A Note on the Leptonic Decay of Hyperons* 

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The leptonic decay modes of pions and K-mesons are studied from the viewpoint of compound model (Sakata model). By comparing these processes (π→μ+ν and K→μ+ν), a conjecture for the leptonic decay of hyperons (e.g., A→p+μ−+ν̄) is given which suggests that the (squared) bare coupling constant of this process is smaller than that of ordinary μ-decay or μ-capture process of nucleons by a factor ~10.

§ 1. Introduction

Recent experimental investigations on the weak interaction of elementary particles seem to suggest that all the leptonic decay processes of non-strange particles are caused by the "V(vector) − A(axial-vector) combination of four-fermion interactions", the scheme of which was proposed by many authors. As is well known, all these interactions have the coupling constants of the same magnitude (1.4×10^{-49} \text{ erg cm}^3) apart from the effects of renormalization due to the strong interactions. Under these circumstances, it may be quite interesting to clarify whether or not the above-mentioned universal V− A scheme can further be extended to the leptonic decay process of strange-particles such as, for example,

\[ A \rightarrow p + μ^- (e^-) + ̄ν \quad \text{(a)} \]

\[ Σ^- \rightarrow n + μ^- (e^-) + ̄ν \quad \text{(a')} \]

At present, however, experimental evidences concerning the process (a) or (a') are not yet sufficient to extract vital information about the structure of decay interactions of this process. Nevertheless, if we suppose that the leptonic decay of K-mesons should be induced by four-fermion interactions of the type (a) and/or (a'), it would be possible to some extent to investigate into the nature of the leptonic process of strange particles by using, as a clue, the transition probability of the decay

\[ K \rightarrow μ + ν \quad \text{(b)} \]

which has been well established by experiments.

In this note, we shall discuss the decay process (b) from the viewpoint of the Sakata model, and in comparing this process with the decay π→μ+ν a con-

* The essential part of this work was prepared for the Kiev Conference (1959). See "Proceedings of the 9-th Conference on High Energy Nuclear Physics" (Kiev, 1959).
jecture will be given that the (squared) unrenormalized coupling constants of the strangeness-non-conserving four-fermion interaction such as (a) should be smaller than those of the ordinary Fermi interactions by a factor 10, and in this sense the "universality" of the $V-A$ scheme is somewhat violated in so far as we are concerned with the leptonic process of strange particles. It is to be mentioned that a similar result has been suggested also by Oneda\textsuperscript{6}) in the phenomenological analysis of various decay processes including $K$-mesons and baryons.

§ 2. Relation between decays $K \rightarrow \mu + \nu$ and $\Lambda \rightarrow p + \mu^- + \nu$ in the Sakata model

The most direct way of clarifying the relation between the processes (a) and (b) is to treat our problem on the basis of the composite model proposed by Sakata in which, for example, the $K^+$-meson is considered to be a compound particle composed of the basic particles $p$ and $\bar{A}$ (anti-$\Lambda$). Let us assume that the decays $\pi \rightarrow \mu + \nu$ and $K \rightarrow \mu + \nu$ are induced by the following local $(V, A)$ four-fermion interactions respectively:

\begin{align}
H_1 &= f_v \left( \bar{\psi}_n \gamma_5 (1 + \gamma_5) \phi_n \right) \left( \bar{\psi}_p \gamma_5 \phi_p \right) + f_A \left( \bar{\psi}_n \gamma_5 \phi_n \right) \left( \bar{\psi}_p \gamma_5 \phi_p \right) + \text{h.c.} \tag{1}
\end{align}

\begin{align}
H_2 &= f_v' \left( \bar{\psi}_n \gamma_5 (1 + \gamma_5) \phi_n \right) \left( \bar{\psi}_p \gamma_5 \phi_p \right) + f_A' \left( \bar{\psi}_n \gamma_5 \phi_n \right) \left( \bar{\psi}_p \gamma_5 \phi_p \right) + \text{h.c.} \tag{2}
\end{align}

with obvious notations. We have only to remark that $f$'s and $\phi$'s are unrenormalized quantities. We assume here $K$-mesons are pseudoscalar as well as pions, and the relative parity of $\Lambda$ and nucleon is even. This parity assignment seems to be preferable according to the recent analyses.\textsuperscript{5}) In our model, pions and $K$-mesons are described by the Bethe-Salpeter wave functions for bound states as follows\textsuperscript{6,7}:

\begin{align}
\varphi_r (x, y) &= \langle \Omega | T \left[ \psi_p (x) \bar{\psi}_n (y) \right] | \pi \rangle \tag{3}
\end{align}

\begin{align}
\varphi_k (x, y) &= \langle \Omega | T \left[ \psi_p (x) \bar{\psi}_n (y) \right] | K \rangle, \tag{4}
\end{align}

where $|\Omega\rangle$ and $|\pi\rangle$ (or $|K\rangle$) respectively denote the vacuum and the one-pion (or one-kaon) state.\textsuperscript{*} Then, in terms of these amplitudes the $S$-matrix for the decay, e.g. $\pi \rightarrow \mu + \nu$, is expressed (in the lowest order perturbation of (1)) by

\begin{align}
\langle \mu, \nu | S | \pi \rangle &= i (2\pi)^4 \delta (p_\mu + p_\nu - p_\pi) \left( \bar{u} (p_\mu) \gamma_\mu \gamma_5 (1 + \gamma_5) u (p_\pi) \right) f_A \text{Tr} [\gamma_\lambda \gamma_5 \varphi_r (0)],
\end{align}

where $p$'s are the momenta of respective particles and $u$ the Dirac wave function, noticing the relation:

\begin{align}
\langle \Omega | \bar{\psi}_n \gamma_\mu \gamma_5 \psi_p (0) | \pi \rangle &= \text{Tr} [\gamma_\lambda \gamma_5 \varphi_r (x, y)] |_{x=y=0} = \text{Tr} [\gamma_\lambda \gamma_5 \varphi_r (0)].
\end{align}

In the same way, for the decay $K \rightarrow \mu + \nu$, we have

\begin{flushleft}
\textsuperscript{*} (3) and (4) are the expressions for positively charged bosons.
\end{flushleft}
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\[ \langle \mu, \nu | S | K \rangle = i (2\pi)^4 \delta (p_\mu + p_\nu - p_K) \langle \bar{u}(p) \gamma_\lambda (1 + \gamma_5) u(p) \rangle f_\lambda \text{Tr}[\gamma_\lambda \gamma_5 \varphi_K(0)]. \]  

(6)

The \( \varphi_\mu \) and \( \varphi_K \) in these formulae are the normalized amplitudes which should be determined by the normalization condition:

\[ i \int_{0}^{1} \langle B | \bar{f}_\mu \gamma_\beta \varphi_\mu | B \rangle \, d\sigma_\mu = 1, \]  

(7)

where \( |B\rangle \) denotes \( |\pi\rangle \) or \( |K\rangle \). For later purposes, we give here a general method to find the normalized amplitude. First, we start with the space-time integration of the expectation value for the current operator density, viz.

\[ \int \langle B | j_\mu (x_0) | B \rangle \, d^4x_0 = \langle B | j_\mu (0) | B \rangle \, V T \]

\[ = \int d^4x_1 \cdots d^4x_4 \bar{\varphi}_H (x_1, x_2) G_\mu (x_1, x_2, x_3, x_4) \varphi_H (x_3, x_4), \]  

(8)

where \( V T \) is the space \( (V) \)-time \( (T) \) volume of the world, \( G_\mu \) the suitable Green's function and \( \bar{\varphi}_H \) the adjoint wave function of \( \varphi_H \). Then, separating the centre-of-mass motion from \( \varphi_H \):

\[ \varphi_H (x, y) = \tau_H (\xi) e^{i P x} \]  

(9)

in which \( \xi = x - y \), \( P \) the total energy-momentum and \( X \) the centre of mass, (8) may be transformed into the form

\[ \langle B | j_\mu (0) | B \rangle = N \int d^4\xi d^4\eta \bar{\varphi}_H (\xi) J_\mu (\xi, \eta ; P) \nu_H (\eta), \]  

(10)

where \( \tau_H (\xi) \) is replaced by \( N^{1/2} \nu_H (\xi) \). In (10), \( N \) denotes the normalization factor and \( \nu_H \) means a norm-undetermined amplitude which is directly determined from the homogeneous equation for the bound states. Defining \( \mathcal{J} (P^2) \) to be

\[ \int \bar{\nu}_H J_\mu \nu_H = 2P_\mu \mathcal{J} (P^2), \]  

(11)

we find the required amplitude:

\[ \tau_H (\xi) = \frac{1}{\sqrt{2P_0 V}} \mathcal{J}^{-1/2} (P^2) \nu_H (\xi). \]  

(12)

For a pseudoscalar boson, \( \nu_H (0) \) generally takes the form

\[ \nu_H (0) = a (P^2) \gamma_5 + b (P^2) \gamma_5 (\gamma P). \]  

(13)*

Accordingly, the terms \( \text{Tr} (\cdots) \) in (5) and (6) can read

* If we define the wave function \( \varphi_B \) by \( \varphi_B = \langle 0 | T (\psi_{\mu} \phi_\nu) | B \rangle (\psi_{\mu}' = \psi_\nu' (or \psi_\mu') \text{ for } |B\rangle = |\pi\rangle \text{ (or } |K\rangle\rangle) \), where \( \psi_{\mu}' = C \psi_\mu \) \( (C: \text{the charge conjugation matrix}) \), we have \( \nu_B = [a \gamma_5 + b \gamma_5 (\gamma P)] C \) instead of (13).
It may be remarked here that the factor
\[ 4 \mathcal{J}(P^2) a(P^2) \bigg|_{P^2 + m^2_B = 0} \] 

is to be considered as a quantity which corresponds to the renormalization constant \( Z_s^{1/2} \) for one-body boson propagator, although in our case it might not be interpreted as the bare particle probability.

Let us turn to experimental situations which provide information about the decays \( \pi \rightarrow \mu + \nu \) and \( K \rightarrow \mu + \nu \). When we describe these processes by the phenomenological Hamiltonian:

\[
H_{\text{eff}}^\pi = \frac{g_s}{\mu_\pi} \left( \bar{\phi}_s \gamma_\lambda (1 + \gamma_5) \phi_s \right) \frac{\partial \phi_s}{\partial x_\lambda} + \text{h. c.} \quad (15)
\]

\[
H_{\text{eff}}^K = \frac{g_K}{\mu_K} \left( \bar{\phi}_K i r (1 + \gamma_5) \phi_K \right) \frac{\partial \phi_K}{\partial x_\lambda} + \text{h. c.}, \quad (16)
\]

where \( \phi_s \) and \( \phi_K \) are the field operators for pions and K-mesons respectively, we find the relation

\[
\left( \frac{g_s}{\mu_\pi} \right)^2 \approx 16 \quad (17)
\]

from the magnitudes of respective transition probabilities.\(^9\) In terms of the expression (14), we obtain instead of (17) the relation

\[
\left( \frac{f_A \beta_s}{f_A \beta_K} \right)^2 \approx 16 \quad (17')
\]

where \( \beta = 4 \mathcal{J}^{-1/2}(\sim \mu_B^2) b(\sim \mu_B^2) \) (see (14)), which correlates the magnitudes of the wave functions of \( \phi_s \) and \( \phi_K \) at the origin to the (unrenormalized) coupling constants in the decay processes (1) and (2).\(^*\)

Up to this stage, we have not introduced any specific assumptions or approximations concerning the structure of pions and K-mesons. The relation (17') is, therefore, valid even for the standard theory of pions and K-mesons which describes these particles as 'elementary'. However, the theoretical predictions for the ratio \( \beta_s / \beta_K \) may be different according to the kind of the model for pions and K-mesons adopted. In any case, it is to be remarked that, if we assume the scheme of "universal V—A Fermi interaction" to hold even for the strangeness-violating leptonic decays, we should have the relation \( \beta_s^2 \approx 16 \beta_K^2 \) from experiments.

\(^*\) A similar attempt has been developed also by S. Tanaka.\(^10\) But since his approach was essentially restricted to the nonrelativistic two-body approximation, no general relations such as (17') were obtained.
We shall now try to estimate the ratio \( \beta_\pi/\beta_K \) according to the Sakata model. As was investigated previously,\(^{11}\) the pions and \( K \)-mesons are described as the bound states resulting from the effective direct interactions of the type:

(i) \((pO\pi) (\bar{n}O\rho)\) for \( \pi^\pm \)

(ii) \((pO'\Lambda) (\bar{\Lambda}O'\rho)\) for \( \Lambda^\pm \),

where \( O \) and \( O' \) denote the Dirac matrices. The general forms of these interactions required for obtaining the pseudoscalar mesons are found to be an arbitrary linear combination of \( P \) and \( A \), if we confine ourselves to the ladder approximation (or chain-approximation\(^7\)).

We can, however, simplify our situation one step further, assuming that the structure of the interaction of (i) and (ii) are the same and the only one type of interactions viz. \( P \) would be dominant. The first assumption is a consequence of a desired symmetry property of the Sakata model recently introduced by Ogawa and others.\(^{12}\) In this proposed symmetry, the three basic particles \( p, n \) and \( \Lambda \) play essentially the same role in the system of strongly interacting particles. In an ideal case in which \( p, n \) and \( \Lambda \) are treated completely on the equal footing, we immediately obtain the result:

\[
\beta_\pi = \beta_K. \tag{18}
\]

If \( \beta \) does not critically depend on \( \mu_p \), we may conclude that \( (\beta_\pi/\beta_K)^2 \approx 1 \) even for the actual case in which the mass difference \( \mu_K - \mu_\pi \) and \( \kappa_1 \) (mass of \( \Lambda \)) \(-\kappa \) (mass of the nucleon) are not equal to zero. In fact, this is the case when we introduce the second assumption (the predominance of pseudoscalar interaction). We feel that this assumption is also a conceivable one, since it provides the effective \( PS(p\rho) \) coupling as the main part of the pion-nucleon interactions.\(^7\) Under these assumptions and following the general prescription of normalization developed at the first stage of this section, the \( \beta_\pi \) and \( \beta_K \) are easily evaluated to be\(^*\)

\[
\beta_\pi = -\frac{\sqrt{2}}{2\pi} \kappa \left( \ln \frac{\lambda^2}{\kappa^3} + \frac{1}{6} \frac{\mu_\pi^2}{\kappa^3} + \cdots \right) \left( \ln \frac{\lambda^2}{\kappa^3} + \frac{1}{2} + \frac{1}{2} \frac{\mu_\pi^2}{\kappa^3} + \cdots \right)^{-1/2} \tag{19}
\]

\[
\beta_K = -\frac{\sqrt{2}}{2\pi} \frac{\kappa + \kappa_1}{2} \left( \ln \frac{\lambda^2}{\kappa_1^3} + \frac{1}{6} \frac{\mu_K^2}{\kappa_1^3} + \cdots \right) \left( \ln \frac{\lambda^2}{\kappa_1^3} + \frac{1}{2} + \frac{1}{2} \frac{\mu_K^2}{\kappa_1^3} + \cdots \right)^{-1/2} \tag{20}
\]

respectively, where \( \lambda \) is the cutoff mass in Feynman's cutoff device \((\lambda \geq \kappa)\). We have omitted in these expressions the terms of higher order of expansion in \( \mu_\pi/\lambda^2 \) or \( \mu_K^2/\kappa_1^3 \) and some terms which depend on \((\kappa_1^3 - \kappa^3)/\kappa_1^3 \) since the corrections due to these terms seem to be too small to modify our discussion. From (19) and (20), one obtains

\* Calculations are made along the same line as developed in reference 7).
(\beta_s/\beta_K)^2 = 0.9 \sim 0.8 \quad (21)

for \ln (\chi^2/\epsilon^2) or \ln (\chi^2/\kappa^2) = 1 \sim 5.

It is to be noted that whenever the magnitude of \beta_s and \beta_K are essentially determined by the binding energies required for constructing pions and K-mesons from the constituent particles, we always obtain the relation \((\beta_s/\beta_K)^2 \approx 1\). Thus, if we accept this relation, implying \(f_4^2 \approx 16f_A^2\), we arrive at the conjecture stated in the preceding section.

§ 3. Discussions

In connection with our arguments, it would be interesting to compare our results with that obtained by using the dispersion technique based on the current meson theories. Recently, Sakita \(^{(3)}\) discussed this problem, applying the method developed by Goldberger and Treiman. \(^{(14)}\) His results seem to be not so different from those obtained in this paper. The essential difference between both theories, however, lies in the fact that his results are concerned with the renormalized coupling constants (for the leptonic decay of hyperons) and depend on the other interaction constants such as of pion-nucleon and K-meson-baryon systems, whereas our conjecture is the one which correlates the unrenormalized constants directly with the magnitudes of wave functions of pions and K-mesons.

The relation between renormalized and unrenormalized constants should be further investigated to clarify the true structure of weak interaction of elementary particles.

If our tentative arguments for the smallness of the strangeness-violating interaction were found to reflect the actual case, the reason for the violation of "universality" of weak interaction would have to be explained from more profound knowledge on the internal structure of elementary particles.

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