Neighborhood of a Whirling Speed—The Effect of Want of Balance," *Philosophical Magazine*, Series 6, vol. 37, p. 304.

3 D. M. Smith, "The Motion of a Rotor Carried by a Flexible Shaft in Flexible Bearings," *Proceedings of the Royal Society*, London, Series A, vol. 142, pp. 92–118.

4 F. C. Linn and M. A. Prohl, "The Effect of Flexibility of Support Upon the Critical Speeds of High-Speed Rotors," *Trans. SNAME*, vol. 59, 1951, pp. 536–553.

5 J. W. Lund and B. Sternlicht, "Rotor-Bearing Dynamics With Emphasis on Attenuation," *Journal of Basic Engineering*, TRANS. ASME, Series D, vol. 84, 1962, p. 491.

Authors' Closure

Mr. Gunter presents a significant discussion of the single-mass rotor including bearing and support stiffness factors. The results may be looked upon as an asymptotic case of the general multimass rotor treated by the authors. The advantage in carefully analyzing the simple systems lies in the understanding one gains about the way in which the system parameters may be expected to enter the solution of the realistically complex system. Sometimes these expectations are misleading, and therein lies the weakness of the simple model. The authors hope that their comparatively elaborate model will prove to be the basis of fruitful analytical work at a level beyond that of the single-mass system.

In answer to the specific questions:

1 Yes. With the digital computer, the speed-response spectra (Fig. 1) of numerous industrial turbomachines have been predicted.

2 The unbalance of each rotor mass is a system parameter. Since a turborotor is assembled without regard to the location of the mass centers, the predicted vibratory performance must be statistical. The computer is programmed to obtain a number of speed-response spectra based on various random-number eccentricity sets.

3 There were convergence problems in the early programs, but it appears that moderate attention to the problem has eliminated the difficulty.

A Note on the Transient Axisymmetric Thermoelastic Problem for the Solid Sphere[1]

G. SONNERMANN.[2] In this paper, the numerical result of Fig. 1 shows very clearly that the thermal penetration into the center of the sphere lags the outside surface thermal transient. Consequently, the concept of utilizing only elementary theory, i.e., ignoring geometry effects, should be feasible.[3, 4]

With the temperature distribution of Warren

$$T(1, u, \tau) = \Sigma B_n(1 - e^{-\beta_n \tau})P_n(u)$$

approximated at $\theta = 0$ ($u = \cos\theta = 1$) by the $n = 0$ term

$$T(1, 1, \tau) = B_0(1 - e^{-\beta_0 \tau})$$

the temperature distribution becomes for small nondimensional times

$$T(1, 1, \tau) \approx B_0\beta_0\tau \quad (1)$$

If one uses for the thermal stress

$$\sigma = E\alpha\Delta T = 2(1 + \nu)G\alpha\Delta T \quad (2)$$

and then substitutes the values of Warren for B_0 and β_0, equation (1) yields

$$T(1, 1, \tau) = 92.2T_0\tau \quad (3)$$

Also

$$\bar{\sigma} = \sigma/G\alpha T_0 = 2(1 + \nu)92.2\tau \quad (4)$$

The peak stress should occur at the time when the outside temperature reaches T_0, i.e., $T = T_0$, which is at $\tau \simeq 0.011$. This ignores the penetration of the thermal front into the sphere and consequently will give a higher thermal stress than one should expect in practice. Substitution of $\tau = 0.011$ into equation 4 yields, with $\nu = 0.3$

$$\bar{\sigma} = 2.64 \quad (5)$$

This value is to be compared to the peak of about 2.2 at the $\theta = 0$ of Warren. The stress computed in this note is 20 percent greater than Warren's value. Although the technique does not yield the details of the thermal-stress distribution, it does bound the value with sufficient accuracy for many practical problems. It has been applied to transient thermal stresses in cylinders.[3] The main attribute of the method is rapid evaluation without the need to conduct either a heat-conduction analysis or a thermal-stress analysis.

Author's Closure

The remarks of Dr. Sonnemann pointing out the conservative nature of an elementary theory for bounding the thermal stresses in a solid sphere should be of practical interest. It should be mentioned that Dr. Sonnemann's equation (1) is only valid for $\beta_0\tau \ll 1$, but is subsequently evaluated at $\beta_0\tau = 1.83$ to arrive at (5). Fortunately, the actual temperature distribution employed has nothing to do with arriving at (5) since (1), (3), (4) merely produce max ΔT = max $T = T_0$ from which (2) renders directly $\bar{\sigma} = 2(1 + \nu) = 2.60$. Equation (5) yields 2.60 rather than 2.64 if three significant figures are carried in τ.

The axisymmetric nature of this problem would appear to make the two-dimensional stress analog of (2) more physically realistic than (2). This presents

$$\bar{\sigma} = 2\left(\frac{1 + \nu}{1 - \nu}\right) = 3.71$$

which is considerably more conservative than (5).

[1] By W. E. Warren, published in the June, 1964, issue of the JOURNAL OF APPLIED MECHANICS, vol. 31, TRANS. ASME, vol. 86, Series E, pp. 348–350.

[2] Senior Technical Assistant to Program Manager, Guidance and Control, Corporate Systems Center, United Aircraft Corporation, Farmington, Conn.

[3] G. W. Reichard, Jr., "Temperature Distributions and Thermal Stresses in Finite Hollow Cylinders," MS thesis, University of Pittsburgh, Mechanical Engineering, 1964.

[4] M. A. Biot, "New Methods in Heat Flow Analysis with Application to Flight Structures," *Journal of the Aeronautical Sciences*, vol. 24, 1957, pp. 857–873.

Longitudinal Impact on Viscoplastic Rods—Linear Stress-Strain Rate Law[1]

P. C. CHOU.[2] The authors have studied the longitudinal impact on rods using a Bingham type of constitutive equation. Their study will certainly be helpful in the evaluation of the strain-rate effect under hypervelocity impact. In evaluating their efforts, the writer would like to offer the following comments:

1 Viscoplastic theory involving only shear has been applied recently to the perforation of a plate due to hypervelocity impact [1, 2].[3] The governing equations and initial and boundary conditions are similar to those used in this paper. The solutions in [1, 2] involve Bessel functions, rather than error functions, because of the axisymmetric nature of the shear problem.

[1] By T. C. T. Ting and P. S. Symonds, published in the June, 1964, issue of the JOURNAL OF APPLIED MECHANICS, vol. 31, TRANS. ASME, vol. 86, Series E, pp. 199–207.

[2] Professor of Aeronautical Engineering, Drexel Institute of Technology, Philadelphia, Pa. Mem. ASME.

[3] Numbers in brackets designate References at end of Discussion.