
**DISCUSSION**

Y. P. Chang

It is gratifying to see in this paper an effort to provide more basic insight concerning the heat transfer in nucleate boiling of saturated liquids from a horizontal surface facing upward. However, the physical interpretation of the visual phenomenon upon which the analysis is based and the result obtained therefrom are worthy of further discussion.

From Figs. 2(d), 2(f), 2(h), and 2(j), it seems difficult to substantiate the postulation that there exist continuous vapor columns near the heating surface. In Fig. 2(j) bubbles are clearly formed at the surface, and large vapor slugs appear at a distance from the heater. The turbulent vapor-liquid mixture above the heater, as shown in Figs. 2(2), 2(f), and 2(h), prevents the observer from identifying what event would have taken place on the central part of the heating surface. In fact, it is this very event which controls the heat transfer. Moreover, the large vapor slugs that appear in Figs. 2(d) and 2(j) cannot be interpreted as continuous vapor columns. Vapor slugs move bodily, taking in vapor by agglomerating other bubbles, but if continuous column the vapor flows as a stream (Fig. 4), requiring a source of vapor supply at the heater.

It must also be noted that all the slugs that appear in Fig. 2 are not in a steady state but change form, size, and position with the time, while the postulated continuous vapor columns are steady as shown diagrammatically in Figs. 1 and 4. Large slugs and bubbles are inherently unstable. They will be distorted and broken up into a smaller number of various sizes. Some of them will be entrained by the turbulent liquid forming a phenomenon more or less similar to the occurrence of “white water” in rapid streams or spillways, while the rest of them will again combine with themselves and/or with those that are newly formed (vapor is generated continuously). If this instability phenomenon occurs near the heater, the heating surface will be splashed by the white water and the heat-transfer rate would approach its maximum. When this phenomenon occurs at some distance from the heating surface, most likely in vigorous boiling as shown in Fig. 2, the heat transfer should be little affected, for the liquid may not be capable of carrying the entrained vapor down to the heating surface. It is this white water which covers up the actual event occurring at the heater. If the heater is nonopaque and photographs can be taken from below, it would be possible to discover that the heating surface would be mostly covered by vapor bubbles instead of continuous vapor columns or jets. These bubbles, however, become disintegrated as soon as they have completed their formation, followed by agglomeration and redissolution.

On the basis of the continuous-column model, equation (21) was obtained for $A_{\gamma}/A$. If the pressure is not too high, the term $\sqrt{\gamma}p_{\gamma}$ in equation (21) can be safely neglected, yielding $\gamma = A_{\gamma}/A \lesssim 1$. This indicates that the heating surface would be practically dry. In other words, there would be practically no solid-liquid contact at the condition of peak heat flux. Were this the case, how could the high heat flux be transferred and where would the tremendous amount of vapor come from?

Finally, the writer is in agreement with the authors’ notation that there are different flow configurations in the entire nucleate boiling process, but not with that regarding the postulation of continuous vapor columns. To facilitate the analysis, the entire range of nucleate boiling may be divided into two regimes: feeble and vigorous boiling. In the former regime bubbles rise in a more or less regular manner. In the latter regime bubbles become disintegrated, followed by agglomeration and redissolution, when they have detached from the heater. The higher the superheat (the main driving force), the earlier this disintegration-agglomeration process will occur above the heater (i.e., at a shorter distance from the heater). The maximum heat-transfer rate would be reached if this process occurs in the proximity of the heater. This model seems not only to be consistent with the authors’ photographic pictures but also to facilitate the analysis of the heat transfer because attention can be focused only to the event occurring immediately next to the heater. It would be of interest to know the authors’ comment on this model.

N. Zuber

This paper is an important addition to the literature on boiling heat transfer and the authors should be warmly congratulated for their effort and contribution. The writer pointed out in [6] that nucleate boiling is characterized by two flow regimes, i.e., the regime of isolated bubbles and the regime of continuous vapor columns. As the authors note the knowledge of the extent of each regime is necessary in the development of any analytical model for nucleate boiling heat transfer. This paper presents equations which predict the limits of these regions.

The writer would like to make two comments in connection with equation (9), i.e., (10), which marks the change from the regime of isolated bubbles to the regime of continuous vapor columns. First, the question arises as to whether or not a clear transition (as indicated by (9), i.e., (10)), exists between the two regimes and second, whether the assumption implied by equations (1) through (7) and leading to (9) were verified experimentally for nucleate pool boiling?

At the Houston Conference the merit of equation (9) was questioned because, as it was observed, the transition between the two regimes is not abrupt but sets in gradually. It is true that an abrupt change, from a regime where isolated bubbles exist only to a regime where vapor columns and vapor patches exist only, is not verified by experiments. Indeed, in pool boiling experiments it can be easily seen that some nucleation centers can generate vapor columns at very low heat flux densities whereas the major-
ity of other nucleating sites are generating isolated bubbles. However, it is true also that as the heat flux is increased and the total bubble population increases, the fraction of active sites which generate continuous vapor columns and vapor patches increases whereas the fraction which generates isolated bubbles decreases. Consequently, at some heat flux density the majority of sites will generate vapor columns and patches.

It is the opinion of the writer that the question of whether or not the transition is abrupt is not as important as the fact that a transition occurs. The experimental data of the authors show this transition; it is also shown by the data of Gaertner and Westwater [21] plotted in Figs. 5, 6, and 7 which are reproduced from [22]. These figures are analyzed in more detail in [6] and [22]; the equation numbers correspond to those of [22]. Fig. 5 shows the diameter of a bubble at departure from the heating surface as a function of the bubble population density. It can be seen that in the region of isolated bubbles the bubble diameter, \( D_b \), is independent of the bubble population density, whereas in the region of interference, i.e., the region of vapor columns and patches, it is a function of \( n/A \). Figs. 6 and 7 show \( D_b \) as function of the heat flux density \( Q/A \), and of the heat-transfer coefficient \( h \), respectively. Both figures clearly show the change from one regime to another. It was noted in [6] and in [22] that this change corresponds to the inflection point of the heat flux density—liquid superheat temperature difference curve, i.e., of the \( Q/A \) versus \( T_{\text{wall}} - T_\text{sat} \) curve.

Equation (9), i.e., (10), derived by the authors predicts (with \( \beta = 50 \) deg) that the transition from the regime of isolated bubbles to the regime of continuous columns occurs at a heat flux of approximately 51,500 Btu/hrft\(^2\) which is in agreement with the data shown in Fig. 6. Although this figure indicates also that the transition takes place over a range of heat flux densities (up to approximately 90,000 Btu/hrft\(^2\)), equation (10) gives a reasonable estimate of this change. It is therefore the opinion of the writer that either equation (9), i.e., (10), or an equation of a similar form is useful and will be useful for estimating the limit of the region of isolated bubbles.

The question relative to the validity of the assumptions which are implied by equations (1) through (7) and which lead to equation (9) is not so important for evaluating the validity and correctness of equation (9) because equations of similar form can be derived [13] (see below) without recourse to some of the assumptions introduced by the authors. The importance of this question arises because of the possibility of using equations (1)–(7) to analyze the turbulent wake flow regime in a bubbling two-phase system. The assumptions which, in the opinion of the writer,
need clarification and verifications are connected with the use of equations (1) and (3) in the present problem. It can be seen that the use of (1) and (3) is essential for obtaining the two equations (not numbered) which follow (4) and proceed (5). These two equations together with (6) and (7) determine then the value of the numerical coefficient in (9), i.e., in (10).

Both equations (1) and (3) were obtained, originally, for a slug flow regime. In this regime the diameter of the bubble is approximately equal to the diameter of the pipe, whereas the length of the bubble may exceed the diameter by several times. In their original form the symbols $D$ and $A$ which appear in (1) and (3) referred to the diameter and cross-sectional area of the pipe.

Equation (3) was derived in [21] by fitting the bubble shape calculated by Dimitrescu [23]; it is valid for the cylindrical portion of the slug. This can be easily seen by rearranging (3) and expressing in terms of the bubble volume $v$, thus, approximately:

$$v = \frac{1}{1.1} A_e \left( L_s - \frac{D}{2} \right)$$  \hspace{1cm} (25)

The factor 1.1 appears in this equation because in the original form of (3) the area $A_e$ referred to the total cross-sectional area of the pipe $A_p$. Thus the term

$$A_{1.1} = A_e$$  \hspace{1cm} (26)

implied that the cross-sectional area of the slug $A_e$ was 90 percent of the pipe cross-sectional area, a relation which was recently verified experimentally by Davidson and co-workers [24]. The bubble radius ($D/2$) is subtracted from the bubble length $L_s$, because (2) applies (see [21]) when $L_s/D > 0.55$, and it does not take into account the spherical cap portion which forms the top of the slug. Because (3) was derived for cylindrical slugs rising in pipes the writer wonders whether it can be used to describe the spherical cap bubbles shown in Fig. 1(a), (b), and (c) and Fig. 2(a), (c), (e), and (i). Perhaps, just before bubbles coalesce they may be considered as cylindrical; this may justify the use of an equation of a form of (3) in this problem. However, additional experimental data would appear desirable to clarify this point and to shed some light onto the mechanism of bubble coalescence.

Equation (1) was determined experimentally for the ease of slug flow; it shows the effect of the wake of a slug on the velocity of rise of a slug which follows it in the pipe. Whether the wake has the same effect in a restricted region (such as a pipe) as it would have in an open region such as in a bubbling system or in nucleate pool boiling needs verification. This point is of considerable importance for an analysis of the process of bubbling in a vessel of large cross section. Mr. J. Hench and the writer have found [25] and [26] that bubble wakes greatly affect the bubble coalescence and liquid circulation. Experimental results show that the form of equations which describes the process depends on whether or not bubbles have trailing wakes. If applicable, an equation of the form of (1) would be extremely helpful in an analysis of the turbulent bubbly regime and in an analysis of bubble coalescence. The writer wonders whether the authors have experimental data which would justify the use of (1) in the problems of turbulent bubbling in a duct or large cross section and in the problem under consideration (nucleate pool boiling)?

Equations of a form similar to equation (9), i.e., (10), can be derived in a somewhat different manner [31] by considering the flow associated with the process of bubbling in a vessel of large cross-sectional area. The latter problem was investigated analytically and experimentally in [25] and [26]. In agreement with previous investigations, it was found that the bubbling process consists of two regimes, "ideal" bubbling and "churn-turbulent" bubbling, (their characteristics and differences are discussed in [25]).

In reference [25] were derived analytically, equations which relate the volumetric gas flow rate to the vapor (or gas) hold-up in the "ideal" bubbling regime, thus

$$Q_g = 1.53 \left( \frac{\sigma_f (\rho_f - \rho_p)}{\rho_f^2} \right)^{1/4} \frac{A}{1 - \alpha} \frac{1}{\alpha}$$  \hspace{1cm} (27)

Other expressions and previously proposed equations are given in [23]. It can be seen from (27) that the maximum value of $Q_g/A$ for "ideal" bubbling occurs when the hold-up is $\alpha = 2/3$, and it is given by

$$Q_g = 1.53 \left( \frac{\sigma_f (\rho_f - \rho_p)}{\rho_f^2} \right)^{1/4} 0.385$$  \hspace{1cm} (28)

It can be shown that the bubble spacing which corresponds to $\alpha = 2/3$ is $a = 1.16 D_b$. Substituting (28) in equation (8) gives the following expression for the heat flux density at the transition

$$(Q_g/A) = 0.50 \rho_f h_f \left( \frac{\sigma_f (\rho_f - \rho_p)}{\rho_f} \right)^{1/4}$$  \hspace{1cm} (29)

which is of a form similar to equation (10) and predicts for water at 1 atm a heat flux density of 40,000 Btu/hr ft².

For the "churn-turbulent" equation which relates the vapor (or gas) flow rate to the hold-up is of the form [25]:

$$Q_g = 1.1 \left( \frac{\sigma_f (\rho_f - \rho_p)}{\rho_f} \right) r_e \frac{1}{1 - K \alpha}$$  \hspace{1cm} (30)

where $r_e$ is the equivalent radius and is related to the bubble volume $V_b$, by $r_e = \frac{3}{4\pi} V_b^{1/3}$. The coefficient $K$ depends on the number and size of orifices, liquid properties, gas flow rate; it represents the effect of the bubble trailing wakes on the induced recirculation and on the three-dimensional flow pattern. It may be noted that by replacing $r_e$ with the diameter of the pipe, $D_b$, and by letting $K = 1$ equation (30) is reduced to the equation for slug flow in pipes. However, the authors of [25] found that it is the dependence of $K$ upon the various parameters together with the induced recirculation which make the "churn-turbulent" bubbling regime quite dissimilar from the ordinary slug flow regime. It is precisely for this reason that, if applicable, an equation of the form of (1) would be of great help to analyze the effect of bubble trailing wakes, i.e., of $K$ on the induced recirculation. It may be also noted that by assuming appropriate values for $\alpha$ and for $K$ equations of a form similar to (10), i.e., to (29), result, differing only in the value of the numerical coefficient.

The fluid dynamic and the heat-transfer process in nucleate pool boiling are analyzed and discussed in some detail in [22]; the writer would like to summarize here the results of [22] because they complement the information presented by the authors.

The Regime of Isolated Bubbles

1. In this regime bubbles do not interfere with each other and at any particular point vapor is produced intermittently.

2. Jakob's description of the liquid flow in nucleate pool boiling: upward with rising bubbles, downward between bubbles, and approximately horizontally along the surface, is similar to Malkus' and Townsend's description of the flow regime caused by up-draught in turbulent natural convection from a horizontal surface. In both cases the heat transfer is effected by the "up-draught" induced circulation.

3. It is shown that the same equations which predict the heat-transfer coefficient and the average turbulent velocity fluctuation in natural turbulent convection from a horizontal surface can be used in the regime of isolated bubbles if the vapor void coefficient, i.e., the vapor hold-up, is taken into account.

4. Equations which relate the vapor hold-up to the heat-transfer coefficient or to the bubble population density and liquid superheat temperature are presented.

5. It is shown that an upper limit exists for the heat-transfer mechanism induced by the "up-draught" circulation.

6. Equations predicting the limiting value of the heat-transfer coefficient, of the heat flux density, and the bubble population...
in the regime of isolated bubbles are presented also. The agreement of values predicted by the analysis presented in [22] is shown in Figs. 5, 6, 7, and 8.

The Region of Interference (The Regime of Continuous Vapor Columns and Patches)

1 In this regime bubbles interfere with each other to form continuous vapor columns and patches.

2 Vapor is continuously produced by vaporization of a pulsating microlayer (proposed and described by Moore and Meeler) at the base of a vapor column or of a vapor patch.

3 In this regime the dominant heat-transfer mechanism is, most probably, the latent heat-transport process and the latent heat transport associated with the large bursts of vapor caused by collapsing vapor patches.

In closing the writer again would like to congratulate the authors for their important contribution to the understanding and knowledge of boiling heat transfer.

References

Authors’ Closure

The authors thank Professor Chang and Dr. Zuber for their interest and stimulating comments. Professor Chang is not in agreement with the proposed analytical model according to which the two regimes in nucleate boiling are that of isolated bubbles and continuous vapor columns. Instead, he proposes the “feeble boiling” regime, which has the same characteristics as the “isolated bubble” regime, and the “vigorous boiling” regime. The “vigorous” boiling model differs from the continuous column one in the sense that, whereas the latter assumes the vapor columns to be stable, the former admits that there are continuous disintegrations, agglomerations, and redisintegrations.

There can be no doubt that the physical picture is as the reviewer points out, i.e., that the columns are unstable and also that the transition from the isolated bubble regime is gradual and not abrupt. That this is recognized by the authors is implied in Section 3B in the next-to-last paragraph. Nevertheless, this analysis postulates that, on the average, the behavior of the real system can be approximated by the model of Fig 1. The continuous vapor column model is admittedly a gross idealization of a very complicated process. However, it is necessary to make this idealization in order to analyze the process and develop a useful correlation equation for the maximum subcooled boiling heat flux. The comparison with experiment seems to justify this idealization.

The primary difference between the many correlation equations for the “burnout” heat flux is in the various ways in which density difference and density ratio appear. When the vapor density is negligible relative to the liquid density, e.g., at low relative pressures, all the equations yield identical results. In order to determine which equation is valid, it will be necessary to obtain maximum heat flux data just below the critical pressure where the vapor and liquid densities approach one another.

Most of Dr. Zuber’s comments are positive and help substantiate the usefulness of our paper. His important questions relate to the applicability of equations (1) and (3) to the boiling problem. It is true that equation (1) was originally derived from measurements of slug flow in pipes. However, these measurements included bubbles of $L/D_p$ as small as 1/4 which is far from “fill” the tube connection. There can be no doubt, however, that for very low bubble-Reynolds numbers, the wake mechanics will be different and, consequently, equation (1) no longer applicable.

As to the use of equation (3), the reviewer is correct in pointing out that an equation of this form does not apply to spherical cap bubbles. The reason why it applies to this problem is that at the time of rapid approach of coalescence, the bubbles are drawn out and assume a bullet-like shape. (This could be verified by high-speed movies, but no such movies were taken at the time of the study.)

In closing, the authors thank the reviewers for their constructive discussion of the paper.