On a Non-local Electromagnetic Model for Electron and Muon Masses*

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In a phenomenological way, a non-local electromagnetic interaction with a Pauli term is assumed in order to explain the whole masses of electron and muon. Qualitative discussions are devoted to the properties of form factors on the assumption of similar internal structures for both particles.

§ 1. Introduction

No remarkable difference of properties between electron and muon, except their large mass difference, has yet been discovered experimentally. In spite of suggestion that the large mass difference would come from the different internal structures of electron and muon, we could not get any definite conclusion from the present available experimental data. ¹)

Concerning the origin of their mass-difference itself, many possibilities have been presented up to now. Schwinger,²) for instance, has raised the possibility of explaining the high muon mass through a strong interaction of this particle with an unobserved iso-singlet σ-meson. Similar ideas have been repeated by other authors.³)⁴) Recently, Marx and Nagy⁵) assumed that the muon has a moderate strong interaction with the kaon doublet, realizing then the muon mass from the electron's. Since there would be no way of detecting this new particle or coupling, they might be equivalent to the introduction of a different structure of the muon from that of the electron.

A possibility that the electron mass has some electromagnetic origin has also been considered frequently. It is well known that we cannot explain the whole masses of fermions entirely by the field reaction of their minimal interactions with the electromagnetic field, as the basic equation of the particles is invariant under the Touschek's transformation.⁶) The only possibility is to introduce some violation term under this transformation. An explanation of the electron mass has already been presented by us and Taketani.⁷) The interaction term thus introduced would make the system unrenormalizable, because it should have a coupling constant with the dimensions of length.***⁸) An advantage of introducing the unrenormalizable

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*** We take natural units: \( \hbar = c = 1 \), and 1 y (yukawa) = 10⁻¹⁵ cm.
interaction is that it may give a sufficiently large effect of the high-energy region where the internal structure of the particle reveals itself. The reactive mass, then, is given by
\[ \delta m \sim \text{const} \cdot f \cdot A^{-(s+1)}, \] (1.1)
where \( f \) is the coupling constant of the unrenormalized interaction with the dimension of [\( L^s \)] \((q > 0)\) and \( A \) is the cutoff energy. Therefore, we have a possibility of explaining the electron and muon masses through this interaction by suitably adjusting their internal structures.

There may be some evidence for believing the existence of such unrenormalizable interaction. The present discrepancies between the theoretical value of the anomalous magnetic moment and the experimental value are*

\[ |\mu_{\text{exp}} - \mu_{\text{theor}}| \lesssim (10^{-5} \sim 10^{-6}) \cdot e/2m_\epsilon \quad \text{for electron,} \]
\[ |\mu_{\text{exp}} - \mu_{\text{theor}}| \lesssim (10^{-3} \sim 10^{-4}) \cdot e/2m_\mu \approx (10^{-6} \sim 10^{-8}) \cdot e/2m_\epsilon \quad \text{for muon.} \] (1.2, 1.3)

This may suggest that we have a possibility of assuming a universal Pauli term with a magnitude of \( \kappa (e/2m_\epsilon) = (10^{-5} \sim 10^{-6}) \cdot e/2m_\epsilon \) for both particles beside the usual minimal interaction as
\[ -L_{\text{int}} \sim e\bar{\psi} \gamma^\mu \psi A_\mu - \frac{e\kappa}{\Delta m_\epsilon} \bar{\psi} \sigma_{\mu\nu} \psi \cdot F_{\mu\nu}, \] (1.4)

where \( \psi, \bar{\psi} \) and \( A_\mu, F_{\mu\nu} \) are the field strengths of electron (or muon) and electromagnetic fields respectively. We take the hermitian representation for the Dirac matrices \( \gamma^\mu \) and \( \sigma^{\mu\nu} = i(\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu)/2 \), and \( g^{00} = 1, g^{11} = g^{22} = g^{33} = -1 \) for the space-time metric.

If we take a universal (local) interaction for electron and muon, the difference between both particles could come only from their different internal structures, which reveal themselves through the non-locality of the above interactions. Then our aim is to analyse the form factors through the interactions**

* The experimental values for the electron magnetic moment are
\[ \mu^{(e)}_{\text{exp}} = 1.001146 \pm 0.000012, 1.001165 \pm 0.000011, 1.001167 \pm 0.000005, \]
in electron Bohr magneton. For the muon we quote the result of the determination of Garwin et al.,
\[ \mu^{(\mu)}_{\text{exp}} = 1.0020 \pm 0.0005 \]
in muon Bohr magneton. The theoretical value of Sommerfield\(^{12}\) up to fourth order is
\[ \mu_{\text{theor}} = 1.0011596 \]
in lepton magneton. Then we have
\[ \mu^{(e)}_{\text{exp}} - \mu_{\text{theor}} = -0.000014 \pm 0.000012, 0.0000054 \pm 0.0000011, 0.0000074 \pm 0.000005 \]
for electron and
\[ \mu^{(\mu)}_{\text{exp}} - \mu_{\text{theor}} = 0.00018 \pm 0.0005 \] for muon.

** Though the way of constructing the hamiltonian is not yet justified, the resultant hamiltonian consists of infinite series by using a usual method. The exact hamiltonian up to the second order needs the terms of higher order.\(^{20}\)
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\[ -\bar{L}_{\text{int}} = \int d^4 x \int d^4 y \left[ e \bar{\psi}(x) \gamma^\mu \psi(x) A_\mu(y) F_1(x - y) \right. \\
\left. - \frac{e \kappa}{4m_e} \bar{\psi}(x) \sigma^{\mu \nu} \psi(x) F_{\mu \nu}(y) F_2(x - y) \right] \]

(1.5)

in connection with the electron and muon masses. As a consistent non-local field theory has not yet been established, our analysis is to be taken tentatively.

In § 2 we get the general restrictions of the form factors so as to realize a positive reactive mass for the electron (or muon). The discussions rest on the second-order perturbation theory. Then, in § 3 we consider the problem of realizing the electron and muon masses and raise some possible examples for the form factors.

§ 2. Electromagnetic self-mass and the general restrictions on the form factors

By calculating the second-order self-mass, we get the result that only the mixed vertices (minimal and Pauli interactions) do contribute in the limit of a zero lepton bare mass. Then, in the assumption of a zero bare mass, we obtain the following expression for the self-mass \( \delta m \):

\[ \delta m = - \frac{e^2}{4\pi^2} \frac{3i\kappa}{4\pi^2 m_e} \int d^4 q \frac{F_1(q^2) F_1(q^2)}{q^2 + i\epsilon}, \quad \epsilon = 0^+, \]

(2.1)

where

\[ F_1(q^2) = \int d^4 x e^{iqx} F_1(x), \quad i = 1, 2. \]

(2.2)

In order to get a finite self-mass, the form factors must be complex functions so as to cancel the divergent integrals, the simplest structures being

\[ F_i(q^2) = \int dM^2 g_i(M^2) \frac{g_i(M^2)}{M^2 - q^2 - i\epsilon}, \]

(2.3)

where the spectral functions must become zero at the limit \( M^2 \to \infty \),

\[ \lim_{M^2 \to \infty} g_i(M^2) \to 0. \]

(2.4)

The first requirement which corresponds to the well-known Feynman cutoff\(^{(14)}\) may destroy the usual concept of an hermitian hamiltonian. But without this requirement, we get a quadratic divergent result regardless of the shape of form factors used. The second one means that we have not a \( \delta \)-like charge at the center of the particle.

Using the spectral representation (2.3), we have

\[ \delta m = - \frac{e^2}{4\pi} \frac{3}{4\pi} \frac{\kappa}{m_e} A \approx 2 \cdot (10^{-2} \sim 10^{-3}) \frac{A}{m_e}, \quad \kappa \geq 0, \]

(2.5)
Then the details and the differences of the internal structures are reduced to the functional $\Delta$.

Since the self-mass must be positive from our standpoint of a zero bare mass, the quantity $\Delta$ must be negative (positive) in accordance with $\kappa > 0$ ($\kappa < 0$). As the present experimental data are not definite enough to determine the sign of the discrepancy $\kappa$, we must consider both possibilities.

i) Case of $\kappa < 0$. There is no further restriction of the form factors. This case can be realized by simply assuming the form factors as

$$g_1(M^2) \geq 0, \quad g_2(M^2) \geq 0 \quad \text{for all } M^2. \quad (2.7)$$

It is sufficient to take only simple Yukawa form factors for both interactions.

ii) Case of $\kappa > 0$. This case may be the more probable one, but we cannot get a positive self-mass without the assumption that at least one of the form factors is indefinite in sign. Since the argument is symmetric for both form factors, we restrict one of the form factors, say $g_2(M^2)$, to be positive definite

$$g_2(M^2) \geq 0 \quad \text{for all } M^2. \quad (2.8)$$

Then the other form factor must have the following form:

$$g_1(M^2) = g_1^+(M^2) - g_1^-(M^2),$$

$$g_1^-(M^2) \neq 0 \quad \text{for some value of } M^2. \quad (2.9)$$

The low-energy data, if it is possible, can restrict the form factors through the relations

$$\int dM^2 \frac{g_1(M^2)}{M^2} = 1, \quad \int dM^2 \frac{g_2(M^2)}{M^2} = \frac{1}{6} \langle r_1^2 \rangle, \quad (2.10)$$

where $\langle r_1^2 \rangle$ means the mean square radius (m.s.r.). At present we have no information concerning its magnitude or sign. The m.s.r. $\langle r_2^2 \rangle$ is positive from our assumption, but the sign of $\langle r_1^2 \rangle$ can be taken to be positive or negative depending on future information. (If experiments are to tell us that $\langle r_2^2 \rangle$ is negative, the role of $g_1$ and $g_2$ in the following discussion should be reversed.) Since the knowledge of the m.s.r. may determine the behavior of the spectral functions at the low mass values, we must consider the two cases of positive and negative $\langle r_1^2 \rangle$.

If the m.s.r. $\langle r_1^2 \rangle$ is positive, which is probable, we can guess that the positive part of the spectral function should be predominant at the low mass value

$$g_1(M^2) \sim g_1^+(M^2) \quad \text{for small } M^2. \quad (2.11)$$

* We take the case of a positive mean square radius, as otherwise the argument becomes more complicated as in the next case.
On the contrary, if the m.s.r. $\langle r_1^2 \rangle$ is negative, though it is less probable, we can imagine that the negative part should be predominant at the low mass value and the positive part might play an important role for the moderate mass values

$$g_1(M^2) \sim \begin{cases} -g_1^-(M^2) & \text{for small } M^2, \\ g_1^+(M^2) & \text{for moderate values of } M^2. \end{cases} (2.12)$$

On the other hand, as the behavior of this spectral function for very large values of $M^2$ might determine the function $J$, (2.6) could be approximated by

$$J \sim \int_{M^2 \geq 0} dM^2 (g_1^+ (M^2) - g_1^- (M^2)) \frac{\log(M^2/A^2)}{M^2 - A^2}, \quad (2.13)$$

where $A^2$ is the effective mass value which comes from the form factor $g_2(M^2)$. Then, in order to get the right sign for the self-mass, the spectral function $g_1(M^2)$ should be

$$g_1(M^2) \sim -g_1^-(M^2) \quad \text{for large } M^2. \quad (2.14)$$

With this requirement, we get

$$0 > J \gtrsim \int_{M^2 \geq 0} dM^2 \frac{\log(M^2/A^2)}{M^2 - A^2} g_1^-(M^2) \gtrsim -\frac{1}{A} \int_{M^2 \geq 0} dM^2 g_1^-(M^2), \quad (2.15)$$

which can determine the lower limit of the strength of the form factor at high energy:\[15\]

$$F_1(q^2) \sim -\frac{1}{q^2} \int dM^2 g_1^-(M^2). \quad (2.16)$$

Combining the requirements (2.11) or (2.12) with (2.14), we can conclude as follows: If the m.s.r. $\langle r_1^2 \rangle$ is positive, the spectral function $g_1(M^2)$ should change its sign odd times. If the m.s.r. $\langle r_1^2 \rangle$ is negative, the spectral function $g_1(M^2)$ should change its sign even times.

The simplest examples of spectral function are illustrated in Figs. 1a and 1b. It is sufficient to take the two-Yukawa combination shape for the former, while for the latter the three-Yukawa combination shape is at least necessary.

![Fig. 1. The simplest examples of the spectral function](https://example.com/fig1)

1a: the case of $\langle r_1^2 \rangle > 0$

1b: the case of $\langle r_1^2 \rangle < 0$
§ 3. Electron and muon masses

We discuss the possibilities of the model of form factors in order to get the electron and muon masses. Whatever the form factors might be, they must give the following orders for the functional $\mathcal{A}$:

$$
\mathcal{A}^{(e)} \simeq +0.5 \cdot (10^8 \sim 10^9) m_e^2 \quad \text{for electron},
$$

$$
\mathcal{A}^{(\mu)} \simeq + (10^{10} \sim 10^{11}) m_\mu^2 \quad \text{for muon},
$$

Though there may be many different possibilities of constructing independent models for the electron and muon, the discussions are hereafter restricted to the cases with some similarity in both particles’ structures.

The simplest case of this kind is the one in which the structures of both particles can be related with each other by a scale transformation

$$
gf^{(e)}(M^2) = gf^{(\mu)}(\lambda M^2). \tag{3.2}
$$

We then get the relations

$$
\langle r_i^2 \rangle^{(e)} = \lambda \langle r_i^2 \rangle^{(\mu)}, \tag{3.3}
$$

Fig. 2. The charge distribution for the case of $\kappa < 0$

$\rho_e$: the electron’s distribution with $\langle r^2 \rangle = (0.1)\AA^2$

$\rho_\mu$: the muon’s distribution with $\langle r^2 \rangle = (0.6 \cdot 10^{-2})\AA^2$
from (2.10) and (2.6). If we take $\kappa$ as universal, we must have $\lambda \approx 200$ in order to get the electron-muon mass difference. This means that the m.s.r. of the electron is far larger than that of the muon, similar to the classical situation.

If we take the assumption of (2.7) for the case of $\kappa < 0$, the m.s.r. of the electron must be

$$\langle r^2 \rangle^{(e)} \approx (0.1y)^2$$

with the approximation

$$\langle r_1^2 \rangle^{(e)} \approx \langle r_2^2 \rangle^{(e)}, \quad A^{(e)} \approx \frac{6}{\langle r^2 \rangle^{(e)}}.$$
The m.s.r. of the muon is then far smaller,
\[ \langle r^2 \rangle^\mu \sim (0.6 \cdot 10^{-5} y)^2, \]  
(3.7)
in virtue of the above argument.

Although the larger m.s.r. of the electron than the muon’s one is not inconsistent with the present information, we should also consider the possibility of assuming the different internal features for the structures with the fixed m.s.r.’s (including the case of \( \langle r^2 \rangle^e \sim \langle r^2 \rangle^\mu \)) and general shape. We illustrate an example for the case of \( \kappa > 0 \).

For the case of \( \langle r_1^2 \rangle > 0 \), we approximate the spectral form factors as
where $\alpha$ and $\beta$ are dimensionless constants with the restriction $1 \geq \alpha \geq \beta \geq 0$. The corresponding form factors in the energy-momentum space are given by

$$F_1(q^2) = \frac{1}{\alpha - \beta} \frac{1 - \beta}{1 - \alpha} \log \frac{1 - \beta}{1 - \alpha},$$

$$F_1(q^2) = \frac{1}{\alpha - \beta} \left( \frac{1 - \beta}{1 - \alpha} \log \alpha - \frac{1 - \alpha}{1 - \beta} \log \beta \right).$$

The function $A$ becomes then

$$A = \frac{6}{\alpha - \beta} \left[ \frac{1 - \beta}{1 - \alpha} \log \alpha - \frac{1 - \alpha}{1 - \beta} \log \beta \right],$$

with the assumption of $\langle r_1^2 \rangle = \langle r_2^2 \rangle = \langle r^2 \rangle$.

If we take

$$\langle r^2 \rangle = (0.1\gamma)^2$$

for the electron and muon, their masses are realized by the following sets of values:

i) $\alpha(e) \simeq \beta(e) \simeq 0.25$, $\alpha(\mu) \simeq \beta(\mu) \simeq 0.02$, (3·13)

ii) $\alpha(e) \simeq \alpha(\mu) \simeq 0.50$, $\beta(e) \simeq 0.08$, $\beta(\mu) \simeq O(10^{-23})$. (3·14)

For the first case, the assumption of $\alpha(e) \simeq \beta(e)$ is taken so as to get a unique solution, while the assumption of $\alpha(e) \simeq \alpha(\mu)$ made in the second case does not give a unique answer. The larger the value of $\alpha$ is, the smaller the corresponding values of $\beta(e)$ and $\beta(\mu)$.

For the case of $\langle r_1^2 \rangle < 0$, the situation is so ambiguous that it does not seem worthwhile to discuss it here.

Some examples of the charge distribution are given in Figs. 2, 3 and 4. The distributions raised in Figs. 2 and 3 are very different for the electron and muon, while the one in Fig. 4 is similar for both particles except for the extremely internal parts. The decision will be given by future experiments up to the region of 0.1\gamma.

§ 4. Discussions and conclusions

The considerations here presented depend on future information concerning the lepton structures. At present, we know neither of the value of the m.s.r. nor of the difference between the theoretical and experimental values of the anomalous
magnetic moments. Since the arguments sensitively depend on these data, we ten-
tatively consider every possible general case, i.e. the cases in which the m.s.r. would
be positive and negative and the discrepancy of the anomalous moment would also
be positive and negative. But the main effort was concentrated in the case of a
positive m.s.r. and \( \mu_{\text{exp}} - \mu_{\text{theor}} > 0 \), because it seemed most probable from the data
now available.

If we accept this case, it is reasonable to presume that the internal structure
has, near the center, a behavior opposite to the one which is shown in the outer
region. For instance, the charge distribution near the center turns out to be nega-
tive, which is very similar to the electromagnetic structure of the nucleon proposed
elsewhere.\(^{15}\)

While the electron mass could be explained by the reasonable value of the
m.s.r. \( \langle r^2 \rangle \sim (0.1\text{fm})^2 \), the explanation of muon mass is still curious from this
argument. Though we could explain it by assuming a strong form factor, why
this should be the case is still an open question. Much more information about
the muon structure should be accumulated, until we can get a definite conclu-
sion.

We started with the assumption of the existence of the discrepancy \( \mu_{\text{exp}} - \mu_{\text{theor}} \),
but we could also take the unrenormalizable interaction for the case \( \mu_{\text{exp}} = \mu_{\text{theor}} \).
However, the answer will then be less reliable, so we omitted to consider about
this case. If the spreading of the leptons could come from the weak interactions,
the electromagnetic interaction would also be singular as the weak interactions.

Concerning the phenomenological aspect of the lepton structure, some know-
ledge of it will be obtained from high energy data. With the development of the
experimental techniques, it is to be expected that the scattering experiments of
identical leptons, for instance, will reveal their structures, avoiding the type of
inconveniences involved, e.g. in the Stanford experiment, where it is difficult to
separate the structure effects due to either nucleon or electron.

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