{mn}\text{) is reducible to the constant matrix (}g_{mn}\text{). There exists in } S \text{ a reducible gravitational field generated by the acceleration of the frame } S \text{ with respect to } \tilde{S}. \text{ We study the behaviour of this field in } S, \text{ then we generalise our results by abstracting from, } i.e. \text{ by lifting, the condition } K. \text{ In doing this we have to study the same processes in two different frames } \tilde{S} \text{ and } S; \text{ so we need a method for translating the results obtained in } \tilde{S} \text{ into results applying in } S; \text{ the absolute differential calculus provides such a method.}

Using such methods, Einstein determined the path of a particle moving freely in a gravitational field. In the frame \( \tilde{S} \) the trajectory of the particle is a straight line whose equations are:

\[
d^2x^i/(dx^0)^2 = 0 (i = 0, 1, 2, 3). \quad \text{Since}
\]
our aim is to 'look' at the same particle from the accelerated frame \( S \), it proves useful to characterise the path in an invariant way, \textit{i.e.} in a way which does not depend on any particular coordinate system. Using the intrinsic parameter \( s \) instead of \( \tilde{x}^i \) (\textit{i.e.} \( t \)), we obtain:

\[(1) \quad \frac{d^2 \tilde{x}^i}{ds^2} = 0 \]

These equations are easily seen to give the integral \( \int ds \) a stationary value. In other words:

\[(2) \quad \delta \left( \int ds \right) = 0 \quad \text{when} \quad \frac{d^2 \tilde{x}^i}{ds^2} = 0 \quad \text{in} \quad \tilde{S}. \]

In view of the fact that \( ds \) is an invariant, this last equation means that, in \( \tilde{S} \) and in any other frame of reference, the trajectory of the particle is a geodesic. From the absolute differential calculus we know that a geodesic in \( S \) satisfies the following equations:

\[(3) \quad \frac{d^2 \tilde{x}^i}{ds^2} + \left\{ \begin{array}{c} \frac{1}{m} \\ \frac{n}{n} \end{array} \right\} \frac{dx^m}{ds} \cdot \frac{dx^n}{ds} = 0 \]

where

\[ \left\{ \begin{array}{c} i \\ mn \end{array} \right\} = g^{iu} [mn, u] = \frac{1}{2} g^{iu} \left( \frac{\partial g_{mu}}{\partial x^m} + \frac{\partial g_{nu}}{\partial x^m} - \frac{\partial g_{mn}}{\partial x^u} \right) \]

Comparing (1) and (3), we conclude that in the accelerating frame \( S \) the path of the particles is no longer a 'straight' line in the ordinary sense of the word: the quantity which deflects the particle from a straight trajectory is the quantity represented by the Christoffel symbol \( \left\{ \begin{array}{c} i \\ mn \end{array} \right\} \). Since \( \left\{ \begin{array}{c} i \\ mn \end{array} \right\} \) is a function of the \( g_{ij} \)'s and since we ascribe the deviation from a straight path to the action of a gravitational field, the latter is represented by the \( g_{ij} \)'s or rather by the partial derivatives \( \partial g_{ij}/\partial x^m \); note that the coefficients \( \left\{ \begin{array}{c} i \\ mn \end{array} \right\} \) vanish if the \( g_{ij} \)'s are constant. The \( g_{ij} \)'s can therefore be looked upon as the gravitational potentials.

So far we have implicitly assumed that the gravitational field is reducible, \textit{i.e.} 'inertial' in the old terminology. We now generalise our results by extending them to the case where there need not exist a global transformation which makes all the \( g_{ij} \)'s constant. Hence, even if the field is irreducible, the path of a free particle is still a geodesic and the metric tensor \( (g_{ij}) \) still represents the gravitational potential.

This method also enabled Einstein to determine the effect of gravitation on other physical phenomena. He wrote the laws governing these phenomena in a generally covariant form, which generally involves the
g_{ij}'s; he then abstracted from the condition $K$ which was originally imposed on the metric tensor. The presence of gravitation manifests itself through the irreducible $g_{ij}'s$ which occur in the new law.

There remained for Einstein the problem of finding the field equations which are satisfied by the $g_{ij}$'s. His attack on the problem was again two-pronged:

(1) The new law would have to yield Poisson's equation $\nabla^2 \phi = k \rho$ as a limiting case. This requirement is identical with the one made in the case of Special Relativistic laws. Because of Poisson's equation Einstein expected his own law to consist of second-order partial differential equations linear in the second derivatives $\partial^2 g_{ij}/\partial x^m \partial x^n$.

(2) We have seen that $g_{ij}'s$ have a dual function: on the one hand they represent the physical gravitational potentials and on the other they are coefficients in the expression of $ds^2$; this second function is a geometrical one. It can be said that Einstein geometrised gravitation or alternatively that he physicalised geometry. Since the field equations describe a geometrical state of affairs, they ought to be independent of any particular frame of reference; i.e. they ought to be generally covariant.

Thus Einstein and Grossmann\(^8\) started from the rather vague assumption that the gravitational field is a geometrical entity ('geometry' is to be understood as synonymous with 'kinematics' or 'space-time geometry'). It had of course long been known that gravity was caused by the presence of massive bodies. Also, by the Special Relativistic equation $E = mc^2$, inertial mass and energy are interchangeable. Finally, by the Principle of Equivalence, inertial mass and gravitational mass are identical (wesensgleich). Putting all these assumptions together, Einstein guessed that gravitation is a geometrical phenomena related to the energy content of space. Grossmann expected the field equations to be of the form $A = B$, where $A$ and $B$ respectively represent the geometry and the energy content of space. He took for granted that the geometry in question is Riemannian and not some more general, i.e. less structured, geometry. In the case of free space, $B$ vanishes, so we are left with the equation $A = 0$. In order to determine $A$, Grossmann considered the tensor $B_{\lambda \mu \nu}$, which Riemann and Christoffel had shown to be essentially relevant to the geometrical properties of space. Grossmann knew that the equations $B_{\lambda \mu \nu} = 0$ imply that the space is flat and hence that the field can be globally transformed away. He also knew that the gravitational field is generally irreducible. So Grossmann weakened the relations $B_{\lambda \mu \nu} = 0$ by using a standard mathematical technique,

\(^1\) For a more detailed account see Zahar [1973].
\(^8\) Einstein and Grossmann [1913].
namely the contraction of a contravariant index with a covariant one. Note that there exists essentially one way of contracting two such indices in $B^\nu_{\lambda \mu}$, because $- B^\nu_{\lambda \mu} = B^\nu_{\lambda \mu}$ and $B^\nu_{\nu \lambda \mu}$ vanishes identically. By using this type of reasoning, Grossmann finally obtained the equations $B^\nu_{\lambda \mu} = R^\nu_{\lambda \mu} = 0$, which are accepted today as the correct field equations for free space. Thus, through giving a precise mathematical formulation of his initial assumption that gravitation is a geometrical phenomenon linked to the energy content of space, Grossman had obtained the much stronger proposition: $R^\nu_{\lambda \mu} = 0$. In fact, as I have already said above, Grossmann's initial assumption was so weak as not even to imply that the geometry to be used is Riemannian. The reason for resorting to Riemannian geometry was its availability at the time as a fully developed mathematical system. This illustrates the point made earlier about the first heuristic role of mathematics in physical discovery.¹

The solution $R^\nu_{\lambda \mu} = 0$ for free space was rejected by its authors for two reasons, both of which turned out to be unfounded. First, Grossmann believed that $R^\nu_{\lambda \mu} = 0$ would not yield the classical equation $\phi^2 = 0$ as a limiting case for weak static fields. This was a relatively simple mathematical error. Secondly, both Einstein and Grossmann thought that, given the appropriate boundary conditions, the ten equations $R^\nu_{\lambda \mu} = 0$ would uniquely determine the ten functions $g^\nu_{\lambda \mu}$. This means that we are not at liberty to choose an arbitrary frame of reference because the functions $g^\nu_{\lambda \mu}$ are generally altered by a change of coordinates. Thus it seems that the Relativity Principle is violated. Hilbert saved the situation by showing that the equations $R^\nu_{\lambda \mu} = 0$ are not all independent; the left hand sides satisfy four identities, which give the exact degree of arbitrariness necessary for the free choice of a frame of reference (four identities corresponding to four coordinates).²

3.2 The Successful Explanation of the Perihelion of Mercury and its Role in the Further Development of the General Theory.

In 1915 Einstein went back to the equations $R^\nu_{\lambda \mu} = 0$ for free space; for the case where non-gravitational energy in the form of a symmetric tensor $T^\nu_{\lambda \mu}$ is present, Einstein found it natural to generalise the equations $R^\nu_{\lambda \mu} = 0$ to $R^\nu_{\lambda \mu} = kT^\nu_{\lambda \mu}$. This generalisation turned out to be untenable. However, using only the equations $R^\nu_{\lambda \mu} = 0$ for the field created by the sun, Einstein explained the precession of Mercury's perihelion. He published this result on the 22nd November 1915. This explanation of a well-known fact was tremendously important for the following reasons: the predicted fact is

² Cf. Einstein [1915e].
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completely novel in the sense that I have previously explained1; that is Einstein did not use the known behaviour of Mercury's perihelion in constructing his theory. In fact this empirical prediction is all the more dramatic because it flows from a hypothesis which is so speculative, so 'metaphysical', that one may wonder whether it belongs to physics or to pure mathematics. Thus, through explaining the 'anomalous' motion of Mercury's perihelion, Relativity Theory superseded its rivals from a strictly empirical point of view.2 This empirical success also proved crucial for the further development of the Relativity Programme.

Einstein realised that the equations \( R_{ij} = kT_{ij} \) are untenable because the right hand side is divergenceless whereas the left hand side is not. Other things being equal, it would have been natural for Einstein to abandon this whole approach, i.e. to reject both the general equations \( R_{ij} = kT_{ij} \) and the special case \( R_{ij} = 0 \). The fact that the equations \( R_{ij} = 0 \) had enabled him to explain the motion of Mercury convinced him that the fault lay not with his overall approach but with the method of generalising the equations \( R_{ij} = 0 \). In other words Einstein kept the equations \( R_{ij} = 0 \) for free space and looked for a new method of generalising these relations.3

At this critical stage Einstein was helped by the mathematical machinery of his system and by the Special relativistic law about the interchangeability of inertial mass and energy. By using variational methods, he extracted from the relations \( R_{ij} = 0 \) a matrix \( t_{ij} \) which obeys a formal conservation law: \( t_{ij} = 0 \), i.e. \( \partial t_{ij}/\partial x^j = 0 \). However, \( t_{ij} \) is not a tensor and hence appears not to be susceptible of any physical interpretation. It seemed as if \( t_{ij} \) ought to be treated as a mere mathematical entity which may be used for purposes of convenience but is otherwise devoid of physical meaning. Realising that he had reached an impasse with the equations \( R_{ij} = kT_{ij} \), Einstein insisted against all odds on interpreting \( t_{ij} \) as a gravitational energy matrix; but energy represents inertial mass and hence also gravitational mass (Principle of Equivalence); thus we reach the surprising physical result that gravitational energy acts as one source of the gravitational field. In passing from \( R_{ij} = 0 \) to \( R_{ij} = kT_{ij} \) Einstein had 'mistakenly' supposed that he was going from a case in which energy was totally absent to a case in which it was not. He had forgotten that even when \( R_{ij} = 0 \), gravitational energy may be present. His solution consisted in

2 Lorentz in his [1900] had produced a theory of gravitation which, however, explained only a small fraction (one tenth) of the residual angle of precession of Mercury's perihelion.
3 Had the General Theory's prediction of Mercury's behaviour not been novel, e.g. had Einstein 'adjusted parameters' in order to obtain the correct experimental results, he would surely not have had such confidence in the equations \( R_{ij} = 0 \), in the face of the breakdown of the more general form \( R_{ij} = kT_{ij} \).
rewriting the equation $R_{ij} = 0$ so as to bring out this dependence of the field on its own energy and then generalising by adding the non-gravitational energy $kT_i$ to the gravitational energy $t_i$. In other words Einstein rewrote $R_{ij} = 0$ in the form $F(t_i, t) = 0$, where: $t = t_i$; he then generalised these equations by adding $kT_i$ to $t_i$ and $kT$ to $t$. Einstein obtained

$$F(t_i + kT, t + kT) = 0$$

which turned out to be equivalent to the currently accepted field equations:

$$R_{ij} = -k \left( T_{ij} - \frac{T}{2} g_{ij} T \right)$$

This illustrates what I called the second role of mathematics in physical discovery.¹

Let me end by reviewing the assumptions and methods which are common to the Special and to the General Theories of Relativity. These common assumptions and methods are the bridges connecting the two stages of the programme. Let me make the obvious point that in both theories Special Relativity holds locally, i.e. it holds in infinitely small domains about each point. Einstein starts the two theories by analysing two well-known ‘facts’ from the same point of view: he analyses the result of the induction experiment and the ‘fact’ that all bodies fall with the same acceleration. Common to both theories is the law concerning the interchangeability of mass and energy. The equation $E = mc^2$ was a dramatic new result implied by Special Relativity; moreover it was precisely this result which led Einstein to transcend the Special Theory and resort to General Relativity as a framework which would embody gravitation. We have also seen that the law about the interchangeability of mass and energy played a crucial role in enabling Einstein to modify his field equations $R_{ij} = +kT_{ij}$. Both the Special and the General Theories make use of the Covariance Principle: Lorentz-covariance in the case of Special Relativity and general covariance in the case of General Relativity. In both stages of the programme scientists exploited the assumption that classical theories ought to be limiting cases of the new relativistic laws. In Special Relativity the law of inertia, Maxwell’s equations, Newton’s second law and the laws of the conservation of energy and momentum were used in order to determine new and different Lorentz-covariant equations. In General Relativity Poisson’s equation was exploited: since Poisson’s equation was to be a limiting case of the law of gravitation, the latter was expected to consist of a system of second order partial differential equations which would be linear in the second order derivatives. The only essentially new method peculiar

to General Relativity was the heuristic use which was made of the Principle of Equivalence and which has no analogue in the Special Theory.

Thus the question asked in the title of this paper, 'Why did Einstein’s Programme supersede Lorentz’s?', has now been answered. Already in 1905 the Relativity programme proved heuristically superior to its classical rival; at a time when the notion of a quasi-material ether was becoming heuristically barren, Einstein provided a powerful new tool for the construction of Lorentz-covariant laws yielding the corresponding classical theories as limiting cases. However, heuristic power gives a measure only of intellectual achievement and not of scientific progress. After all, science is empirical. Special Relativity by itself did not empirically supersede Lorentz’s programme. Bucherer’s experiment confirmed both Lorentz’s and Einstein’s hypotheses and Kaufmann’s experiment disconfirmed them both. Indeed, before the advent of General Relativity the scientific community (e.g. Planck, Poincaré, Bucherer, Kaufmann and Ritz) spoke of the Lorentz-Einstein theory and contrasted it with the more classical theories of Abraham and Ritz: they regarded the theories of Lorentz and Einstein as observationally equivalent.

It was only when Einstein’s programme yielded General Relativity that it superseded Lorentz’s empirically by successfully explaining the ‘anomalous’ precession of Mercury’s perihelion. This explanation constitutes empirical progress because, according to my amended definition of ‘novel fact’, the behaviour of Mercury, although well-known, is nonetheless a novel fact predicted by General Relativity.

This new (General Relativistic) phase in which empirical success was achieved was, as it happened, more speculative than the previous (Special Relativistic) phase. In this later phase Einstein strengthened his earlier heuristic and thus arrived at a covariant theory of gravitation (he had been unable to accommodate gravitation within the confines of Special Relativity).

Nevertheless there is a strong continuity between the Special Theory and the General Theory. The latter can be regarded as a more powerful realisation of essentially the same outlook and the same heuristic which had previously led Einstein to Special Relativity. During the earlier phase the deep differences between Lorentz and Einstein remained primarily heuristic (and of course metaphysical) ones. It was only with the development of the General Theory that the underlying conflict between the two programmes was reflected at the empirical level: with regard to Mercury’s perihelion, the bending of the light rays and the red shift, General Relativity made predictions which were never matched by Lorentz’s (or Ritz’s) theories.

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4 Cf. Einstein [1915c]. 5 Cf. Einstein [1912].
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