Progress of Theoretical Physics. Vol. 23, No. 5, May 1960

Classification of Composite Bosons in the Sakata Model

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(Received January 14, 1960)

Assuming the Sakata model \((p, n, A)\) are basic, all other strongly interacting particles are composite particles) and neglecting moderately strong interactions which contribute to \(N-A\) mass splitting, we find the complete symmetry between three fundamental fields (referred to as global symmetry). Under this global approximation, classification of two baryon pair states—which are supposed to represent physical mesons—is described.

§ 1. Introduction

Recently, Ogawa and the present author\(^1\) have independently developed a composite theory of strongly interacting particles, which is a special case of the so-called Sakata model\(^2\). We assume that all strongly interacting particles are composed of three basic particles (bare \(p, n\) and \(A\)), and introduce two different stages of approximations, the global and charge independent approximations (see I and II).** If we ignore the moderately strong interactions (MSI) responsible for \(N-A\) mass splitting, we find a complete symmetry among three basic particles. We refer to this symmetry property as to the global symmetry. The approximation in which MSI's are neglected shall be called the global approximation. In the charge independent approximation, MSI's are taken into account but electromagnetic and weak couplings are still neglected.

In the global approximation, basic fields, \(B=(p, n, A)\), have two invariant groups (the isospin rotation and the permutation \(n\leftrightarrow A\)). Thus (bare) \(p, n, A\) can be regarded as the three-dimensional irreducible representation of our groups. Mesons*** are supposed to be composed of baryon pairs.\(^3\) Group theoretically, meson states would be equivalent to, and hence be contained in, the representations \(\overline{B} \times B, \overline{B} \times B \times \overline{B} \times B\), etc. The simplest representation corresponding to \(\overline{B} \times B\) and its irreducible decomposition have already been given.\(^3\) \(9(-3 \times 3)\)-dimensional representation can be decomposed into 1- and 8-dimensional representations, and the latter has been assigned in I and II as \(\pi\) (pions), \(K, \overline{K}\) (kaons), and \(\pi'\) (the isosinglet, non-strange, neutral meson). However, the \(\pi\pi\) resonance state \((I=J=1)\), suggested recently,\(^4\) and the Dubna-particle,\(^5\) if they exist, are not contained.

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** See references 1) and 3).

*** Baryons have been discussed fully in I and II.
in this simplest choice of "configurations". Therefore we are going to investigate
in this paper the representation equivalent to \( \bar{B} \times B \times \bar{B} \times B \) (which corresponds,
roughly speaking, to the two-baryon-pair states).

Perhaps it would be appropriate to state again the philosophy underlying
our analysis (see I, II). First of all, we assume that the global approximation
does make sense and the MSI's give rise to relatively small and smooth changes
in the physical situation. The very strong interactions (which do not split \( N \)-\( A \)
masses) are supposed to be chosen in such a way that only a very small number
of particles corresponding to the irreducible representations be allowed to appear
as stable bound states and/or resonance levels: All members of the irreducible
group represent the physical particles (baryons or mesons) - in the global approxi-
mation — and must have the same spin-parity as well as identical mass in the global
approximation. The introduction of MSI's splits the members in the irreducible
set into several isomultiplets, which correspond to physical particles found in nature.

We describe the results of irreducible decomposition of \( \bar{B} \times B \times \bar{B} \times B \) in Section
2, and give some discussions in Section 3.

§ 2. Irreducible decomposition of \( \bar{B} \times B \times \bar{B} \times B \)

The representation corresponding to \( \bar{B} \times B \times \bar{B} \times B \) has \( 3 \times 3 \times 3 \times 3 = 81 \)
dimensions, and can be decomposed into the following irreducible sets in the global
approximation, (I) — (IV) : \( 1 \times 2 + 8 \times 4 + 27 + 10 \times 2 = 81 \).

( I ) Trivial representation \( (g(0)) \), containing only one particle.
   It appears twice, and is equivalent to \( \pi_0 \) in II.

(II) 8-dimensional representation \( (g(8)) \)
   It appears 4 times, and is equivalent to \( \pi, K, \bar{K}, \pi' \) in II.

(III) 27-dimensional representation \( (g(27)) \). Its members can conveniently be
   expressed as \( A_{I_1}^{I_2} \), where \( I, S \) and \( Q \) are the isospin, the strangeness and
   the electric charge in units of protonic charge. There are \( A^{20}, A^{3/2}, A^{-3/2}, A^{12}, A^{10}, A^{8}, A^{6}, A^{4}, A^{2}, A^{0} \).

   Under the permutation \( m \rightarrow \lambda \) of basic fields (called the G-transformation in
   II), these particles transform as follows :

   \[
   Q = 2 \\
   A^{20} \rightarrow A^{12} \\
   A^{12} \rightarrow \frac{1}{\sqrt{6}} A^{3/2} + \frac{5}{6} A^{1/2} \\
   Q = 1 \\
   A^{10} \rightarrow \frac{1}{6} A^{3/2} + \frac{5}{6} A^{1/2} \\
   A^{10} \rightarrow \frac{1}{\sqrt{6}} A^{3/2} + \frac{5}{6} A^{1/2} \\
   A^{1/2} \rightarrow \frac{1}{6} A^{3/2} + \frac{5}{6} A^{1/2} \\
   
   
\]
There appear two 10-dimensional representations, \( g^{(10)} \) and \( \bar{g}^{(10)} \). However, each particle in \( \bar{g}^{(10)} \) is the antiparticle of that belonging to \( g^{(10)} \), respectively. \( g^{(10)} \) contains the following members \((B^{10})\): \( B^{3/2}, B^{1/2}, B^{1}, B^{0}, B_{\pm} \) and \( B_{\pm}^{\pm} \) \( (4+2+3+1=10) \). The \( G \)-transformation for \( B_{\pm}^{\pm} \) is given as follows:

\[
\begin{align*}
Q &= 0 \\
A_{5/2}^{1/2} &\rightarrow \sqrt{\frac{5}{6}} A_{1}^{0} + \sqrt{\frac{1}{6}} A_{1}^{0} \\
A_{3/2}^{3/2} &\rightarrow A_{-}^{1/2} \\
A_{2}^{10} &\rightarrow \frac{1}{6} A_{2}^{10} + \frac{\sqrt{5}}{2 \sqrt{3}} A_{2}^{10} + \frac{\sqrt{5}}{3} A_{2}^{09} \\
A_{2}^{10} &\rightarrow \frac{\sqrt{5}}{2 \sqrt{3}} A_{2}^{10} + \frac{1}{2} A_{2}^{10} - \frac{1}{\sqrt{3}} A_{2}^{09} \\
A_{2}^{09} &\rightarrow \frac{\sqrt{5}}{3} A_{2}^{10} - \frac{1}{\sqrt{3}} A_{2}^{10} + \frac{1}{3} A_{2}^{09} \\
A_{2}^{3/2} &\rightarrow \frac{2}{3} A_{2}^{3/2} - \frac{\sqrt{5}}{3} A_{2}^{3/2} - \frac{2}{3} A_{2}^{3/2} \\
Q &= -1 \\
A_{2}^{20} &\rightarrow \frac{1}{\sqrt{6}} A_{2}^{20} + \sqrt{\frac{5}{6}} A_{2}^{1/2} \\
A_{2}^{10} &\rightarrow \sqrt{\frac{5}{6}} A_{2}^{1/2} - \sqrt{\frac{1}{6}} A_{2}^{1/2} \\
A_{2}^{3/2} &\rightarrow \frac{1}{\sqrt{6}} A_{2}^{3/2} + \sqrt{\frac{5}{6}} A_{2}^{3/2} \\
A_{2}^{1/2} &\rightarrow \sqrt{\frac{5}{6}} A_{2}^{3/2} - \sqrt{\frac{1}{6}} A_{2}^{3/2} \\
A_{2}^{3/2} &\rightarrow A_{2}^{3/2} \\
Q &= -2 \\
A_{2}^{20} &\leftrightarrow A_{2}^{20} \\
A_{2}^{3/2} &\leftrightarrow A_{2}^{3/2} \\
\end{align*}
\]
It should be noticed that particles, as well as their antiparticles, are contained in the same irreducible groups, except for \( g(10) \) and \( g(10) \). Therefore if the particles corresponding to \( g(10) \) are supposed to exist, \( g(10) \) should be taken into account simultaneously.

§ 3. Discussions

In the preceding section we have found 4 different classes of possible boson states. We must now discuss which of these groups of bosons should be identified as the mesons existing in nature and what follows from a special choice. At least we must pick up a group which contains bosons with \((I=1, S=0)\) and \((I=1/2, S=\pm 1)\). There are three possibilities \( g(8) \), \( g(27) \) and \( g(10) \) (plus its anti, \( g(10) \)).

The possibility of picking up \( g(8) \) has already been discussed fully (see I and II), and no further comments will be given here.

\( g(27) \) contains too many members and will not be welcome by anybody.*

The most interesting possibility is provided by \( g(10) \) and its anti \( g(10) \). There are two kinds of particles, \( B^{10} \) and \( B^{10} \), which have \( I=1, S=0 \), which have equal masses in the global approximation, and are the antiparticles of each other. Nevertheless, by a suitable choice of MSI’s we can assure that

\[
B_1^{10} = \frac{B^{10} + B^{10}}{\sqrt{2}} \quad \text{and} \quad B_2^{10} = i \frac{B^{10} - B^{10}}{\sqrt{2}}
\]

should be properly called the physical particles. Then we can establish the following property,

\[
B_1^{10} \rightarrow B_1^{10}
\]

\[
B_2^{10} \rightarrow -B_2^{10}
\]

under the charge conjugation (cf. the relationship between \( K^0, \bar{K}^0 \) and \( K^\pm, \bar{K}^\pm \)). There are no a priori reason why \( B_1^{10} \) and \( B_2^{10} \) should have equal masses in the charge independent approximation. There are two interesting assignments one can think of.

(a) We can try to identify \( B_1^{10} \), \( B^{1/2-1} \) and \( B^{1/2-1} \) as \( \pi, K \) and \( \bar{K} \), respectively. Then all \( B^{1/2} \) must have spin-parity 0− even in the charge independent approximation. Notably, we can identify pseudoscalar \( B^{2-2} \) and \( B^{2} \) as the Dubna particles whose possible existence has been suggested recently. There are also remaining three:

* We have excluded the possibility of the Dubna particles being isotriplet.
$B_{1}^{10}$, $B_{3/2}^{1+}$ and $B_{3/2}^{-1}$. They must be either stable or quasistable (say, resonance states appearing in collision processes).

(b) Another possibility is also worth mentioning. We assume that $\pi$, $K$ and $\bar{K}$ belong to $g^{(8)}$, whereas $B_{1}^{10}$ is nothing but the $\pi\pi$ resonance state ($J=I=1^{-}$). Then all particles of resonance levels containing $g^{(10)}$ and $\bar{g}^{(10)}$ must have spin-parity $1^{-}$. There are two meson-meson resonance levels, $B_{1}^{10}$ and $B_{1}^{10}$, which might be responsible for the double humps in $\pi^{-}\rho$ total cross section around the so-called second resonance. $B_{3/2}^{1+}$ and $B_{3/2}^{-1}$ should be regarded as $\pi K$ or $\pi \bar{K}$ resonances perfectly analogous to the $\pi\pi$ resonance level (unless they are stable). Finally, we can conclude that the Dubna particles, $B_{0}^{2}$, are of vector type.

It is extremely interesting to see the spin-parity of Dubna particles experimentally, provided, of course, that such particles do exist. We can predict that the Dubna particles must be either pseudoscalar or vector.

Since rather comprehensive discussions on our composite theory have already been given in I and II, we do not think it necessary to add here any further comments on our subject.

References

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