Authors' Closure

a) As pointed out by Reed, Fichman and Pnueli's (1985) work is an improvement[1] on Rogers and Reed (1984). Indeed, in addition to the elastic-plastic deformation at the center of the circle of contact, we have considered the plastic deformation at the edge of that circle. This plastic deformation always exists during impact, unlike plastic dissipation at the center of the circle, which appears only after the yield point there has been reached. We do not show how to calculate ϵ_{pl_1}, the dissipation work at the center of the circle, because it is not needed to obtain the condition for the two particles sticking together. Once the yield point at the center has been reached – any surplus kinetic energy is dissipated. Thus, only the energy dissipated at the edge of the circle, ϵ_{pl_2}, is required. This result has a very interesting implication: particles with diameters smaller than some critical values always stick together, whatever their velocities might be.

b) We regret that there is a type-setting error and the second term on the right-hand side of our equation (7) has been omitted. This equation (7) should read:

$$p_o = \frac{3}{2} k\delta a_1 - \frac{P_1}{2} \quad (1)$$

This equation agrees with Reed's comments.

c) Another typographical error has caused a 2/3 constant to appear in the first term of our equation (9), which corrected should read (the whole equation):

$$\delta = \frac{a_1^2}{R} \pm \frac{2}{3}\sqrt{\frac{6\gamma\pi}{k}}\, a_1^{\frac{1}{2}} \quad (2)$$

Unfortunately, this error was made before the paper was submitted, and influenced other expressions. R in equations (11), (12), (29) ÷ (32) must be corrected to:

$$R = \frac{2}{3} \cdot \frac{R_1 R_2}{R_1 + R_2} \quad (3)$$

d) Finally, Rogers and Reed (1984) does not have an equation (13) in it, and equation (5) in Reed's Comments should be:

$$P_1 = P_o + 3\pi\gamma R + \sqrt{6\pi R P_o + (3\pi\gamma R)^2}$$

and not as written; which just shows that mistakes will happen. In conclusion, we are very grateful to Dr. Reed for his comments, which did bring about these necessary corrections.

References

Same as in Reed's Comments.

A New Rate Principle Suitable for Analysis of Inelastic Deformation of Plates and Shells[2]

J. N. Reddy[3]. Much of the paper is a review of well-known variational principles of elasticity which can be found in a number of books (e.g., see Oden and Reddy, 1976). The 'new variational principle' presented in the paper is not new, and can be found in the monograph by Oden and Reddy (1976). More specifically, the functional Π_M in equation (11) of the paper is exactly the same as that in equation (4.115) on page 115 of this reference. The monograph also contains a number of related (fourteen in total) functionals for linear elasticity (on pp. 114–119) (Oden and Reddy, 1974), and variational principles for viscoelasticity (on pp. 143–169) and nonlinear elasticity and inelasticity (on pp. 173–189). Also, the title of the paper is not justified because the authors have not presented any analytical discussion of the specialization of the elasticity principles to plates and shells. Only a qualitative discussion of a possible extention is presented.

References

Oden, J. T., and Reddy, J. N., "On Dual Complementary Variational Principles in Mathematical Physics," *Int. J. Engng. Sci.*, Vol. 12, 1974, pp. 1–29.

Oden, J. T., and Reddy, J. N., *Variational Methods in Theoretical Mechanics*, Springer-Verlag, Berlin, 1976 (2nd Ed., 1982).

Authors' Closure

We thank Professor Reddy for his interest in our work [1]. We regret that the work of Oden and Reddy did not come to our attention prior to publication of our paper and that we ended up rediscovering their variational principle. The review of existing variational principles, which occupied half a page of our paper, was given in order to set the stage for what was to follow. These, of course, were clearly referenced ([5] and [6] of our paper). Further, we purposely restricted ourselves, in this paper, to a qualitative discussion of the application of our principle to inelastic shells. We have indicated in the paper that "a strictly two-dimensional formulation containing vector and tensor fields referred to the base vectors of the undeformed shell midsurface" would be published elsewhere ([8] of our paper). This paper [2] has just been published. Professor Reddy had communicated his concern about our paper, in a private letter to one of us, soon after it was published last September. We immediately replied to him and sent him a preprint of our Acta Mechanica paper. In view of this, we are really quite surprised to find his continuing concern regarding an analytical treatment of this variational principle for inelastic shells, as voiced in the last two sentences of his discussion.

References

1 Mukherjee, S., and Kollmann, F. G., 1985, "A New Rate Principle Suitable for Analysis of Inelastic Deformation of Plates and Shells," ASME Journal of Applied Mechanics, Vol. 52, pp. 533–535.

2 Kollmann, F. G., and Mukherjee, S., 1985, "A General Geometrically Linear Theory of Inelastic Thin Shells," *Acta Mechanica*, Vol. 57, pp. 41–67.

The Problem of Minimizing Stress Concentration at a Rigid Inclusion[1]

G. S. Bjorkman, Jr.[2] and R. Richards Jr.[3] The objective of this paper, as stated by Professor Wheeler, "is an optimization problem aimed at finding the best shape for a rigid inclusion imbedded in an elastic matrix of infinite extent, if the stress concentration is to be minimized." It should be emphasized that the solution to this problem had already been obtained earlier by the writers (Bjorkman and Richards, 1979; Richards and Bjorkman, 1980). In these two works the writers found that the rigid-inclusion shape which satisfied the harmonic field condition (i.e., the condition that the first invariant of the original stress (or strain) field remain unperturbed everywhere in the field) in a biaxial field is an ellipse whose axes are inversely proportional to the principal normal strains of the original field (i.e., $a/b = \epsilon_2/\epsilon_1$) irrespective of plane stress or plane strain. This is precisely the result Professor Wheeler obtains in equation (4.1) in less-

[1]But do not rely on it. Their paper was submitted long before Rogers and Reed's was published.

[2]By S. Mukherjee and F. G. Kollmann, and published in September 1985 issue of ASME Journal of Applied Mechanics, Vol. 52, pp. 533–535.

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[1]By L. Wheeler and published in the March 1985 issue of the ASME Journal of Applied Mechanics, Vol. 52, pp. 83–86.

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