The New Viewpoint of the High Energy Elastic Scattering of Nucleons from Nuclei

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In the previous notes, we showed that the scattering of nucleon by deuteron or He\(^4\) can be obtained by the superposition of the nucleon-nucleon scattering amplitudes multiplied by the sticking factor. Others also reached this conclusion.

This important conclusion is obtained by the experimental checks of light nuclei, lighter than C. In heavy nuclei, however, the multiple scattering effect will be important. And it is supposed that we cannot derive the same conclusion from heavy nuclei as got by the light nuclei. Then we must perform tedious calculations to reproduce the experimental data of the scattering by heavy nuclei. In fact, if we calculate the high energy elastic scattering under the assumption of the direct two-body collision, we get the results somewhat larger than the experimental values, though we can reproduce the main feature, as stated in the previous note. This is seen in the dotted line of Fig. 1, in the case of the elastic scattering of 95 Mev protons from typical nuclei.

We take the Saxson-type density distribution as described in the previous note. The parameters are \(R=1.32 \times A^{1/3} \times 10^{-13}\) cm, \(a=0.49 \times 10^{-13}\) cm.
In this calculation, the Coulomb effect is neglected. But it is proved that the Coulomb effect cannot reduce the calculated value to the experimental value, except in the case of carbon.

The purpose of the present paper is to point out the following interesting theoretical finding, in relation to the above discussion.

$$\lambda = \frac{\sum \langle \hat{\xi}_i | \varphi_c \rangle \langle \varphi_c | \sum_p V_p | \psi_c (p) \rangle \langle \varphi_c | \hat{\xi}_i \rangle + \sum_{mn} \langle \hat{\xi}_i | \varphi_m \rangle \langle \varphi_m | \sum_p V_p | \psi_n (p) \rangle \langle \varphi_n | \hat{\xi}_i \rangle}{\left( \langle \varphi_i | \sum_{p'} V_{p'} | \psi_i (p') \rangle + \sum_{mn} \langle \hat{\xi}_i | \varphi_m \rangle \langle \varphi_m | \sum_{p'} V_{p'} | \psi_n (p') \rangle \langle \varphi_n | \hat{\xi}_i \rangle \right)}.$$  

Here $V_p$ represents the interaction in free space of the incident and the target nucleon specified by $p$. $1/N \langle \varphi_i + \hat{\xi}_i \rangle$ is the wave function of the initial state. $N$ is the normalization factor. $\xi_i$ represents the product of the incident free nucleons and the correlations between the nucleons in the target nucleus in its ground state. We assume that the correlation in the target nucleus is described at least by two-particles-jump from the Fermi sea. $\varphi_i$ is the product of the wave function of the incident (free) nucleon and that of the correlation free part (under the Fermi sea) of the target nucleons. Here the target nucleons are distorted by the average potential, but the incident particle is not distorted. $|\Psi_n(p)\rangle$ describes the collision of the incident and the target nucleon $p$. $|\Psi_n(p)\rangle$ is the solution of the following equation,

$$|\Psi_n(p)\rangle = |\psi_n\rangle + \frac{1}{E_n + i\eta - (K + \tilde{U})} V_p |\psi_n(p)\rangle.$$  

$K$: the total kinetic energy operator. $	ilde{U}$: the sum of the single particle potential that the target nucleons feel in the nucleus.

Theoretically, we can derive the fact that in the high energy elastic scattering where the effect of the binding can be neglected in the multiple scattering processes, the main terms of the multiple scattering together with the direct two-body interaction can be represented as if it were the direct interaction with the reduced strength. After a long calculation, the reduction factor $\lambda$ is found to be

The value of $\lambda$ is expected to be smaller and reaches some constant value with the increasing mass number of the target nucleus, because in light nuclei the target is appreciably different from the Fermi gas and because in heavy nuclei it is expected that only a limited and a definite number of particles can have an appreciable correlation to the representative particle $p$. Thus in heavy nuclei the mass number dependence of both the numerator and the denominator of $\lambda$ is linear to $A$. Then $\lambda$ is independent of mass number in heavy nuclei.

In comparison with the experiments we can confirm this expectation. In fact, we get that the value of $\sqrt{|\lambda|^2}$ is 1.00 for C, 0.75 for Al, 0.55 for heavier nuclei than Cu. With this reducing factor we get the solid curves in Fig. 1.

This result will present the very important knowledge as we treat the other high energy nuclear reactions. The details of this note will be presented in this journal.
Comparison with experiment. The small circles are the experimental values presented by Gerstein, Niederer and Strauch (Phys. Rev. 108 (1957), 1427). (The author thanks for their sending experimental data.) The solid curve represents the theoretical value with reducing factor as indicated in the text. The dotted curve shows the calculated value with use of the impulse approximation. (We calculated the $t$-matrix using the pion theoretical phase shifts.)
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