Using mathematical modelling to inform on the ability of stormwater ponds to improve the water quality of urban runoff

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Abstract This paper concerns the mathematical modelling of flow and solute transport through stormwater ponds. The model is based on appropriate lumped system conservation equations that are solved using standard numerical techniques. The model was used to route a first flush pollution scenario through a cylindrical pond for 16 combinations of elevation and diameter of a submerged pipe outlet, in conjunction with a high level weir. Higher pipe elevations and smaller pipe diameters created larger pond volumes and hence led to greater dilution of the pollutant. In contrast, lower pipe elevations created larger storage volumes, leading to better flow attenuation. Interestingly, larger pipe diameters improved peak flow attenuation, even though the storage used decreased.

Keywords Dilution; flow attenuation; mathematical modelling; solute transport; stormwater ponds

Introduction

Stormwater ponds are frequently used to attenuate flows derived from surface water drainage and, depending on their design, they provide a greater or lesser degree of water quality treatment. Detention basins (or dry ponds) store stormwater for relatively short periods of time and provide little water quality treatment, whereas retention ponds (or wet ponds) store stormwater for much longer, which enables most of the suspended material to settle out, thus significantly reducing the turbidity of the outflow. Since much of the pollutant load is bound to the sediment (Ellis and Hvitved-Jacobsen, 1996), the removal of the suspended sediment further reduces the impact on the receiving watercourse. In addition, a retention pond’s permanent pool of water provides the sort of conditions that allows pollutants to degrade or to become more securely bound to plant or mineral substrates. However, some pollutants exist in solution (and do not settle out) and some sediment particles may be so small that they pass through the retention pond before settling, taking any bound pollutants with them. The latter is particularly likely to happen when the inflow to, and the outflow from, the pond are both high, since under these conditions short-circuiting is more likely to occur, caused by the development of a momentum driven preferential flow path through the pond.

To date, little attention has been devoted to the fate of solutes and fine sediments in retention ponds, and little is known about: (a) the degree of physical dilution that is achieved by the mixing of the inflow with the water in the permanent pool; and (b) how sensitive the dilution is to the outlet configuration. The aim of the work described in this paper is to address these issues by undertaking simulations of solute transport through a simplified retention pond. The simulations were carried out using a mathematical model of a retention pond that was developed by the study team.
Model development
The mathematical model consists of two components: a flow model and a solute transport model; and in both cases the pond is modelled as a deterministic, lumped system. In the work described in this paper the pond was assumed to be cylindrical, having a single inlet and a dual outlet system.

Flow model
Flow through the pond is modelled using a standard storage routing method (Shaw, 1994; Mays, 2001) that is based on the following equation that describes the conservation of volume of water:

\[
\frac{dV}{dt} = Q_i - Q_o
\]

where \(V\) is the volume of water in the pond (often termed storage), \(Q_i\) is the volumetric flow rate of water entering the pond (inflow) and \(Q_o\) is the volumetric flow rate of water leaving the pond (outflow). Equation (1) is solved to give the temporal variation (hydrograph) of outflow, assuming that the inflow hydrograph is known. The outflow passes through two outlet devices that are described by hydraulic head-discharge equations. Thus the outflows are actually calculated from predicted water levels in the pond. The outlet devices considered here are: a submerged horizontal pipe (modelled as a submerged orifice) located at various elevations above the base of the pond and a v-notch weir located at a fixed elevation above the base of the pond. The hydraulic equations for these two devices are (Chadwick and Morfett, 1998):

(a) submerged pipe

\[
Q_{sp} = aC_D \sqrt{2gh} = K_{sp}H^{0.5}
\]

where \(Q_{sp}\) is the flow through the pipe, \(a\) is the cross-sectional area of the pipe, \(C_D\) is the coefficient of discharge, \(g\) is the acceleration due to gravity, \(H\) is the head and \(K_{sp}\) is a constant;

(b) v-notch weir

\[
Q_{vnw} = \frac{8}{15} C_D \sqrt{2g \tan(\gamma/2)}H^{2.5} = K_{vnw}H^{2.5}
\]

where \(Q_{vnw}\) is the flow through the weir, \(\gamma\) is the weir angle, \(K_{vnw}\) is a constant and the other symbols are as previously defined.

The head for the submerged pipe is the difference between the water level in the pond, \(y\), and the pipe elevation, \(y_{sp}\); the head for the v-notch weir is the difference between the water level in the pond and the weir crest elevation, \(y_{vnw}\). Clearly, outflow through the pipe occurs whenever \(y > y_{sp}\), and outflow through the weir occurs whenever \(y > y_{vnw}\). The total outflow, \(Q_o\), is simply the sum of the pipe and weir outflows. Modelling the pipe outlet using equation (2) implies certain assumptions and simplifications that may not be true at all times in practice (Urbonas and Stahre, 1993; Mays, 2001). Nevertheless, equation (2) has the attraction of simplicity.

Equation (1) can be solved in several ways. Since inflow hydrographs can rarely be represented by a simple function, a numerical approach is usually adopted. Traditionally, hydrologists have employed the storage-indication method (Shaw, 1994; Mays, 2001) although there are many standard numerical algorithms for ordinary differential equations that could be used instead (Mason and Stocks, 1987; Quinney, 1987; Griffiths and Smith, 1997).
1991; Chapra and Canale, 1998). The latter route is followed here mainly because these methods can also be used for the solute transport model.

Expressing equation (1) in a standard time-weighted finite difference form gives:

\[
\frac{V^{n+1} - V^n}{\Delta t} = \theta (Q_i - Q_o)^{n+1} + (1 - \theta) (Q_i - Q_o)^n
\]  \hspace{1cm} (4)

where superscripts \( n + 1 \) and \( n \) refer to values evaluated at two times separated by the time step, \( \Delta t \), and \( \theta \) is a time weighting parameter \((0 \leq \theta \leq 1)\). It can be assumed that any term evaluated at time \( n \) is known, so that only terms evaluated at time \( n + 1 \) need to be calculated. The left-hand side of equation (4) represents the change in volume that occurs during the time step. This can be expressed in terms of the surface area of the pond, \( A \), and the water level, \( y \), as follows:

\[
V^{n+1} - V^n = \bar{A} (y^{n+1} - y^n)
\]  \hspace{1cm} (5)

where the \( \bar{\cdot} \) overbar indicates an average value during the time step. Combining equations (4) and (5) and re-arranging gives:

\[
y^{n+1} = B - \frac{\Delta t \theta}{A} Q_{sp}^{n+1}
\]  \hspace{1cm} (6)

where:

\[
B = y^n + \frac{\Delta t(1 - \theta)}{A} (Q_i - Q_o)^n + \frac{\Delta t \theta}{A} Q_{i}^{n+1}
\]  \hspace{1cm} (7)

All the terms on the right-hand side of equation (7) are known, either because they are all evaluated at time \( n \) or, in the case of the third term, because all values of the inflow are known. Replacing the unknown outflow in equation (6) by the sum of equations (2) and (3) and re-expressing the outlet heads gives:

\[
y^{n+1} = B - \frac{\Delta t \theta}{A} \left( K_{sp} (y^{n+1} - y_{sp})^{0.5} + K_{vw} (y^{n+1} - y_{vw})^{2.5} \right)
\]  \hspace{1cm} (8)

Generally, equation (8) is non-linear in \( y^{n+1} \) and needs to be solved via iteration. When \( \theta = 0 \), however, a direct or explicit solution is possible. This corresponds to the Euler algorithm (Mason and Stocks, 1987; Quinney, 1987; Griffiths and Smith, 1991; Chapra and Canale, 1998). It is well known that the simplicity of this algorithm needs to be balanced against its relatively poor accuracy and potential for instability. Taking \((0 < \theta \leq 1)\), however, though incurring extra computational cost, yields a more robust solution. Following standard practice, \( \theta = 0.5 \) (corresponding to the trapezium method (Quinney, 1987; Griffiths and Smith, 1991) or the Crank–Nicolson method (Griffiths and Smith, 1991)) was adopted. This gives a theoretically more accurate solution than other values of \( \theta \), and the algorithm has good stability characteristics.

Again, following standard practice, Newton–Raphson iteration (Griffiths and Smith, 1991; Chapra and Canale, 1998) was used to cater for the non-linear nature of equation (8). To simplify the detail of the iteration, the surface area used in equations (7) and (8) was taken as the area corresponding to \( y^n \). To maintain the highest accuracy, this surface area could, instead, have been evaluated as the average of the areas corresponding to \( y^n \) and \( y^{n+1} \), however, such a refinement was not considered to be necessary because it was anticipated that relatively small time steps would be used in the simulations.
Solute transport model

The solute transport model is based on a similar equation to the flow model, but in this case the equation describes the conservation of mass of solute:

\[
\frac{d(CV)}{dt} = Q_i C_i - Q_o C_o \tag{9}
\]

where \(C\) is the concentration of the solute in the pond, \(C_i\) is the solute concentration in the inflow and \(C_o\) is the solute concentration in the outflow. Equation (9) is solved to give the temporal variation of \(C_o\), assuming that volumes and flows are known from the flow model and that the solute concentration in the inflow is known. Using a similar finite difference form, as before, gives:

\[
\frac{(VC)^{n+1} - (VC)^n}{\Delta t} = \theta (Q_i C_i - Q_o C_o)^{n+1} + (1 - \theta) (Q_i C_i - Q_o C_o)^n \tag{10}
\]

Assuming that the pond is well mixed, so that solute that enters the pond is instantaneously and uniformly mixed throughout the water in the pond, then \(C_o\) can be replaced by \(C\). Thus the pond is assumed to behave as a continuously stirred tank reactor (Chapra, 1997; Mihelcic, 1999). After some re-arrangement, equation (10) becomes:

\[
C^{n+1} + \theta \Delta t Q_o C^{n+1} = C^n (V - (1 - \theta) \Delta t Q_o) + \theta \Delta t (Q_i C_i)^{n+1} + (1 - \theta) \Delta t (Q_i C_i)^n \tag{11}
\]

Hence, \(C^{n+1}\) is found easily since all the other terms are known and there are no non-linearities. As before, \(\theta\) was taken as 0.5, to enhance the accuracy of the calculation.

Simulations

The simulations were designed to identify some of the major influences on solute concentration in the outflows from retention ponds. The (cylindrical) geometry of the pond, the location and characteristics of the v-notch weir and the inflow hydrograph remained fixed whilst the elevation of the submerged pipe and its diameter were varied. Several, different temporal distributions of solute (pollutographs) in the inflow were considered, but here interest focuses on just one.

The pond radius was specified as 10 m. The crest of the weir was located 2 m above the base of the pond, the weir angle was specified as 90° and the coefficient of discharge was specified as 0.6 (Chadwick and Morfett, 1998). The pipe was located at one of four elevations above the base of the pond (0 m, 0.5 m, 1.0 m, 1.5 m) and for each elevation four pipe diameters were considered (0.05 m, 0.1 m, 0.15 m and 0.2 m). The pipe’s coefficient of discharge was specified as 0.6 (Chadwick and Morfett, 1998). The inflow hydrograph was an isosceles triangle of duration 3.2 hours and peak flow of 175 l/s. Thus the inflow volume was 1008 m³, compared to a maximum available storage volume (below the weir crest) of 628 m³. For each simulation the initial water level was set equal to the elevation of the submerged pipe. Note that the case where the pipe is located at the base of the pond corresponds to a detention basin, while the other three cases correspond to retention ponds.

The pollutograph considered here consisted of a short pulse of solute occurring during the rising limb of the inflow hydrograph and portrays a “first flush” type of scenario. In all cases, the initial concentration of solute in the pond was specified as zero. A total of 16 simulations were carried out covering all the combinations of pipe elevation and diameter. Following some initial sensitivity tests, a time step of 0.025 h was used in all the simulations.
Results and discussion
The main characteristics of the flow attenuation and water quality treatment shown by the simulations are described in the following sections. Attention focuses on the peak outflow and the peak solute concentration in the outflow. The conservation of flow volume and solute mass was checked for all the simulations and was found to be excellent.

Flow attenuation
A typical example of the flow and water level variations from the simulations is shown in Figure 1. Three outflows are shown: the outflow through the pipe, the outflow through the weir and the total outflow. Following the start of inflow, outflow initially occurs only through the pipe until the water level reaches the weir crest. Thereafter, outflow occurs through both outlets until, on the recession, the water level falls below the weir crest, after which outflow only occurs through the pipe.

The peak outflow is directly related to the maximum water level because both outlet devices are driven by their hydraulic heads. Generally, the maximum water level achieved depends on the pipe elevation, the pipe diameter and the inflow volume, however, since the latter was kept constant here, the combination of the former two controlled the flow attenuation achieved.

An interesting way of showing all the results together is given in Figure 2, in which the peak outflow is plotted against its time of occurrence. Since in all cases the peak outflow should occur when the outflow hydrograph crosses the recession limb of the inflow hydrograph, all the data should fall on the inflow recession limb. Figure 2 shows that this is indeed the case, and indicates that there are no gross errors in the simulations. The spread of the data shows the range of peak flow attenuation achieved over the sixteen combinations of pipe elevation and diameter. The data is grouped according to pipe elevation, showing that, in broad terms, attenuation improves (peak outflow decreases) with decreasing pipe elevation. This reflects the greater storage volume that is available when the pipe outlet is located closer to the base of the pond.

Figure 3 shows the same data in a more detailed way, and exposes the relationship between peak flow attenuation, storage, pipe elevation and pipe diameter. More specifically the figure shows the peak flow ratio (defined as the ratio of peak outflow to peak inflow) plotted against the maximum storage used (defined as (maximum water level reached – initial water level) × surface area of pond) for all the simulations. This shows that for the same pipe elevation, increasing attenuation is found with increasing

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**Figure 1** Flows and water level for 0.1 m diameter pipe located 0.5 m above base of pond
pipe diameter, but the storage used decreases. This reflects the lower maximum water
levels achieved (because with a larger diameter pipe more water is discharged from the
pond and a smaller volume is stored). Indeed, when the 0.2 m pipe is located at 0 m and
0.5 m above the base of the pond the water level does not reach the weir crest because of
the large capacity of the pipe outlet. These two cases then provide identical flow attenu-
ation (using the same storage volume), and are to some extent anomalous.

Water quality treatment
A typical example of the pollutographs and pond volume variations from the simulations
is shown in Figure 4. Once pollutant enters the pond it immediately appears in the out-
flow because the pond is assumed to be completely mixed. The outflow concentration
then rises and reaches a peak at the cessation of the pollutant inflow, following which it
gradually reduces to a constant level. This recession phase is caused by dilution of the
pollutant in the pond by the further inflow of clean water. Once the inflow stops, the pol-
lutant concentration in the outflow remains constant until eventually the outflow ceases,
after which the outflow concentration is set to zero.

Figure 2 Peak flow attenuation for all simulations plotted for different pipe elevations

Figure 3 Relationship between peak flow attenuation, storage and outlet pipe configuration; the legend
shows pipe diameters and the dashed lines indicate pipe elevation
The relationship between peak outflow concentration, storage, pipe elevation and pipe diameter is shown in Figure 5, which shows the peak solute ratio (defined as the ratio between peak outflow solute concentration and peak inflow solute concentration) plotted against the maximum pond volume (defined as maximum depth achieved \( \times \text{surface area of pond} \)). This is a more appropriate measure to use here than the maximum storage used in Figure 3 because the pollutant is mixed within the whole volume of the pond (including the permanent pool below the elevation of the pipe). Clearly, higher dilution of the incoming pollutant is achieved when there is a larger volume of water in the pond, i.e. better dilution is found when the pipe outlet is located further above the base. The figure also shows that increased dilution is found when the pipe diameter decreases. This happens because the volume of water in the pond tends to be greater when the pipe outlet capacity is reduced. There is a greater sensitivity to pipe diameter when the pipe is at a lower elevation.

**Conclusions**

Simulations of flow through cylindrical retention ponds having a submerged pipe outlet and a higher level weir outlet were generally consistent with the conventional idea that flow attenuation improves as available storage increases. Peak flow attenuations were in
the range 0.36–0.96. Interestingly, by increasing the diameter of the pipe outlet, peak flow attenuation was improved even though a reduced storage volume was used. Simulations of the dilution of a dissolved first flush pollutant load resulted in the ratio of the peak concentration in the outflow to the peak concentration in the inflow being in the range 0.2–0.8. Larger dilutions corresponded with larger pond volumes, created either by a higher elevation, or by a smaller diameter, of the submerged pipe. There is an interesting conflict between achieving good dilution and good flow attenuation because the former requires a large percentage of the pond volume to be occupied by water at the start of the inflow, whereas the latter requires a small percentage of the pond volume to be initially occupied by water.

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References