Multistage stochastic linear programming model for daily coordinated multi-reservoir operation

Yongdae Lee, Sheung-Kown Kim and Ick Hwan Ko

ABSTRACT

Operation planning for a coordinated multi-reservoir is a complex and challenging task due to the inherent uncertainty in inflow. In this study, we suggest the use of a new, multi-stage and scenario-based stochastic linear program with a recourse model incorporating the meteorological weather prediction information for daily, coordinated, multi-reservoir operation planning. Stages are defined as prediction lead-time spans of the weather prediction system. The multi-stage scenarios of the stochastic model are formed considering the reliability of rainfall prediction for each lead-time span. Future inflow scenarios are generated by a rainfall–runoff model based on the rainfall forecast. For short-term stage (2 days) scenarios, the regional data assimilation and prediction system (RDAPS) information is employed, and for mid-term stage (more than 2 days) scenarios, precipitation from the global data assimilation and prediction system (GDAPS) is used as an input for the rainfall–runoff model. After the 10th day (third stage), the daily historical rainfall data are used following the ensemble streamflow prediction (ESP) procedure. The model is applied to simulate the daily reservoir operation of the Nakdong River basin in Korea in a real-time operational environment. The expected benefit of the stochastic model is markedly superior to that of the deterministic model with average rainfall information. Our study results confirm the effectiveness of the stochastic model in real-time operation with meteorological forecasts and the presence of inflow uncertainty.

Key words | daily reservoir operation, ESP, GDAPS, RDAPS, scenario generation, stochastic programming

Yongdae Lee
Graduate School of Information Management and Security,
Korea University, 5-1 Anam-Dong,
Sungbook-Gu,
Seoul 136-701,
Korea

Sheung-Kown Kim (corresponding author)
Division of Information Management Engineering,
Korea University, 5-1 Anam-Dong,
Sungbook-Gu,
Seoul 136-701,
Korea
Tel.: +82 2 3290 3385
Fax: +82 2929 5888
E-mail: *kimsk@korea.ac.kr*

Ick Hwan Ko
Korea Institute of Water and Environment,
Korea Water Resources Corporation,
462-1 Jeonmin-dong, Yuseong-Gu,
Daejeon 305-730,
Korea

INTRODUCTION

Coordinated, multi-reservoir system, operation planning is essentially the task of the optimum allocation of water among the dams in the system to determine the storage level and release quantity of each reservoir over the planning period. Most of the analytic frameworks for multi-reservoir system operation involving deterministic optimization and simulation models were established in the 1970s and early 1980s (Yazicigil *et al.* 1983; Can & Houck 1984). Yeh (1985), Wurbs (1993) and Labadie (2004) provided extensive lists of references on the use of simulation and optimization methods in reservoir-system operation. However, we cannot ignore uncertainties such as the uncertainty of parameter values, input data, uncertainty associated with projections of future demands, or uncertainty of inflow. Sensitivity analysis or parametric programming techniques were used to analyze the risk and uncertainty of parameter or input data. These techniques can help estimate the extent to which we need to reduce these uncertainties. In this study, we use a scenario-based, stochastic linear programming approach to cope with the inflow uncertainty because it is a clear source of uncertainty. We assume that demand uncertainties in water supply planning are less significant than the inflow uncertainty. Setting water demand as a constraint is sufficient, since we know that a hydrologic basin cannot deliver more water than is available, and demand requirements are usually determined by the historical records, and set by contracts.

doi: 10.2166/hydro.2008.007

Downloaded from https://iwaponline.com/jh/article-pdf/10/1/23/386254/23.pdf
by guest

Many stochastic optimization models such as stochastic dynamic programming (SDP) (Loucks *et al.* 1981; Yakowitz 1982; Stedinger *et al.* 1984), chance-constrained programming (CCP) (ReVelle *et al.* 1969; Houck 1979), two-stage stochastic linear programming (SLP) with recourse (Seifi & Hipel 2001) and multi-stage stochastic programming (Pereira & Pinto 1985; Jacobs *et al.* 1995; Watkins *et al.* 2000; Kracman *et al.* 2006) have been studied to deal with inflow uncertainty. SDP models can generate effective and feasible release policies for every possible reservoir storage state in each month. However, SDP usually requires discrete state variables and is limited by the so-called "curse of dimensionality" (Yeh 1985; Pereira & Pinto 1985). CCP is focussed on the reliability of the solution, which sets a minimum requirement on the probability of constraint satisfaction. This problem can be converted to a general deterministic linear programming problem, but it is often non-convex unless the right-hand side has a log-concave multivariate probability density function (Prékopa 1995). Hence, SLP seems to be the most practical approach to deal with the inflow uncertainty.

Two-stage SLP with recourse has been applied for long-term planning under uncertainty since it was first proposed by Dantzig (1955) and Beale (1955). In the first stage, the non-anticipative planning level decisions ("here and now") are usually determined independently of the subsequent variations in random scenarios, and then the control or operational decisions are made with consideration for the feasibility in the second stage, which is called a "recourse" action. The two-stage SLP can be expanded to a multi-stage problem. A finite number of the scenarios, corresponding to sequences of realizations of random variables at each stage, must be specified to apply the multi-stage SLP with recourse (Dupačová *et al.* 2000). The multi-stage SLP for reservoir operation seems to be a viable approach if we can identify stages at which a decision is required, and when release decisions taken at any stage of the decision process do not depend on future realizations of the inflows or on future release decisions. However, it has the shortcoming that the problem size of the deterministic equivalent of multi-stage SLP is usually very large and grows exponentially with the number of decision stages and linearly with the number of scenarios at each stage (Seifi & Hipel 2001). To overcome these difficulties, many solution methods such as the L-shaped algorithm (Van Slyke & Wets 1969; Wets 1988), Benders decomposition approaches (Jacobs *et al.* 1995), Dantzig–Wolfe decomposition (Dantzig & Madansky 1961) and interior-point methods (Ponnambalam *et al.* 1989; Seifi & Hipel 2001) have been proposed. However, with the increasing computing power and development of many solution packages (CPLEX, MSLiP-OSL, OSL-SP, etc.), the choice or construction of a suitable model, that takes into account the nature of the real-life problem, input data characteristics, software availability and computer technology, has become more important than algorithmic development (Dupačová *et al.* 2000).

Most of the SLP models for operation planning of coordinated multi-reservoirs are applied to long-term operational planning (Pereira & Pinto 1985; Jacobs *et al.* 1995; Watkins *et al.* 2000; Kracman *et al.* 2006). Seifi & Hipel (2001) suggested the use of two-stage SLP with a simple recourse model, using the interior-point method for the monthly operation of the Great Lakes. In this study, we suggest the use of multi-stage SLP with recourse for daily reservoir operation. Although the suggested model looks similar to other SLP models since it follows the general structure of multi-stage SLP, the problem nature of our model is quite different from others. Our model is unique in that the stages are defined as prediction lead-time spans of the weather prediction system, multiple objectives are utilized, and the model is designed for direct application to real-time operation.

Most of the studies on daily operation or real-time operation use deterministic approaches (Yazicigil *et al.* 1983; Mujumdar & Ramesh 1997; Eschenbach *et al.* 2001; Turgeon 2005; Kim *et al.* 2005). However, deterministic models are incapable of considering hedging decisions and the risk of water supply shortage or flooding against the inherent uncertainty of the inflow. In this study, we attempted to use SLP for the daily real-time operation simulation of multiple reservoirs in the Nakdong River Basin in Korea, and investigated possibility of using SLP for real-time daily operation. The probable benefits are evaluated and the results are summarized in the fourth section. Jacobs *et al.* (1995) suggested the use of a stochastic optimization model for medium-term optimal operation planning for PG&E's hydroelectric generation. Unlike ours, there is no report on the real benefits that might be obtained as a result of implementing the multi-stage SLP for medium-term planning operation. They formulated a scenario tree and forecasted the streamflow by using the characteristics of snowpack; furthermore, they

Downloaded from https://iwaponline.com/jh/article-pdf/10/1/23/386254/23.pdf
by guest

indeed defined the stages based on the accuracy of streamflow forecasting. However, their model is designed to find optimal scheduling of hydroelectric generation and is unsuitable for multi-reservoir systems such as the Korean hydrologic basin, where water supply is the primary purpose and snowpack is not a significant factor.

Since we wanted to have a multi-stage SLP model for real-time daily reservoir operation planning, we built the multi-stage scenario tree considering both the accuracy of short-term, mid-term or long-term meteorological forecasts, but unlike Watkins *et al.* (2000) or Kracman *et al.* (2006), we generated streamflow scenarios using the rainfall–runoff model with meteorological forecasting information in order to preserve the spatial and serial correlations. In this study, we formulate a multiperiod, multi-stage, SLP model to derive an efficient daily release plan for each reservoir and we suggest the use of it for daily, coordinated, multi-reservoir operation planning. The multi-stage scenarios for the stochastic model are formulated considering the reliability of the future inflow prediction for a specified forecasting lead-time. The future inflows are generated by a rainfall–runoff model based on the rainfall forecasts. In Korea, we rely on two climate modelling systems which are both operated by the Korea Meteorological Administration (KMA). One is the global data assimilation and prediction system (GDAPS) and the other is the regional data assimilation and prediction system (RDAPS). GDAPS is a numerical climate modelling system designed to predict weather condition by dividing the global atmospheric domain into vertical levels and horizontal resolutions using ocean wave profile and synoptic observations, including satellite retrieval data. RDAPS is also a numerical modelling system with high resolution 3D grids designed to predict regional weather condition, using GDAPS as its temporal boundary condition. It uses more enriched and detailed regional asynoptic data, such as wind and moisture profile, temperature, automatic weather station and radar data, to predict local precipitation. The type of rainfall prediction model for each prediction lead-time is selected based upon the viability and accuracy of the prediction. For short-term (2 days) rainfall forecasting, RDAPS is applied. The GDAPS is used for mid-term (10 days) rainfall forecasts. After the tenth day (third stage), the daily historical rainfall data are applied. The model becomes essentially a large-scale, linear programming model (comprising over 180,000 columns and 120,000 rows), and was solved using the mathematical programming solver, CPLEX 9.0. In the following section, the model formulation is presented. The third section presents a case study in which a simulation of daily reservoir operation for the Nakdong River Basin in Korea under uncertainty is performed under the real-time operational environment. In the fourth section, we evaluate the value of the stochastic model based on Birge's measure (Birge & Louveaux 1997). The conclusions and future extensions are presented in the final section.

MODEL FORMULATION

To derive an efficient daily release plan for each reservoir, a multi-period, multi-stage, SLP with recourse model is formulated. The planning decision involves determining the daily release plan, while the scenario-dependent control decisions on storage, hydroelectric energy generation, water supply, etc., constitute recourse actions to minimize any infeasibility and maximize the operational benefit. The multi-stage scenarios for the stochastic model are generated by a rainfall–runoff model based on the meteorological rainfall forecasts with consideration for the accuracy of future rainfall prediction for each forecasting lead-time span. The objective function of the model is defined as the weighted sum of multiple objectives. The weighting factors of each objective are assigned preemptively. The constraints sets include flow conservation of the multi-period, dynamic network flow, physical limitation of nodes and arcs, and non-anticipativity constraints.

Multi-stage inflow scenario tree

In order to elaborate the inflow uncertainty in the decision process explicitly, scenario-based multi-stage stochastic optimization is applied. A multi-stage scenario tree enables us to keep track of uncertain decision states following the model of rainfall prediction.

The accuracy of two-day rainfall forecasts has been improved by the increased meteorological information available due to the significantly better understanding of the climatic system. Accordingly, the accuracy of the

Downloaded from https://iwaponline.com/jh/article-pdf/10/1/23/386254/23.pdf
by guest

streamflow forecast has also been improved with the advance of computer modelling technology in hydrological science. However, rainfall forecasting is still not reliable if the prediction lead-time is longer than 10 days.

The multi-stage inflow scenario tree is built considering both the accuracy and reliability of short-term meteorological forecasts. The accuracy of the streamflow forecasting is dependent upon the quality of the rainfall forecasting for different lead-time spans. In the model, stages are defined as time spans corresponding to the available meteorological forecast information, as shown in Figure 1.

In the first stage (first 2 days), the rainfall forecast using information from RDAPS is applied. As RDAPS provides only 2 days' forecasts twice a day, we set 2 days as the time span of the first stage. In the second stage (from the third to the tenth day), the rainfall forecasts from the GDAPS are linked to the previous data, and the historical minimum and maximum rainfall information are supplemented because the accuracy of GDAPS forecasts are not sufficient. As GDAPS forecasts can provide meteorological information for 10 days once a day, 10 days were set as the end of the time span of the second stage. After the tenth day (third stage), the rainfall forecasts are unreliable so the observed daily precipitation data from every year available in the historical records were concatenated to obtain the ESP (Day 1985).

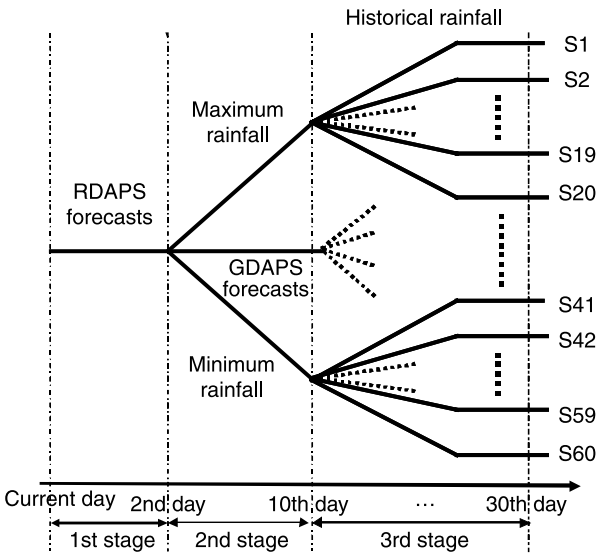

Figure 1 | The multi-stage scenario tree.

Although the multiple scenarios based on the historical inflow after the tenth day in the third stage are not reliable, it is crucial for non-anticipating decisions to attain the final storage target, taking into account the risk of serious flooding or drought in the sequential decisions. For the case of the real-time operation, only the first-stage decision is adopted with the good prospect of achieving the final storage target for each streamflow scenario. Therefore, the model's planning horizon is designed from today to the end of the month, and more than 10 days of multiple scenarios are required.

The statistics of the 2-day RDAPS rainfall forecasts are compared with automatic weather system (AWS) observations in Table 1 (KWRA (Korea Water Resources Association) 2003). In Table 1, the average correlation coefficient (CC) between RDAPS and AWS is 0.7358, which confirms the appropriateness of RDAPS forecasts for water resources planning. The statistics of the 10-day GDAPS rainfall forecasts

Table 1 | Statistics of RDAPS and AWS

Forecasting time (UTC)	AWS (mm)	RDAPS (mm)	CC	RMSE
2003.07.18.00	1.9007	1.9855	0.9250	2.0240
2003.07.18.12	0.4702	0.5153	0.8226	0.4691
2003.07.19.00	0.2666	0.7106	0.2295	1.0305
2003.07.19.12	0.3493	0.6524	0.8784	0.5558
2003.07.20.00	2.5275	0.8098	0.6271	5.6127
2003.07.20.12	4.7120	4.7120	0.7707	4.8136
2003.07.21.00	7.5025	6.0338	0.2743	8.8929
2003.07.21.12	7.1726	6.2250	0.7954	4.6269
2003.07.22.12	1.7903	0.4587	0.9925	3.6038
2003.07.23.00	0.0590	0.0635	0.6054	0.1286
2003.07.23.12	0.0439	0.0505	0.8466	0.0619
2003.07.24.00	0.0439	0.3362	0.9251	0.7330
2003.07.24.12	0.0479	0.1087	0.8723	0.1822
Average	2.0682	1.7432	0.7358	2.5181

Downloaded from https://iwaponline.com/jh/article-pdf/10/1/23/386254/23.pdf
by guest

Table 2 | Statistics of GDAPS and AWS

Forecasting time (UTC)	AWS (mm)	GDAPS (mm)	CC	RMSE
2003.07.13.12	5.2513	3.3185	0.6321	7.3821
2003.07.18.12	7.2818	9.5823	0.7542	6.3851
2003.07.21.12	5.7626	9.4032	0.6325	7.8541
2003.08.29.12	4.3854	0.2678	0.5478	7.8845
Average	5.6687	5.6429	0.6416	7.3764

are summarized in Table 2 (KWRA 2003). In Table 2, the average CC between GDAPS and AWS is 0.6416, which means the GDAPS forecasts are less reliable than the RDAPS forecasts, but is still useful information for mid-term forecasts.

The procedures to generate the inflow scenario using the rainfall forecasts information are represented in Figure 2. The rainfall forecast information was inputted into the rainfall–runoff model to generate a single streamflow forecast time series. The rainfall–runoff model is the rainfall–runoff forecasting system (RRFS) which was developed based on the streamflow synthesis and reservoir regulation model (SSARR) (US Army Corps of Engineers 2006) by the Water Resources Research Institute of Korea Water Resources Corporation (KOWACO). The rainfall forecasts have a spatial correlation. The streamflow generated based on the RRFS rainfall–runoff simulation, using the observed rainfall data for one month during the warm-up period, is found to preserve the serial and spatial correlations.

For real-time operation, the necessary component is the first-stage decision which is determined independently of future hydrologic events but the feasibility of the decision must retain consideration for the variation in future hydrologic scenarios. Nonanticipativity constraints in the model necessitate that decisions at the divergence points in the scenario tree must coincide for the scenarios which have a common path up to that point (Kracman *et al.* 2006). This ensures that the release decisions are made independently of the unknown hydrologic random event. Meanwhile, the future control decisions for storage and hydropower generation in the recourse action are specific to each inflow scenario in order to ensure the feasibility of the decision.

In real-time operation, the first-stage decisions on the release of each reservoir are applied. The next day, the rainfall forecasts are updated and the process of the prediction by multi-stage and scenario-based, stochastic linear program with a recourse model and the succeeding simulation run is repeated until the end of the month, as was suggested by Kim *et al.* (2000).

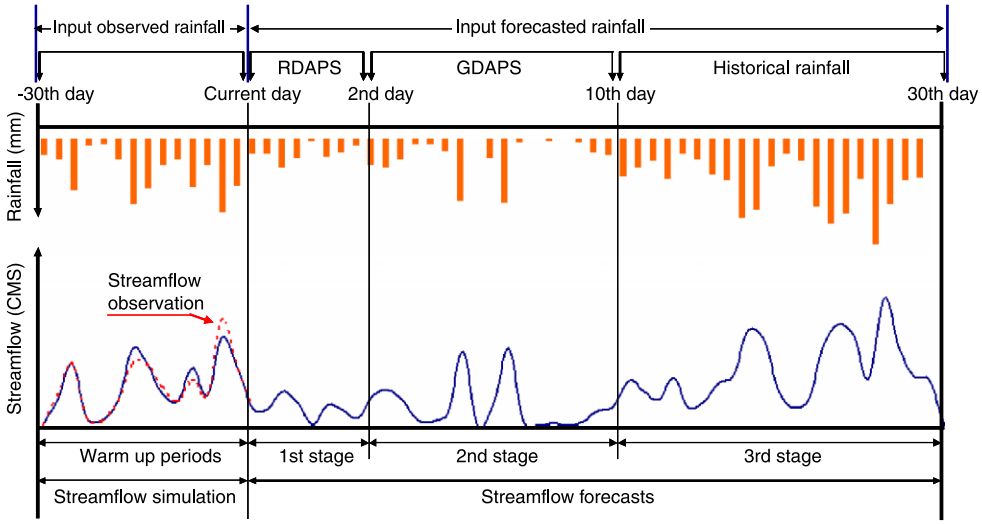

Figure 2 | The forecasted streamflow scenario generated by RRFS model based on the rainfall forecast.

Downloaded from https://iwaponline.com/jh/article-pdf/10/1/23/386254/23.pdf
by guest

Framework of the model

The deterministic equivalent of our multi-stage SLP conceptual model is written as follows:

$$\min {c_1}^{\mathrm{T}} x_1 + \sum_{k_2=2}^{N_2} p_{k_2} {c_{k_2}}^{\mathrm{T}} x_{k_2} + \sum_{k_3=N_2+1}^{N_3} p_{k_3} {c_{k_3}}^{\mathrm{T}} x_{k_3} \quad (1)$$

$$\text{s.t.}\, A_1 x_1 = b_1, \quad (2)$$

$$B_{k_2} x_1 + A_{k_2} x_{k_2} = b_{k_2}, \quad k_2 \in M_2, \quad (3)$$

$$B_{k_2} x_{ap(k_3)} + A_{k_3} x_{k_3} = b_{k_3}, \quad k_3 \in M_3, \quad (4)$$

$$\hat{x}^{ap(k_L)} - \hat{x}_{k_L}^{ap(k_L)} = 0, \quad k_L \in M_L, \quad L = \{2,3\}, \quad (5)$$

$$x_1, x_{k_2}, x_{k_3} \geq 0, \; M_2 = \{2,\ldots,N_2\}, \; M_3 = \{N_2+1,\ldots,N_3\} \quad (6)$$

where M_2, M_3 are the set of indices of divergence arcs at stages 2 and 3, respectively, in the scenario tree; k_L is the path (the set of arcs) at the Lth stage in the scenario tree; x_{k_L} is the set of decision vectors, such as release and storage of each reservoir at the Lth stage; C_{k_L} is a vector of cost coefficients for x_{k_L}. In order to relate this conceptual representation to our specific application, the vector of cost coefficients at the Lth stage (C_{k_L}) becomes the composite cost coefficients composed of costs for shortage of water demand, spill, storage and hydroelectric energy objectives. A more detailed description is summarized later in this section; $ap(k_L)$ is the immediate ancestor path (set of arcs) of the Lth stage of the k_L path, $ap(k_3) \in k_2$, and $ap(k_2) = 1$; p_{k_L} are the marginal probabilities of choosing the path in k_L at stage L and $\sum_{k_L \in M_L} p_{k_L} = 1$. The probability p_s of realizing specific path S at each stage can be obtained by multiplication of the marginal probability and the conditional probabilities. It is assumed to be $1/n$, $(P_s = 1/n(M_L))$, where $n(M_L)$ is the number of elements (arcs) of the set M_L at the Lth stage. p_s is the probability of individual scenarios S and becomes pk_3 in this third-stage case; A and B are matrices defining the constraint of the problem; and b represents the right-hand side under path k_L. Equation (5) enforces nonanticipativity by requiring that all paths that have the same immediate ancestor maintain the same decision. In Equation (5), $\hat{x}^{ap(k_L)}$ is the immediate ancestor variable that ensures that the planning decision vectors maintain the same decision for all paths having the same immediate ancestor at the Lth stage, and $\hat{x}_{k_L}^{ap(k_L)} \in x_{K_L}$ is the set of planning decision vectors (the release of each reservoir) of each path k_L at the Lth stage in the scenario tree.

The detailed model is presented in the appendix.

This multi-stage SLP model can be so large that special solution algorithms like the L-shaped method may be required (Van Slyke & Wets 1969; Wets 1988). Many solving programs (CPLEX, MSLip-OSL, OSL-SP, etc.) are currently available for this type of model (Dupačová *et al.* 2000). In this study, we used CPLEX 9.0.

Objective function

The model has multiple objectives. The objective function is defined as the weighted sum of each objective function value in Equation (7):

$$\min Obj = \sum_{t=1}^{2} \sum_{i=1}^{7} c_1^i Z_i^t + \sum_{t=3}^{10} \sum_{\omega \in \Omega(t)} p_2^\omega \sum_{i=1}^{7} c_2^i Z_i^{t,\omega} + \sum_{t=11}^{T} \sum_{\omega \in \Omega(t)} p_3^\omega \sum_{i=1}^{7} c_3^i Z_i^{t,\omega} \quad (7)$$

where c_L^i is the cost coefficient vector of the ith objective described in Table 3 at the Lth stage; and $Z_i^{t,\omega}$ is the ith objective value in Table 3 at time period t with scenario ω. In our application, stages are defined as prediction lead time

Table 3 | Priority of the objectives and basic weights of each objective

Obj. no.	Priority number	Description of objectives	Basic weights
z_1	1	Minimize the infeasibility	10^{11}
z_2	2	Satisfy the minimum requirement flow	10^9
z_3	3	Minimize the shortage of water demand	10^7
z_4	4	Attain the final target storage	10^6
z_5	5	Minimize the spill at the basin outlet	10^4
z_6	6	Maximize the storage	10^3
z_7	7	Maximize the hydroelectric energy generation.	10^2

Downloaded from https://iwaponline.com/jh/article-pdf/10/1/23/386254/23.pdf
by guest

spans, and the first-stage decision is made during the time period from $t = 1$ to $t = 2$. And at the first stage, we have only one scenario. The planning variable at the first stage is the release decision at each dam, which is made before the inflows are revealed, whereas the second-stage decisions are allowed to adapt to the revealed inflow except the ones that were determined at the first stage and set by the non-anticipativity constraint. The objective functions and their priorities and basic weights are summarized in Table 3. Some of the objectives are presented for the purpose of specifying a target for the goal programming.

Highest preemptive precedence is granted to minimize the constraint violation for hedging against varying inflow scenarios in the stochastic model. The second precedence is given to the minimum requirement flow objective at each control point. The minimum requirement flow is an operational constraint, but we specified it as the goal to be attained by applying an extremely high penalty in order to avoid the infeasibility problem. The shortage of water demand is also minimized by applying a very high weight to the penalty.

The daily operation model is supposed to attain monthly storage targets of each reservoir at the end of the month. We assumed that the monthly target storage can be assessed by a stochastic monthly operating model in the higher hierarchy of the decision process. Either two-stage SLP model (Lee *et al.* 2006) or the sampling SDP model (Kim *et al.* 2007) based on ESP can be used to provide the monthly storage targets. For successful daily operation, the trade-off analysis between storage maximization and release for hydroelectric energy generation has to be made in multi-objective analysis as demonstrated by Kim *et al.* (2005). However, such analysis is not the focus of this study.

The primary purpose of reservoir operation in Korea is water conservation for safe water supply. Therefore, we tried to minimize the spillage at the basin outlet from the perspective of basin-wide water conservation rather than at each dam site individually (Kim & Park 1998*a*, *b*; Kim *et al.* 2000). We also included the storage maximization objective. Keeping a high water level may increase the future safe water supply capability. To maximize hydroelectric energy generation, an iterative linear approximation technique associated with hydroelectric energy generation is used. The hydroelectric energy is calculated using the equation $\mathrm{kWh}_t = \gamma \times Q_t \times H_t \times \varepsilon \times \mathrm{hours}_t$. This calculation involves a nonlinear term, which is described by the product of two variables, head (H_t) and flow (Q_t). So the hydroelectric energy function is approximated by $\overline{\mathrm{kWH}_t} = \gamma \times \left\{q_t^0 \times H_t + h_t^0 \times Q_t - h_t^0 \times q_t^0\right\} \times \varepsilon \times \mathrm{hours}_t$ as suggested by Loucks *et al.* (1981), which is the first-order linear approximation of the hydroelectric energy function. Starting with the estimated average head, h_t^0, and flow, q_t^0, we solve the linearized mathematical model iteratively until we find a convergent optimal pair of head (H_t) and flow (Q_t) parameters, which are used to identify the acceptable optimal average flow and head corresponding to the coordinated optimal release and storage levels among the dams in the basin. The results of the hydroelectric energy generation are usually well matched with those in the historical records, if we use appropriate turbine head-flow-efficiency curves and assume efficient operation, during the post-analysis phase (Kim 1999).

Model constraints

The model is formulated as the multi-stage SLP model based on the multi-period optimization with embedded dynamic network flow. The network flow diagram of the Nakdong River Basin in Korea is represented in Figure 3.

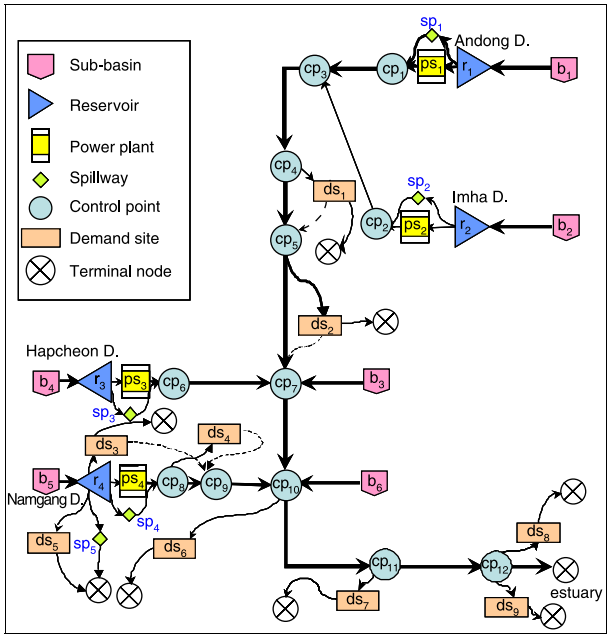

Figure 3 | Network flow diagram of Nakdong River Basin in Korea.

Downloaded from https://iwaponline.com/jh/article-pdf/10/1/23/386254/23.pdf
by guest

We classified the nodes into eight types which are represented in the legend of Figure 3. Each node has its own characteristics. The sub-basin is the super-source of the network flow. The reservoir storage is described by the flow into carryover nodes, which stores water from one period to the next. The use of carryover nodes therefore means that the network flow can be modelled by the multi-period, dynamic network model. The storage capacity of the reservoir is also divided into multiple zones, each of which is represented by a segment of the carryover nodes corresponding to the storage zones of the reservoir (Kim *et al.* 2000).

The hydroelectric energy is calculated at power plant nodes. These power plants have a maximum flow capacity. Whenever the release of the reservoir is larger than the flow capacity of the power plant and the water surface elevation is higher than the spillway crest, the overflow is spilled through the spillway. The release from the reservoir and local flow from the sub-basin are gathered into the control points, where the violation in the minimum required flow and the flow over the flooding level are suppressed using the goal programming technique. The demand sites represent the place or the demand sector for the delivery of water. The consumptive use of the water at the demand site flows to the terminal nodes. The flow from the final control points FIXED GRAPHICS to the terminal nodes is defined as the excess outflow at the basin outlet, and high weighting factors are applied to reduce any such excessive outflow from the basin for the purpose of basin-wide water conservation (Kim & Park 1998*a*, *b*; Kim *et al.* 2000)

The detailed objective function and constraint equations are presented in the appendix. In this section, the constraints sets are summarized.

The constraints are grouped into six sets:

(a) Flow conservation constraints.
(b) Physical limitation for each node and arc.
(c) Goal programming constraints.
(d) Hydropower linearization constraints that are the first-order linear approximation of the hydroelectric energy function.
(e) Spillage constraints to prevent spillage unless the water storage level is higher than the spillway crest elevation. Most Korean dams have spillways with radial gates which allow the overflow release to be controlled. No dams are equipped with river outlets or sluiceways, except for hydroelectric and emergency release. Hence, spillage is modelled specifically by mixed integer variables which prevent spillage when the storage level is under the spillway crest elevation.
(f) Non-anticipativity constraints for the daily release decision at each divergence point in the scenario tree. These constraints have to coincide for the scenarios which have a common path up to each divergence point.

CASE STUDY: NAKDONG RIVER BASIN IN KOREA

We applied the model to the daily operation of the Nakdong River Basin, multi-reservoir system in Korea.

Basin outlook

The Nakdong River Basin is the second largest watershed in the Korean peninsula, as shown in Figure 4. It occupies approximately 23,817 km^2, which constitutes about 24% of the South Korean land area. It is located in the temperate region, with four distinct seasons. The annual average precipitation is about 1,231 mm, two-thirds of which falls between June and September. There are 7 dams (5 multi-purpose and 2 water supply) operating in parallel. In this study, we considered the 4 major multipurpose dams, excluding the 3 small dams.

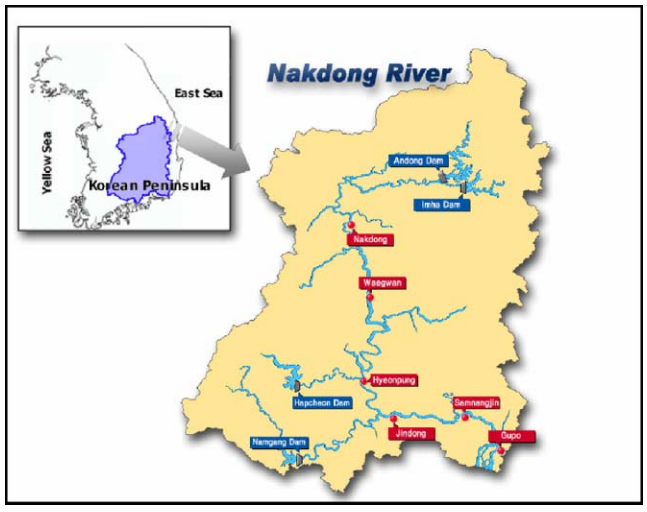

Figure 4 | Nakdong River Basin in Korea.

Downloaded from https://iwaponline.com/jh/article-pdf/10/1/23/386254/23.pdf
by guest

Scenario generation

The streamflow scenarios are generated for the daily stochastic model using RRFS with meteorological forecast information. The first-stage scenarios are generated by RRFS with RDAPS rainfall forecast input data. The second-stage scenarios by RRFS with GDAPS rainfall forecast information are supplemented by the historical maximum and minimum rainfall data, because RRFS may not cover all possible historical extreme situations. The third-stage ESP scenarios generated by RRFS with 20-year daily rainfall data are supplemented by the historical maximum and minimum rainfall.

RDAPS and GDAPS data are obtained from KMA and analyzed by the KOWACO Weather Information Service System, the meteorological information data analysis system of KOWACO (KWRA 2003).

The accuracy of the generated streamflow is represented in Figure 5. Figure 5 shows the mean absolute percent error (MAPE) of the generated streamflow using the historical mean, maximum and minimum rainfall, RDAPS rainfall forecasts and GDAPS rainfall forecasts. MAPE is the average of the absolute values of the relative error rate between generated streamflow and observed streamflow. It represents forecasting error, regardless of seasonal flow fluctuation.

The accuracy of the generated streamflow is very low due to the uncertainty of the streamflow in nature, as shown in Figure 5. However, MAPE of the streamflow generated from meteorological forecasts of RDAPS and GDAPS is lower than that of historical rainfall. The streamflow generated using the RDAPS rainfall forecasts tends to have the smallest error among all forecasts. However, the forecasting lead-time span of RDAPS information is only 2 days. For the mid-term (up to 10 days), GDAPS information is available. The streamflow from GDAPS rainfall forecasts is better than that from the historical average or maximum: however, it is not sufficiently accurate from a practical point

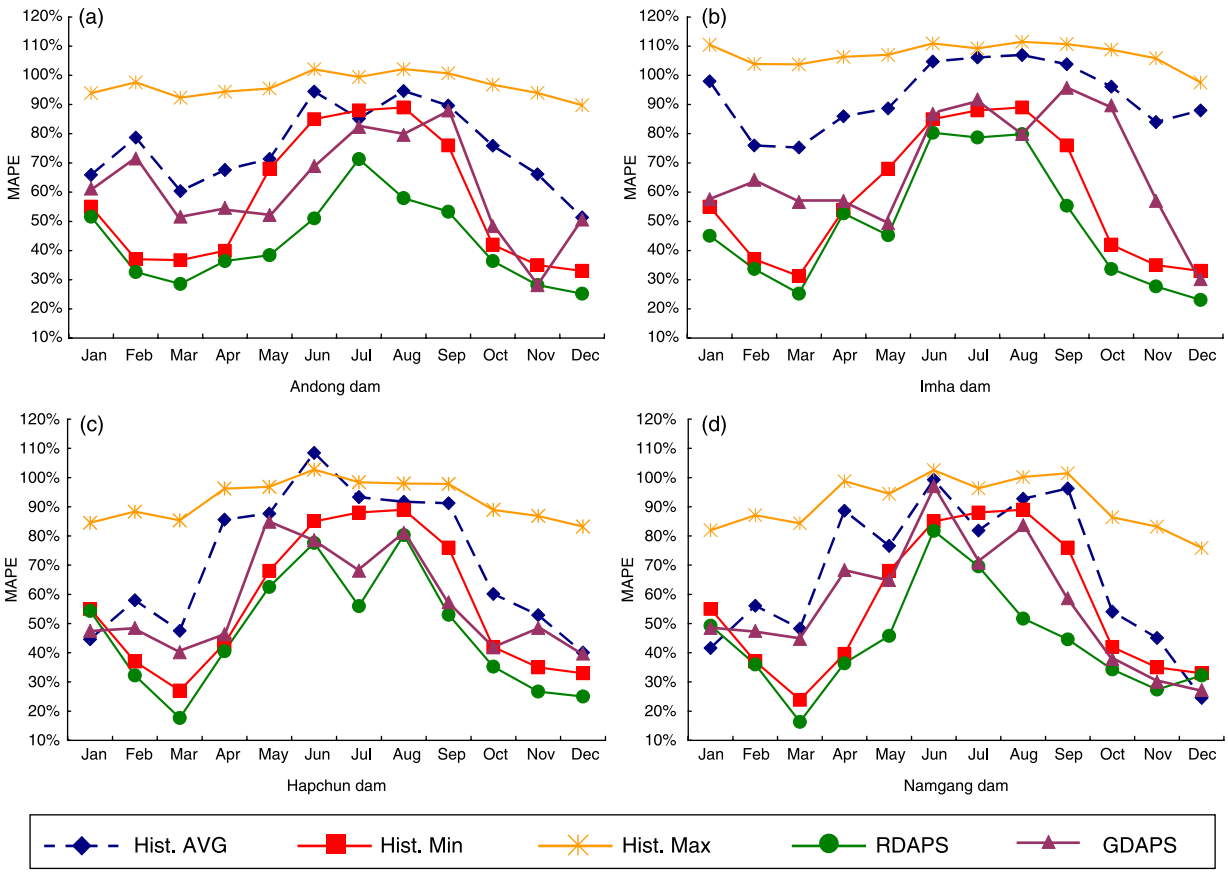

Figure 5 | MAPE of the generated streamflow using RRFS.

Downloaded from https://iwaponline.com/jh/article-pdf/10/1/23/386254/23.pdf
by guest

of view. The effect of the meteorological information will be discussed later, based on the comparison of the simulation results with and without meteorological information.

Although the streamflow generated from historical maximum and minimum rainfall may be inaccurate, it is included in the scenario tree to reduce the risk of serious flooding or drought during the process of choosing the non-anticipating decisions. The added value of updating the non-anticipating decision is discussed in the fourth section.

Solution procedure

The model was constructed using a Microsoft Visual C++6.0 application with the ILOG Concert technology library 2.0. It comprises over 123,658 columns and 84,129 rows for 100 scenarios. Since the model is a large-scale linear programming model, we could solve the model using the linear programming solver ILOG CPLEX 9.0.

As the number of scenarios increases, the model becomes very large, as shown in Tables 4 and 5. In Table 4, when spillage control constraints are excluded, the model was constructed for 60 scenarios (1 scenario for the first stage, 3 scenarios for the second stage and 20 scenarios for the third stage) and solved in 5 s (19,322 dual simplex iterations, 2 s CPLEX solving time) with a Pentium 4 PC (3.2 GHz CPU and 1GB RAM). Even when the number of scenarios was increased to 300, it was solved in only 3 min 40 s (112,229 dual simplex iterations).

When spillage control constraints are included, the model becomes a mixed-integer programming model with a consequent exponential increase in the solution time, as shown in Table 5. For the same 60 scenarios, the model was solved in 8 min (217,182 dual simplex iterations, 469 s CPLEX solving time), which is longer than the case with spillage control constraints excluded. Nevertheless, this 8 min solution time remains in the acceptable range. However, when the number of scenarios was increased to 300, the computation time lengthened to more than 30 h, which is unacceptable for the daily planning model.

For more than 60 scenarios, it is recommended to exclude spillage control constraints, even though this can make spill occur when it is physically impossible. For example, when one of the parallel reservoirs has good storage whose water level is lower than the spillway crest elevation, while the other dam's storage is insufficient, it may force the water spill downstream in order to meet downstream requirements so as to derive the first-stage solution, even if water cannot be delivered downstream. The model may derive impracticable solutions in this case, but it remains rare because in practice we use SLP primarily for water supply operation during the non-flood season. To overcome such an impracticable solution, the solution needs to be corrected by a simulation program.

For further research, special solution algorithms like the L-shaped method may be required when cases with a large number of scenarios are modelled with spillage control constraints included.

RESULTS AND DISCUSSION

We perform a real-time simulation under uncertainty, in which we assume that historical inflow will occur during

Table 4 | Problem size and the solution time without spillage control constraints

Number of scenarios	Columns	Rows	Non-zero elements	Number of iterations	Cplex CPU time (s)	Total time (s)
1 (1 × 1 × 1)	3,648	3,362	8,685	418	0.02	0.87
9 (1 × 3 × 3)	34,795	24,474	83,907	3,191	0.18	1.70
15 (1 × 3 × 5)	104,011	71,388	251,067	14,949	1.52	3.94
60 (1 × 3 × 20)	173,227	118,302	418,227	19,332	1.55	4.66
100 (1 × 5 × 20)	346,267	235,586	836,127	38,786	5.17	13.26
300 (3 × 5 × 20)	1038,352	704,320	2507,596	112,229	50.92	220.09

Downloaded from https://iwaponline.com/jh/article-pdf/10/1/23/386254/23.pdf
by guest

Table 5 | Problem size and the solution time with spillage control constraints

Number of scenarios	Columns	Rows	Non-zero elements	Number of iterations	Cplex CPU time (s)	Total time (s)
1 (1 × 1 × 1)	3,798	3,662	8,897	543	0.87	1.72
9 (1 × 3 × 3)	36,145	27,174	85,816	11,668	17.02	18.68
15 (1 × 3 × 5)	106,261	75,888	254,249	86,517	118.27	122.10
60 (1 × 3 × 20)	182 227	136,302	430,955	217,182	468.96	477.48
100 (1 × 5 × 20)	361,267	265,586	857,340	847,768	1,326.08	1,338.94
300 (3 × 5 × 20)	1083,352	794,320	2571,236	4420,930	110,046.72	110,260.21

the study period. However, for real-time operation with the stochastic model, the future inflow scenarios are generated using RRFS based on the meteorological information. For real-time simulation, the daily release plan for each reservoir is obtained by the stochastic model, and then used as today's release plan in a reservoir simulation model (in the real situation, the use of a multi-reservoir operation simulation run may not be necessary but the actual reservoir operation will be carried out regardless). The multi-reservoir simulation model may verify the release plan of the stochastic model, and if there is any physical violation, the release plan may be modified to meet the limitation. The actual storage at the end of the day is calculated based on the real inflow (the historical inflow was presumed as the real inflow in the real-time simulation process). A discrepancy between the actual storage attained with the real inflow and the desired storage calculated by the stochastic model is unavoidable due to the inflow uncertainty. To resolve this discrepancy, the reservoir operation plan from the stochastic model is updated every day with the actual storage data until the end of the study period. Upon implementation of the first-stage decision on the release of each reservoir with the assumed real inflow, the updated actual storage from the reservoir simulation becomes the initial storage for the stochastic model to calculate the release plan for the next day (Kim *et al.* 2000).

We evaluate the added value gained by using a stochastic model in a real-time environment based on the value of information concept (Birge & Louveaux 1997). The expected value of perfect information (EVPI) and the value of the stochastic solution (VSS) are evaluated. EVPI is the difference between WS (wait-and-see, the solution of perfect information) and the stochastic solution of the recourse problem (RP), which measures the value of perfect information about the future. WS is the expected or average return, in the long run, if we have had perfect information before a decision was made. To calculate this value, we choose the best alternative for each presumed scenario and multiply its payoff with the probability of occurrence of that scenario. As a result, WS is the ideal return we can achieve. RP can be defined as the expected or average return of the nonanticipativity decision with a stochastic model when the assumed scenarios in the stochastic model are reproduced. To evaluate this value, we choose the best decision for hedging against varying inflows and calculate the expected value when the best decision is applied to all presumed scenarios. RP is the return attainable in the long run.

VSS is the difference between RP and EV (the solution from the expected value problem). It measures the benefit of the stochastic model compared to the decision with no information. EV can be defined as the expected value with no information in the long run, in the absence of any information before a decision is made. To calculate this value, we choose the best solution for average inflow of the presumed scenarios and calculate the expected value when the solution is applied to all presumed scenario.

WS, RP and EV are defined as the expected return of the operation results using the model when the projected scenarios in the stochastic model are reproduced based on

Downloaded from https://iwaponline.com/jh/article-pdf/10/1/23/386254/23.pdf
by guest

the value of information concept (Birge & Louveaux 1997). However, it is almost impossible to expect that the inflow scenarios assumed will be reproduced exactly in the daily operation. In real-time operation, the release plans are updated every day. Therefore, although the meaning is not exact in the truest sense, we interpreted the "expected return" in WS, RP and EV as being the results of the real-time operation using the release plan from the model; WS with the perfect rainfall information, RP with the probabilistic forecasting information on the multi-stage, scenario tree and EV with the historical average rainfall information.

We compared the real-time simulation results of the stochastic model with those of deterministic models that were divided into two cases: one with no information (use only the average inflow information) and the other with perfect information. To evaluate the value of the meteorological forecasts, we performed two separate, real-time simulation studies according to the existence of the forecasted meteorological information. The first study is for the 20 years from January 1983 to December 2002, during which no forecasted meteorological information is available. We therefore used the historical minimum rainfall data instead of RDAPS and GDAPS forecasts. The second study is for the 3 years from January 2003 to December 2005, in which we assumed that the RDAPS and GDAPS forecast information is available. For all studies, the historical storage at the end of the month is assumed as the monthly storage targets of each reservoir, to evaluate the pure effect of the daily stochastic planning model in the real-time simulation.

The results of real-time simulation without meteorological information

We performed the real-time simulation for the 20 years from January 1983 to December 2002 without meteorological information in order to investigate the effect of the stochastic model with RRFS forecasting. The results are summarized in Table 6, which shows the VSS results when we apply the daily SLP model to the Nakdong River Basin multi-reservoir system in Korea in the real-time operational environments. EV, RP and WS are the expectations of the operational results of using the model. The stochastic solution (RP) reduced the water supply deficit and the spill amount by 21 MCM and 905 MCM, respectively, compared to the case of using average inflow, which is shown in the VSS column. The RP rate shows the incremental performance of RP relative to the case without any information. It is calculated as the ratio of the incremental performance of EV to the difference between EV and WS. Average storage of the RP rate reaches 43% of the perfect information (WS) results.

A box plot of the results of the real-time simulation for 20 years is depicted in Figure 6. The variance of EV average storage is larger than that of RP and WS, which indicates that EV is more influenced by the future inflow uncertainty. The spill results confirm that WS and RP can cope with the flooding condition of the high flow scenario effectively by reserving appropriate flood control capacity of the reservoir in advance. The amount of hydroelectric energy generation of RP and WS is also larger than that of the EV case for all inflow scenarios.

Table 6 | Results of the real-time simulation under uncertainty without meteorological information from January 1983 to December, 2002 (units: MCM)

Objectives	EV	RP	WS	EVPI(WS vs. RP)	VSS(RP vs. EV)	RP rate*
Deviation from the final storage target	36	4	–	4	32	89%
Water supply deficit**	21	–	–	–	21	100%
Amount of spill**	1,542	637	346	291	905	76%
Average storage***	1,403	1,416	1,433	17	13	43%
Hydroelectric energy(GWh)**	515	531	542	11	16	59%

* RP rate = (RP — EV)/(WS — EV), ** yearly total in the basin, *** daily total in the basin

Downloaded from https://iwaponline.com/jh/article-pdf/10/1/23/386254/23.pdf
by guest

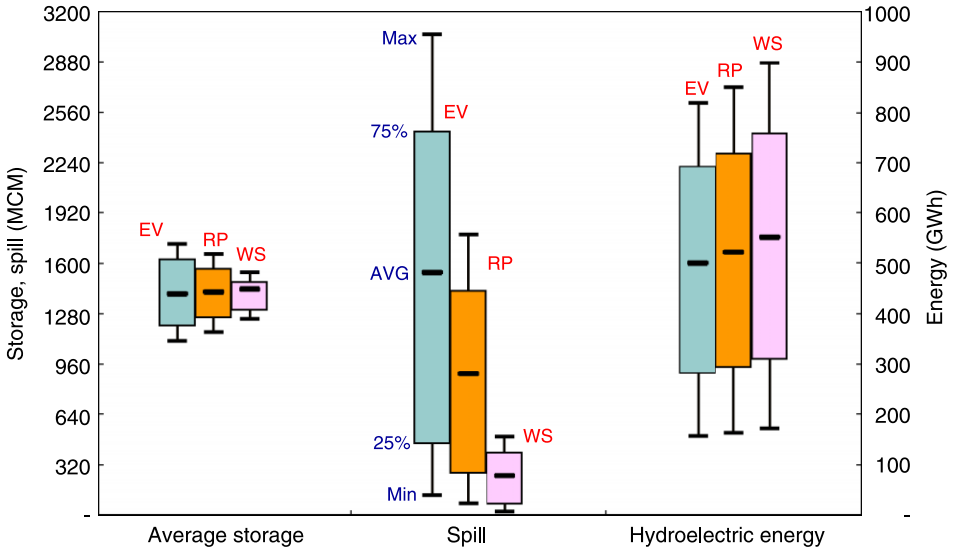

Figure 6 | Box plot of the results of the real-time simulation under uncertainty.

This result demonstrates the effect of the nonanticipativity decision of the stochastic model and the added information gained from using the RRFS forecasting ability while preserving the serial correlation.

The added value of the meteorological information with SLP

In order to examine the effect of the meteorological information on the SLP results, a real-time simulation is performed for the 3 years from January 2003 to December 2005 and the results are summarized in Table 7. Table 8 presents a comparison of the results with and without meteorological information.

As shown in Table 7, the RP results reduce the water supply deficit and the spill amount by 3 MCM and 909 MCM, respectively, compared to EV, as shown in the VSS column. The performance increase of the RP results on the spill amount accounts for up to 93% of the WS results, confirming the effectiveness of using the stochastic model (RP) in real-time operation to avoid spill occurrences. The results in the other objective categories also demonstrate the usefulness of the stochastic model under uncertainty.

As shown in Table 8, the comparison of performance with and without meteorological information confirms the improved result obtained with the added value of the meteorological information. The spill amount is decreased by 293 MCM (30% of the RP rate) using the improved streamflow forecast generated with the meteorological information. The average storage and hydroelectric energy generation are

Table 7 | Results of the real-time simulation under uncertainty with meteorological information from 2003 to 2005 (units: MCM)

Objectives	EV	RP	WS	EVPI (WS vs. RP)	VSS (RP vs. EV)	RP rate *
Deviation from the final storage target	18	1	–	1	17	94%
Water supply deficit**	3	–	–	–	3	100%
Amount of spill**	1,633	724	657	67	909	93%
Average storage***	1,423	1,431	1,441	10	8	44%
Hydroelectric energy(GWh)**	517	541	548	7	24	77%

*RP rate = (RP — EV)/(WS — EV), ** yearly total in the basin, *** daily total in the basin

Downloaded from https://iwaponline.com/jh/article-pdf/10/1/23/386254/23.pdf
by guest

Table 8 | The comparison of results with and without meteorological information from 2003 to 2005 (units: MCM)

Objectives	Without meteorological information RP	RP rate	With meteorological information RP	RP rate	Improvements RP	RP rate
Deviation from the final storage target	1	94%	1	94%	–	0%
Water supply deficit**	–	100%	–	100%	–	0%
Amount of spill**	1,017	63%	724	93%	− 293	30%
Average storage***	1,430	39%	1,431	44%	1	6%
Hydroelectric energy(GWh)**	531	45%	541	77%	10	32%

*RP rate = (RP — EV)/(WS — EV), ** yearly total in the basin, *** daily total in the basin

increased by 1 MCM and 10 GWh, respectively, compared to the case without meteorological information.

In addition, we evaluated the cases in which only one meteorological forecast is availableL RDAPS or GDAPS. In the RDAPS case (i.e. employing RDAPS forecasting information at the first stage) we applied the historical average rainfall instead of GDAPS forecasts at the second stage because the forecasting lead-time of RDAPS is only 2 days. In the GDAPS case, we used GDAPS information all the way from the first stage to the second stage without RDAPS information. The results are summarized in Table 9.

It shows that the use of only RDAPS information or only GDAPS information is more efficient than that of using no meteorological information. The results of using only GDAPS information decreased spill by 30 MCM and increased hydroelectric energy generation by 1 GWh, respectively, compared to the case without meteorological information. The results of using only RDAPS information also decreased spill by 269 MCM and increased hydroelectric energy generation by 8 GWh compared to the case without meteorological information. When we compared the results of using only RDAPS information and that of using only GDAPS information, the RDAPS case is more efficient than the GDAPS case because the accuracy of RDAPS is more reliable than GDAPS, even if GDAPS has a longer forecasting lead-time span. Moreover, the accuracy of the short-term forecasts may affect the results more because we apply only a first day release decision in the real-time simulation process and update the decision every day. Finally, the subsequent application of GDAPS information to the RDAPS result, as originally suggested in the model, produces the most efficient results.

Table 9 | The comparison of results for each case of using meteorological information from 2003 to 2005 (units: MCM)

Objectives	Without meteorological information	With only GDAPS information	With only RDAPS information	With RDAPS and GDAPS information
Deviation from the final storage target	1	1	1	1
Water supply deficit*	–	–	–	–
Amount of spill*	1,017	987	748	724
Average storage**	1,430	1,430	1,430	1,431
Hydroelectric energy(GWh)*	531	532	539	541

*Yearly total in the basin, ** daily total in the basin

Downloaded from https://iwaponline.com/jh/article-pdf/10/1/23/386254/23.pdf
by guest

CONCLUSION

In this study, a multi-period, multi-stage SLP with recourse model is developed to derive the daily release plan of each reservoir in a coordinated multi-reservoir system. Stages are defined as prediction lead-time spans. The multi-stage scenarios for the stochastic model are formulated considering the reliability of rainfall radar prediction for a specified forecasting lead-time. The future inflows are generated by RRFS based on the meteorological rainfall forecasts. The forecasting model for each stage is selected based on the viability and accuracy of the prediction. For short-term (2 days) rainfall forecasting, RDAPS is applied. For mid-term (up to 10 days) forecasting, precipitation from GDAPS is used as an input for a rainfall–runoff model in the ESP procedures. After the tenth day (third stage), the daily historical ensemble rainfall data are applied.

The model is applied to a simulation of the daily reservoir operation of the Nakdong River Basin multi-reservoir system in Korea, under a real-time operational environment. The expected benefit of the stochastic model is analyzed quantitatively based on Birge & Louveaux (1997) value of information measure. We also evaluate the added value gained by using meteorological information. We perform two separate, real-time simulation studies according to the existence of the forecasted meteorological information. The first study is for the 20 years from January 1983 to December 2002, during which no forecasted meteorological information is available and we therefore use minimum rainfall data instead of RDAPS and GDAPS forecasts. The stochastic solution (RP) reduces the water supply deficit and the spill amount, and increases the average storage and hydropower generation compared to the case of using average inflow. This result demonstrates the effect of the nonanticipativity decision of the stochastic model and the added information gained from utilizing the RRFS forecasting ability without meteorological forecast information, while preserving the serial correlation. The second study is for the 3 years from January 2003 to December 2005, during which we assume the availability of the RDAPS and GDAPS rainfall forecasting information. From this result, we could estimate the added value gained from using the meteorological forecasting information. The meteorological forecasts obtained with SLP increase the hydroelectric energy generation while reducing the spill, confirming the effectiveness of using the stochastic model in real-time operation with meteorological forecasts in the presence of uncertainty.

In this study, the historical storage at the end of the month was used as the monthly storage target for each reservoir, to evaluate the pure effect of the daily stochastic planning model in a real-time simulation. The monthly stochastic model is required for long-term storage planning, while the model we developed here is capable of short-term real-time operation. Special solution algorithms like the L-shaped method may be required when cases with a large number of scenarios are modelled with spillage control constraints included.

ACKNOWLEDGEMENTS

This research was supported by a grant (code 1-6-2) from the Sustainable Water Resources Research Center of 21st Century Frontier Research Program and partly by the Brain Korea 21 Project in 2007.

REFERENCES

Beale, E. M. L. 1955 On minimizing a convex function subject to linear inequalities. *J.R. Statist. Soc. Ser. B* **17** (2), 173–184.

Birge, J. R. & Louveaux, F. 1997 *Introduction to Stochastic Programming*. Springer-Verlag, Berlin.

Can, E. K. & Houck, M. H. 1984 Real time reservoir operating by goal programming. *J. Wat. Res. Plann. Mngmnt.* **110** (3), 297–309.

Dantzig, G. B. 1955 Linear programming under uncertainty. *Mngmnt Sci.* **1** (3), 197–206.

Dantzig, G. B. & Madansky, A. 1961 On the solution of two-stage linear programming under uncertainty. In *Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability*. University of California Press. Berkeley, CA.

Day, G. N. 1985 Extended streamflow forecasting using NWSRFS. *J. Wat. Res. Plann. Mngmnt.* **111**, 147–170.

Dupačová, J., Consigli, G. & Wallace, S. W. 2000 Scenarios for multistage stochastic programs. *Annals. Oper. Res.* **100**, 25–53.

Eschenbach, E. A., Magee, T., Zagona, E., Goranflo, M. & Shane, R. 2001 Multiobjective operations of reservoir systems via goal programming. *J. Wat. Res. Plann. Mngmnt.* **127** (2), 108–120.

Houck, M. H. 1979 A chance constrained optimization model for reservoir design and operation. *Wat. Res. Res.* **15** (5), 1011–1015.

Downloaded from https://iwaponline.com/jh/article-pdf/10/1/23/386254/23.pdf
by guest

Jacobs J., Freeman, G., Grygier, J., Morton, D., Schultz, G., Staschus, K. & Stedinger, J. 1995 Stochastic optimal coordination of river-basin and thermal electric systems (SOCRATES): a system for scheduling hydroelectric generation under uncertainty. *Annals Oper. Res.* **59**, 99–133.

Kim, S. K. 1999 Hydro energy from multiple reservoir operation. In *HYDRO'S FUTURE Technology, Markets, and Policy, Proceedings of the WATERPOWER'99 Conference July 6-9, 1999, MGM Grand Hotel and MGM Grand Conference Center, Las Vegas, NV* (ed. P. A. Brookshier). Water Resources Session 23: Decision Support Systems II, No 2.

Kim, S. K., Lee, Y. D., Kim, J. H. & Ko, I. H. 2005 A multiple objective mathematical model for daily coordinated multi-reservoir operation. *Wat. Sci. Technol. Wat. Supply* **5** (3-4), 81–88.

Kim, S. K. & Park, Y. J. 1998*a* Coordinated multireservoir operating model. In *May 26–29 1998 Spring Meeting, American Geophysical Union, Boston, MA*.

Kim, S. K. & Park, Y. J. 1998*b* A mathematical model for coordinated multi-reservoir operation. *J. Korea Wat. Res. Assoc.* **31** (6), 779–793 (in Korean).

Kim, S. K., Park, Y. J. & Kim, J. H. 2000 Simulation of the real-time multiple reservoir operating environment with a multi-reservoir operation optimization model. In *ASCE's 2000 Joint Conference on Water Resources Engineering and Water Resources Planning and Management*.

Kim, Y. O., Eum, H. -I., Lee, E. G. & Ko, I. H. 2007 Optimizing operational policies of a Korean multi-reservoir system using sampling stochastic dynamic programming with ensemble streamflow prediction. *J. Wat. Res. Plann. Mngmnt* **133** (1), 4–14.

Kracman, D. R., McKinney, D. C., Watkins, D. W. Jr & Lasdon, L. S. 2006 Stochastic optimization of the Highland Lakes System in Texas. *J. Wat. Res. Plann. Mngmnt.* **132** (2), 62–70.

KWRA (Korea Water Resources Association) 2003 *Study on Watershed-based Seasonal Forecast Rainfall for Dam Operation Application*. Korea Water Resources Corporation (in Korean). Daejeon.

Labadie, John W. 2004 Optimal operation of multireservoir systems: state-of-art review. *J. Wat. Res. Plann. Mngmnt.* **130** (2), 93–111.

Lee, Y. D., Kim, S. K. & Ko, I. H. 2006 Two-stage stochastic linear programming model for coordinated multi-reservoir operation. In *The 2006 Operations Management Conference about Operating Reservoirs in Changing Conditions sponsored by EWRI of ASCE, August 14-16, 2006, Sacramento, CA*.

Loucks, D. P., Stedinger, J. R. & Haith, D. A. 1981 *Water Resources Systems Planning and Analysis*. Prentice-Hall. Englewood Cliffs, NJ.

Mujumdar, P. P. & Ramesh, T. S. V. 1997 Real-time reservoir operation for irrigation. *Wat. Res. Res.* **33** (5), 1157–1164.

Pereira, M. V. F. & Pinto, L. M. V. G. 1985 Stochastic optimization of a multireservoir hydroelectric system: a decomposition approach. *Wat. Res. Res.* **21** (6), 779–792.

Ponnambalam, K., Vannelli, A. & Unny, T. E. 1989 An application of Karmarkar's interior-point linear programming algorithm for multireservoir operations optimization. *Stochast. Hydro. Hydraul.* **3**, 17–29.

Prékopa, A. 1995 *Stochastic Programming*. Kluwer, Dordrecht.

ReVelle, C., Joeres, E. & Kirby, W. 1969 Linear decision rule in reservoir management and design. 1: Development of the stochastic model. *Wat. Res. Res.* **5** (4), 767–777.

Seifi, A. & Hipel, K. W. 2001 Interior-point method for reservoir operation with stochastic inflows. *J. Wat. Res. Plann. Mngmnt.* **127** (1), 48–57.

Sen, S. & Higle, J. L. 1999 An introductory tutorial on stochastic linear programming models. *Interfaces* **29** (2), 33–61.

Stedinger, J. R., Sule, B. F. & Loucks, D. P. 1984 Stochastic dynamic programming models for reservoir operation optimization. *Wat. Res. Res.* **20** (11), 1499–1505.

Turgeon, A. 2005 Daily operation of reservoir subject to yearly probabilistic constraints. *J. Wat. Res. Plann. Mngmnt.* **131** (5), 342–350.

US Army Corps of Engineers 2006 *Streamflow Synthesis and Reservoir Regulation Model*. Available at: http://www.nwd-wc.usace.army.mil/report/ssarr.htm

Van Slyke, R. & Wets, R. 1969 L-shaped linear programs with application to optimal control and stochastic programming. *SIAM J. Appl. Math.* **17**, 638–663.

Watkins, D. W., McKinney, D. C., Lasdon, L. S., Nielsen, S. S. & Martin, Q. W. 2000 A scenario-based stochastic programming model for water supplies from the Highland Lakes. *Int. Trans. Oper. Res.* **7** (3), 211–230.

Wets, R. J.-B. 1988 Large scale linear programming techniques. In *Numerical Techniques for Stochastic Optimization* (ed. Y. S. Ermoliev & R. J. -B. Wets), Springer, New York. pp. 65–93.

Wurbs, R. A. 1993 Reservoir-system simulation and optimization models. *J. Wat. Res. Plann. Mngmnt.* **119** (4), 455–472.

Yakowitz, S. 1982 Dynamic programming applications in water resources. *Wat. Res. Res.* **18** (4), 673–696.

Yazicigil, H., Houck, M. H. & Toebes, G. H. 1983 Daily operation of a multipurpose reservoir system. *Wat. Res. Res.* **19** (1), 1–11.

Yeh, W. W.-G. 1985 Reservoir management and operations models: a state-of-the-art review. *Wat. Res. Res.* **21** (12), 1797–1818.

First received 3 July 2006; accepted in revised form 20 August 2007

Downloaded from https://iwaponline.com/jh/article-pdf/10/1/23/386254/23.pdf
by guest

APPENDIX (MATHEMATICAL MODEL)

Definition of sets

CL^u: Set of lower nodes connected to an upper node u in the basin network

CU_l: Set of upper nodes connected to a lower node l in the basin network

CP: Set of control points

DS: Set of demand sites

N: Set of nodes

PS: Set of power stations

RE: Set of reservoirs

SB: Set of sub-basins

SP: Set of spillways

TN: Set of terminals (sink node)

TP: Set of time periods

$\Omega(t)$: Set of scenarios (arc in the scenario tree) at time period t.

Definition of variables

$D_d^{t,\omega}$: Deficit of the water demand at demand site $d \in$ DS, at time period $t \in$ TP with scenario ω

$E_p^{t,\omega}$: Hydroelectric energy generation of powerplant $p \in$ PS, at time period t with scenario ω

$ET_n^{t,\omega+}$: Surplus to target of node $n \in$ N, at time period t with scenario ω

$ET_n^{t,\omega-}$: Deficit from target of node $n \in$ N, at time period t with scenario ω

$Q_{u,l}^{t,\omega}$: Flow from node u (upper node) to node l (lower node) at time period t with scenario ω

$R_r^{t,ap(\omega)}$: Release of reservoir $r \in$ RE, at time period t having $ap(\omega \in \Omega(t)$ as the ancestor path (planning decision variable)

$S_r^{t,\omega}$: Storage of reservoir $r \in$ RE, at time period t with scenario ω

$V_r^{t,\omega+}$ and $V_r^{t,\omega-}$: Violation of the control constrains at reservoir r at time period t with scenario ω

$Z_i^{t,\omega}$: ith objective value at time period $t \in$ TP with scenario ω

Definition of constants

a_r^t, b_r^t: The coefficients of linearized storage-elevation function of reservoir $r \in$ RE

$aa(\omega \in \Omega(t)) \in \Omega(t-1)$: The immediate ancestor arc of scenario ω at time period t. It is required to keep track of the carryover storage.

$ap(\omega \in \Omega(t))$: The immediate ancestor path (set of arcs) of scenario ω at the stage belonging to the time period t. It is defined for the nonanticipativity constraints because the planning decision (the daily release plan) has to coincide for the scenarios which have a common path up to each divergence point

c_L^i : The cost coefficient vector of the ith objective at the Lth stage

dmd_d^t: The water demand of demand site $d \in$ DS, at time period t

$eh_p^{t,\omega}$: The estimated head of powerplant $p \in$ PS, at time period t with scenario ω

$eq_p^{t,\omega}$: The estimated flow of powerplant $p \in$ PS, at time period t with scenario ω

$init_s_r$: The initial storage of reservoir $r \in$ RE

lb_n^t: The lower bound of node $n \in$ N, at time period t

mrf_c^t: The minimum requirement flow of control point $c \in$ CP, at time period t

p_L^ω: The marginal probabilities of choosing scenario ω at stage L

rfr_d: The return flow rate from demand site $d \in$ DS, at time period t

$ro_b^{t,\omega}$: The runoff to a sub-basin, $b \in$ SB, at time period t with scenario ω

$spillable_str_s$: The reservoir storage at the spillway crest elevation of spillway $s \in$ SP

$tailwater_p$: The tailwater elevation of powerplant $p \in$ PS

ub_n^t: The upper bound of node $n \in$ N at time period t

$\Gamma(eh_p^{t,\omega}, eq_p^{t,\omega})$: The hydroelectric energy generation coefficient of efficiency hill diagram $= 9.8 \times 24 \times$ efficiency of estimated head at estimated flow of powerplant $p \in$ PS at time period t

Downloaded from https://iwaponline.com/jh/article-pdf/10/1/23/386254/23.pdf
by guest

Objectives

$$\min Obj = \sum_{t=1}^{2}\sum_{i=1}^{7} c_1^i Z_i^t + \sum_{t=3}^{10}\sum_{\omega\in\Omega(t)} p_2^\omega \sum_{i=1}^{7} c_2^i Z_i^{t,\omega} + \sum_{t=11}^{T}\sum_{\omega\in\Omega(t)} p_3^\omega \sum_{i=1}^{7} c_3^i Z_i^{t,\omega} \quad \text{(A1)}$$

- Minimize the infeasibility

$$Z_1^{t,\omega} = \sum_{r\in RE}\{V_r^{t,\omega+} + V_r^{t,\omega-}\} \quad \text{(A2)}$$

- Satisfy the minimum requirement flow

$$Z_2^{t,\omega} = \sum_{c\in CP} ET_c^{t,\omega+} \quad \text{(A3)}$$

- Minimize the shortage of water demand

$$Z_3^{t,\omega} = \sum_{d\in DS} ET_d^{t,\omega-} \quad \text{(A4)}$$

- Attain the final target storage

$$Z_4^{T,\omega} = \sum_{r\in RE}\{ET_r^{T,\omega+} + ET_r^{T,\omega-}\} \quad \text{(A5)}$$

- Minimize the spill at the basin outlet

$$Z_5^{t,\omega} = \sum_{s\in SP}\sum_{u\in CU_s} Q_{u,s}^{t,\omega} \quad \text{(A6)}$$

- Maximize the storage

$$Z_6^{t,\omega} = -\sum_{r\in RE} S_r^{t,\omega} \quad \text{(A7)}$$

- Maximize the hydroelectric energy generation

$$Z_7^{t,\omega} = -\sum_{p\in PS} E_p^{t,\omega} \quad \text{(A8)}$$

Constraints

(a) Flow conservation constraints

- Sub-basins

$$\sum_{t\in CL^b} Q_{b,l}^{t,\omega} = ro_b^{t,\omega} \quad \text{for } b\in SB, \quad \omega\in\Omega(t), \quad t\in TP \quad \text{(A9)}$$

- Reservoirs

$$S_r^{t,\omega} + \sum_{l\in CL^r} Q_{r,l}^{t,\omega} - V_r^{t,\omega+} + V_r^{t,\omega-} + = S_r^{t-1,aa(\omega)} + \sum_{u\in CU_r} Q_{u,r}^{t,\omega} \text{ for } r\in RE, \quad \omega\in\Omega(t), \quad t\in TP \quad \text{(A10)}$$

$$S_r^0 = init_s_r \quad \text{for } r\in RE \quad \text{(A11)}$$

- Demand sites

$$\sum_{u\in CU_d} Q_{u,d}^{t,\omega} + ET_d^{t,\omega-} = dmd_d^t \quad \text{for d}\in DS, \quad \omega\in\Omega(t), \quad t\in TP \quad \text{(A12)}$$

$$rfr_d \sum_{u\in CU_d} Q_{u,d}^{t,\omega} = Q_{d,c}^{t,\omega} \quad \text{for d}\in DS, \quad c\in CL^d\cap CP, \quad \omega\in\Omega(t), \quad t\in TP \quad \text{(A13)}$$

- For other nodes (CP, PS, OL, SP)

$$\sum_{u\in CU_n} Q_{u,n}^{t,\omega} = \sum_{l\in CL^n} Q_{n,l}^{t,\omega} \quad \text{for } n\in CP\cup PS\cup SP, \omega\in\Omega(t), t\in TP \quad \text{(A14)}$$

(b) Physical limitation for each node and arc

- Bounds on the storage

$$lb_r^t \le S_r^{t,\omega} \le ub_r^t \quad \text{for r}\in RE, \quad \omega\in\Omega(t), \quad t\in TP \quad \text{(A15)}$$

- For other nodes (CP, PS, OL, SP)

$$lb_n^t \le \sum_{u\in CU_n} Q_{u,n}^{t,\omega} \le ub_n^t \quad \text{for } n\in CP\cup PS\cup SP, \quad \omega\in\Omega(t), \quad t\in TP \quad \text{(A16)}$$

(c) Goal programming constraints

- Attaining the target storage

$$S_r^{T,\omega} - ET_r^{T,\omega+} + ET_r^{T,\omega-} = tar_s_r \quad \text{for} \quad \text{r}\in RE, \quad \omega\in\Omega(t) \quad \text{(A17)}$$

Downloaded from https://iwaponline.com/jh/article-pdf/10/1/23/386254/23.pdf
by guest

- Satisfy the minimum requirement flow

$$\sum_{u\in CU_c} Q_{u,n}^{t,\omega} + ET_c^{t,\omega^-} \geq mrf_c^t \text{ for } c\in CP,\ \omega\in\Omega(t),\ t\in TP \quad \text{(A18)}$$

(d) Hydropower linearization constraints for each powerstation which are the first-order linear approximation of the hydroelectric energy function

$$E_p^{t,\omega} = \Gamma(eh_p^{t,\omega}, eq_p^{t,\omega}) \times \{eh_p^{t,\omega} Q_{r,p}^{t,\omega} + eq_p^{t,\omega} H_p^{t,\omega} - eh_p^{t,\omega} eq_p^{t,\omega}\} \quad \text{for } p\in PS,\ \omega\in\Omega(t),\ t\in TP \quad \text{(A19)}$$

$$H_p^{t,\omega} = \left(a_r^t S_r^{t,\omega} + b_r^t\right) - tailwater_p \quad \text{for} \quad p \in PS,\ r \in RS \cap CU_p,\quad \omega \in \Omega(t),\quad t \in TP \quad \text{(A20)}$$

(e) Spillage control constraints

$$Q_{r,s}^{t,\omega} \leq BS_s^{t,\omega} \times ub_s^t \quad \text{for} \quad s \in SP,\quad r \in CU_s \cap RS,\quad \omega \in \Omega(t),\quad t \in TP \quad \text{(A21)}$$

$$BS_s^{t,\omega} \leq S_r^{t,\omega}/spillable_str_s \quad \text{for} \quad s \in SP,\ r \in CU_s \cap RS,\quad \omega \in \Omega(t),\quad t \in TP \quad \text{(A22)}$$

(f) Non-anticipativity constraints on release decision

$$R_r^{t,ap(\omega)} = \sum_{l\in CL^r} Q_{r,l}^{t,\omega} \quad \text{for} \quad r \in RE,\quad \omega \in \Omega(t),\quad t \in TP \quad \text{(A23)}$$

Downloaded from https://iwaponline.com/jh/article-pdf/10/1/23/386254/23.pdf
by guest