Stochastic approach to the derivation of emission limits for wastewater treatment plants
D. Stransky, I. Kabelkova and V. Bares

ABSTRACT

Stochastic approach to the derivation of WWTP emission limits meeting probabilistically defined environmental quality standards (EQS) is presented. The stochastic model is based on the mixing equation with input data defined by probability density distributions and solved by Monte Carlo simulations. The approach was tested on a study catchment for total phosphorus (P$_{\text{tot}}$). The model assumes input variables independency which was proved for the dry-weather situation. Discharges and P$_{\text{tot}}$ concentrations both in the study creek and WWTP effluent follow log-normal probability distribution. Variation coefficients of P$_{\text{tot}}$ concentrations differ considerably along the stream ($c_v = 0.415$ – $0.884$). The selected value of the variation coefficient ($c_v = 0.420$) affects the derived mean value ($C_{\text{mean}} = 0.13$ mg/l) of the P$_{\text{tot}}$ EQS ($C_{\text{90}} = 0.2$ mg/l). Even after supposed improvement of water quality upstream of the WWTP to the level of the P$_{\text{tot}}$ EQS, the WWTP emission limits calculated would be lower than the values of the best available technology (BAT). Thus, minimum dilution ratios for the meaningful application of the combined approach to the derivation of P$_{\text{tot}}$ emission limits for Czech streams are discussed.

Key words | combined approach, dilution ratio, mixing equation, Monte Carlo simulation, stochastic modelling, wastewater treatment plants

INTRODUCTION

Wastewater discharges into surface waters have to be controlled by a combined approach using control of pollution at source through the setting of emission limit values and of environmental quality standards (EQS) by the year 2015 (Directive 2000/60/EC).

In the Czech Republic the combined approach will be put into practice starting January 2010. In the Czech legislation the combined approach is specified as “a method of determining the target emission limits based on the requirement to meet simultaneously the emission and pollution standards as well as the target status of water quality in a water course, having regard to the best available technologies in production and to the available technologies in wastewater disposal” (Order No. 229/2007 Coll.). Thus, a water quality authority shall determine the effluent criteria as site-specific emission limits up to the level of common emission standards laid in the Order.

However, a method of the derivation of emission limits is complicated by the fact that the environmental quality standards (ambient water quality criteria) for most of the receiving water quality parameters are defined probabilistically as $C_{\text{90}}$, i.e. concentrations that must not be exceeded in more than 10% of annual samples, whereas the emission standards are mainly set as mean values $C_{\text{mean}}$ and maximum permissible concentrations $C_{\text{max}}$.

This paper introduces a stochastic approach to the calculation of WWTP emission limits reflecting the statistical nature of water quality parameters (Novotny 2004). The procedure enables to calculate:

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• the maximum emission (as mean and max concentrations) from the WWTP to the receiving water of a known concentration distribution to comply with the probabilistic environmental quality standard (EQS),
• mean concentration or \( C_{90} \) to be achieved in the receiving water above the WWTP to comply with the probabilistic EQS after mixing with the WWTP effluent of a known concentration distribution,
• percentage of the fulfilment of the EQS.

The procedure is demonstrated on an example of the WWTP Caslav and Brslenka Creek for total phosphorus (P\(_{\text{tot}}\)). Finally, the application of the combined approach to WWTPs situated on small water bodies is discussed.

**METHODS**

**Study catchment**

Caslav is a small town east of Prague, Czech Republic. Present loading of its WWTP equals 6,500 PE. The WWTP discharges to Brslenka Creek at km 7.8. The total length of the creek is 31.3 km and catchment area 250 km\(^2\) with a predominantly rural character. The reach studied stretched from km 11.45 (below the first village above Caslav) to km 1.05.

**Monitoring programme**

Water quality and quantity both in the creek and the WWTP effluent were monitored during one year (11/2005 – 10/2006). Discharges were monitored continuously in two profiles (the WWTP effluent and the creek upstream of the WWTP) whereas grab water samples were taken approximately once per two weeks (as randomly as possible in the day and week cycle) in eleven monitoring profiles. The profiles were set to cover water quality upstream of Caslav, the effect of the Podmestsky Pond, tributaries and point sources in Caslav, effect of the WWTP and self-purification in the creek (**Figure 1**). Also water quality data provided by the WWTP Caslav operator were evaluated for years 2004 – 2006.

**Hydrological regime**

Hydrological regime was evaluated for the hydrological year 2006. The mean daily discharge in Brslenka Creek was 3,20 l/s and median 1,00 l/s whereas the mean daily outflow from the WWTP was 26 l/s and median 25 l/s. The mean daily outflow from the WWTP exceeded the mean daily discharge in the creek for 33 days in the period investigated.

**Water quality**

Brslenka Creek is significantly polluted already above Caslav, especially by nutrients and faecal bacteria originating from agriculture and domestic wastewaters from small municipalities without sewer systems. Critical parameters, the values of which are further increased by the effluent of the WWTP Caslav, appear to be P\(_{\text{tot}}\) and NH\(_4\)-N (**Figure 2** left). The desired compliance of the P\(_{\text{tot}}\) immisssion standard \( C_{90} = 0.2 \text{ mg/l} \) was not reached already above Caslav and markedly decreased below the Kosmos treatment pond and WWTP Caslav (**Figure 2** right).

The WWTP Caslav effluent meets the COD, BOD and suspended solids emission standards with a large margin of safety and the NH\(_4\)-N emission standard closely; however, it does not comply with the P\(_{\text{tot}}\) emission standard (the mean P\(_{\text{tot}}\) effluent concentrations were 3.6–4.2 mg/l whereas the emission standard is 3 mg/l).

**Stochastic model**

The approach is presented on the example of P\(_{\text{tot}}\) as the most critical parameter in the catchment. However, it can be used for any water quality parameter with a probabilistically defined EQS.

**Mixing equation**

The mixing equation of stream water and WWTP effluent loads is enhanced for a stochastic nature of the input data...
by consideration of their probability density distributions:

\[(Q_{US} + Q_{WWTP})C_{DS} = Q_{US}C_{US} + Q_{WWTP}C_{WWTP}\] (1)

where:
- \(C_{US}\) [mg/l] is concentration in the river upstream of the WWTP effluent,
- \(Q_{US}\) [m³/s] is discharge in the river upstream of the WWTP effluent,
- \(Q_{WWTP}\) [m³/s] is discharge from the WWTP,
- \(C_{WWTP}\) [mg/l] is concentration of the WWTP effluent,
- \(C_{DS}\) [mg/l] is concentration in the river downstream of the WWTP effluent.

The equation is solved for a desired variable and further calculations are performed by Monte Carlo simulations. Values of input variables are repeatedly generated according to their probability density distribution functions and results are statistically evaluated. The basic assumption of the stochastic approach is the independence of the concentration of the water quality parameter of interest from discharge. The model applies outside of the mixing zone, which size depends on the outfall structure, characteristics of the effluent and receiving water flow characteristics.

**Independency of input variables**

The analysis of the measured data proved the independency assumption to be fulfilled for \(P_{tot}\) in the WWTP effluent (correlation coefficient \(R = -0.31\)). In case of Bršlenka Creek upstream of the WWTP the independence assumption is true for low discharges (i.e. dry weather flow) \((R = 0.3)\) whereas not true for the whole data set including discharges increased due to storm water runoff or snow-melt. The correlation coefficient considering also these values raised to \(R = 0.6-0.8\) (Figure 3 left) as also higher concentrations of \(P_{tot}\) originating from the rural catchment.
were measured (Figure 3 right). However, as the Czech legislation (Government Order No. 229/2007 Coll.) requires samples to be taken in a “usual situation”, i.e. not during floods or storm events, these data can be omitted and the independency assumption can be accepted. In case of a significant correlation of discharges and concentrations, the model would have to be adapted accordingly (e.g. Song & Brown 1990).

Statistical distribution of data

Discharge and P\text{tot} concentrations both in Brslenka Creek and WWTP Caslav effluent follow log-normal probability distribution with the shape parameter \( \sigma_g \) greater than 1.2 (Table 1) (Limpert et al. 2001).

Variation coefficients \( c_v \) (ratios of arithmetical standard deviation \( \beta \) and arithmetical mean \( \alpha \)) of P\text{tot} concentrations in individual monitoring profiles of Brslenka Creek differ considerably (\( c_v = 0.415 - 0.884 \)). The highest values were caused by an irregular discharge from storm water tanks and by a significant WWTP effluent fluctuation due to overdimensioned return sludge pumps.

Monte Carlo simulations

The necessary number of simulations (randomizations) was tested and proved to be 1825 (\( p \)-value \( \leq 0.01 \)). This corresponds with the findings of Burges & Lettenmaier (1975) and Scavia et al. (1981).

Scenarios

The scenarios studied (Table 2) assume the improvement of the water quality in Brslenka Creek so that the P\text{tot} EQS \( C_{90} = 0.2 \text{mg/l} \) is achieved upstream of the WWTP. Then, the percentage of not exceedance of the P\text{tot} concentration \( C_{90} = 0.2 \text{mg/l} \) below the WWTP is calculated for the present state (scenario No. 1) and for varying requirements on the WWTP effluent (scenarios No. 2–4). In the last scenario (No. 5) the P\text{tot} emission limit for the WWTP is derived under the assumption of the simultaneous compliance of the P\text{tot} EQS both upstream and downstream of the WWTP.

Parameters of the P\text{tot} log-normal distribution matching the EQS were determined for the mean variation coefficient of least affected profiles 1–3 \( c_v = 0.420 \) and \( C_{90} = 0.2 \text{mg/l} \).

### Table 1 | Log-normal distribution of parameters

<table>
<thead>
<tr>
<th>Data</th>
<th>Geometric mean ( \mu_g ) (median)</th>
<th>Geometric st. dev. ( \sigma_g ) (shape parameter)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q_{U/S} ) [l/s]</td>
<td>83.36</td>
<td>3.356</td>
<td>Discharge corresponding to wat. qual. sampling excluding extreme hydrologic events</td>
</tr>
<tr>
<td>( C_{U/S} ) [mg/l]</td>
<td>0.16–0.29</td>
<td>1.489–1.703</td>
<td>Range for profiles 1–6</td>
</tr>
<tr>
<td>( Q_{WWTP} ) [l/s]</td>
<td>26.24</td>
<td>1.245</td>
<td>Mean daily discharges</td>
</tr>
<tr>
<td>( C_{WWTP} ) [mg/l]</td>
<td>3.09</td>
<td>1.490</td>
<td></td>
</tr>
<tr>
<td>( C_{D/S} ) [mg/l]</td>
<td>0.73–0.99</td>
<td>1.730–2.039</td>
<td>Range for profiles 8–11</td>
</tr>
</tbody>
</table>

### Table 2 | Scenarios of the WWTP Caslav effluent concentrations studied

<table>
<thead>
<tr>
<th>Scenario</th>
<th>WWTP effluent ( P_{\text{tot}} ) (mg/l)</th>
<th>Derivation of log-normal parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Present state</td>
<td>( C_{\text{mean}} = 3.3 ) ( C_{\text{max}} = 6.6 )</td>
<td>WWTP monitoring</td>
</tr>
<tr>
<td>2 Emission standard (Government Order No. 229/2007 Coll.)</td>
<td>( C_{\text{max}} ) considered as not exceeded in 99.7%</td>
<td></td>
</tr>
<tr>
<td>3 Best available technology (BAT) for WWTPs 2,001–10,000 PE (Guideline to the Government Order No. 229/2007 Coll.)</td>
<td>( C_{\text{mean}} = 2 ) ( C_{\text{max}} = 5 )</td>
<td></td>
</tr>
<tr>
<td>4 Emission limit by the watershed authority</td>
<td>( C_{\text{max}} ) considered ( 2 \times C_{\text{mean}} ) and as not exceeded in 99.7%</td>
<td></td>
</tr>
<tr>
<td>5 Emission limit when EQS upstream + downstream fulfilled</td>
<td>( C_{\text{max}} ) considered ( 2 \times C_{\text{mean}} ) and as not exceeded in 99.7%</td>
<td></td>
</tr>
</tbody>
</table>
RESULTS AND DISCUSSION

Scenarios

The improvement of water quality in Brslenka Creek upstream of the WWTP would not result in a significant upgrading of the water quality downstream of it (Table 3, scenarios No. 1–4). For the present WWTP effluent quality only 3.2% EQS compliance would be reached. Even obeying the very strict emission limits issued by the local watershed authority would not lead to the P_{tot} EQS fulfillment (only 42.3% of samples would comply). The EQS would be met only in case the mean WWTP effluent concentration drops from present 3.3 mg/l to 0.17 mg/l and the maximum concentration from 6.6 mg/l to 0.34 mg/l. The emission limit derived is much lower than effluent concentrations of the best available technology for this WWTP category issued by the Czech Ministry of Environment.

Coefficient of variation

The derived mean value $C_{\text{mean}}$ of the P_{tot} EQS is affected by the variation coefficient. Thus, $C_{\text{mean}}$ based on the monitoring data is 0.13 mg/l, whereas the Guideline to the Government Order No. 229/2007 Coll. recommends the value of 0.15 mg/l, corresponding to $c_v = 0.220$ (i.e half variation compared to the minimum variation in Brslenka Creek). For regularly monitored water bodies where the P_{tot} EQS $C_{90}$ is met (467 profiles out of totally 1006 profiles in the national database on The water management information portal, 2007), the mean variation coefficient $c_v$ reaches the value of 0.271 with a relatively high standard deviation of 0.130 (Table 4) which corresponds to the 90% confidence interval of $C_{\text{mean}}$ between 0.12 and 0.18 mg/l.

Dilution ratio

The unfavourable dilution ratio of Brslenka Creek and WWTP discharges (4:1 for median values) plays an important role in the WWTP effluent quality requirements. Table 5 shows minimum dilution ratios for different stream concentrations including the 90% confidence interval for which it is reasonable to derive P_{tot} emission limits for WWTPs by the combined approach. Emission limit was set at the level of BAT (Table 3) and the P_{tot} EQS $C_{90} = 0.2$ mg/l was characterized by $c_v = 0.271$ (Table 4).

In case of the P_{tot} concentration in the stream water $C_{90} = 0.036$ mg/l, which is reached in about 1% of 1006 monitored profiles of Czech rivers, the derivation of emission limits by the combined approach can be applied if the dilution ration exceeds 27:1 whereas it has no sense if the dilution ratio does not reach 8:1 (i.e. BAT values as emission limits can be used directly). In the intermediate

### Table 3 | WWTP emissions and percentage of not exceedance of the P_{tot} EQS $C_{90} = 0.2$ mg/l in Brslenka Creek below the WWTP Caslav

<table>
<thead>
<tr>
<th>Scenario</th>
<th>WWTP effluent P_{tot} (mg/l)</th>
<th>$C_{\text{mean}}$</th>
<th>$C_{\text{max}}$ %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present state</td>
<td>3.3</td>
<td>6.6</td>
<td>3.2</td>
</tr>
<tr>
<td>Emission standard</td>
<td>3</td>
<td>8</td>
<td>3.8</td>
</tr>
<tr>
<td>Best available technology</td>
<td>2</td>
<td>5</td>
<td>7.1</td>
</tr>
<tr>
<td>Emission limit by the watershed authority</td>
<td>0.5</td>
<td>1.0</td>
<td>42.3</td>
</tr>
<tr>
<td>Emission limit when EQS upstream + downstream fulfilled</td>
<td>0.17</td>
<td>0.34</td>
<td>90.0</td>
</tr>
</tbody>
</table>

### Table 4 | Effect of the variation coefficient $c_v$ on the distribution shape (ratio of $C_{90}$ and $C_{\text{mean}}$)

<table>
<thead>
<tr>
<th>$c_v$ source</th>
<th>P_{tot} $c_v$ [-]</th>
<th>$C_{\text{mean}}$ [mg/l]</th>
<th>$C_{90}$ [mg/l]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Guideline to the Government Order No. 229/2007 Coll.</td>
<td>0.220</td>
<td>0.15</td>
<td>0.20 (EQS)</td>
</tr>
<tr>
<td>Brslenka Creek upstream of Caslav</td>
<td>0.420</td>
<td>0.13</td>
<td></td>
</tr>
<tr>
<td>Derived from the national database for streams complying P_{tot} EQS</td>
<td>0.271 (0.117–0.512) 90% conf. int.</td>
<td>0.14 (0.180–0.120) 90% conf. int.</td>
<td></td>
</tr>
</tbody>
</table>
cases an individual assessment is necessary. For water bodies with $C_{90} = 0.19 \text{ mg/l}$ (about 40% of rivers in the database) the lower bound of the dilution ratio for the usage of the combined approach increases to 129:1 and the upper bound to 453:1.

### CONCLUSIONS

In the paper stochastic approach to the derivation of emission limits for WWTP effluents by the combined approach based on Monte Carlo simulations was introduced. This approach respects the stochastic nature of the processes involved and provides a useful tool for the watershed loading analysis.

The application of the procedure on the study catchment showed the following.

- The independence of input data must be tested prior the model application, especially when wet weather data are included.
- Parameters of the data statistical distribution are strongly site specific (as was shown for the variation coefficient of stream $P_{\text{tot}}$ concentrations) and require monitoring to decrease model output uncertainties.
- The emission limits derivation by the combined approach is reasonable when certain dilution ratios of the river water and WWTP discharge are exceeded, otherwise BAT values as emission limits can be applied directly. However, uncertainties in the determination of these dilution ratios are relatively high.
- Other pollution sources (especially agricultural) must be tackled otherwise the attainment of permissible pollution is impossible in many Czech rivers.

### ACKNOWLEDGEMENTS

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