Prediction of scour depth at culvert outlets using neural networks

S. L. Liriano and R. A. Day

ABSTRACT

Scour at culvert outlets is a phenomenon encountered world-wide. Research into the problem has mainly been of an experimental nature, with equations being derived for particular circumstances. These traditional scour prediction equations, although offering the engineer some guidance on the likely magnitude of maximum scour depth, are applicable only to a limited range of situations. A model for the prediction of scouring that is generally applicable to all circumstances is not currently available. However, there is a substantial amount of data available from research over many years in this area. This paper compares current prediction equations with results obtained from two Artificial Neural Network models (ANN). The development of a basic feed forward artificial neural network trained by back-propagation to model scour downstream of culvert outlets is described. A supervised training algorithm is used with data collected from published studies and the authors' own experimental work. The results show that the ANN can successfully predict the depth of scour with a greater accuracy than existing empirical formulae and over a wider range of conditions.

Key words | artificial neural networks, culverts, scour

S. L. Liriano
R. A. Day (corresponding author)
Water Engineering Research Group,
University of Hertfordshire,
College Lane,
Hatfield,
Herts AL10 9AB,
UK

INTRODUCTION

Accurate prediction of the dimensions of scouring downstream of hydraulic structures is required to ensure foundations are properly designed and prevent damage to the structure as a result of undermining. Although work has been conducted on scouring downstream of culverts since the 1970s, a widely accepted generic equation encapsulating a range of outlet sizes, flow rates, bed materials and channel and outlet geometries is not available.

The aim of this research is to compare the use of traditional empirical equations for predicting scour depth with artificial neural network models. Two neural networks are compared, one using the raw variables as the causal inputs and the second following a dimensionless analysis of the variables used in the first model.

The majority of previous research in this area has focused on developing models to predict the size and extent of scour holes formed as a result of submerged jets impinging on uniformly graded granular beds. Studies such as those by Opie (1967), Rajaratnam & Berry (1977), Rajaratnam (1981), Ruff *et al.* (1982), Rajaratnam & MacDougall (1983), Blaisdell & Anderson (1988), Abida & Townsend (1991), Lim (1995), Chiew & Lim (1996) and others have resulted in a selection of empirical equations for predicting scour depth, and in some cases length and width for particular hydraulic conditions. Breusers & Raudkivi (1991) and Hoffmans & Verheij (1997) presented detailed literature reviews of research undertaken into scour at culvert outlets and a further review can be found in Liriano (1999). The work conducted by Opie at Colorado State University (Opie 1967) remains significant as experimental data up to the largest outlet size used was 914 mm. Rajaratnam & Berry (1977) report scour depth as a function of densimetric Froude number, the first example of scour depth being related to densimetric Froude number known to the authors and the majority of scour predictors now make use of this dimensionless parameter. A 5 year study into scouring downstream of culverts was conducted by Ruff *et al*. (1982) and a significant amount of data was collected with outlet shape, size, tailwater depth, sediment grading and size all being variables. However, in spite of over 40 years of research into scouring downstream of culvert outlets the effects of tailwater depth, outlet shape and sediment grading are not well understood, with conflicting conclusions from different papers. Therefore the aim of this paper was to develop a model that could be applied to all circumstances.

DATASET

The dataset used in this study consists of the authors' original data (Liriano 1999) in addition to data kindly made available by Dr S. Y. Lim (Associate Professor, Nangyang Technical University, Singapore), Mr F. Ade (formerly at the University of Alberta, Canada) and Dr T. R. Opie (formerly at Colorado State University, USA). Further data has been added from published datasets from Bohan (1970), Rajaratnam & Berry (1977), Rajaratnam (1981), Ruff *et al*. (1982), Rajaratnam & MacDougall (1983), Ali & Lim (1986), Abida & Townsend (1991), Ade & Rajaratnam (1998) and Aderibigbe & Rajaratnam (1998). Table 1 shows the datasets used, the number of results included and the hydraulic conditions and sediments used in the respective experiments. In total 273 results were available which were divided into a training set of 215 results and a testing set of 31 results. A further 27 results were kept for validation purposes.

It can be noted from the table below that the distribution of the data is not uniform across all variables. In particular there is less data on rectangular and square outlets than on round outlets and very little data on graded sediments.

ANN

An artificial neural network is a computational tool that is able to acquire, represent and compute a mapping from one multivariate space of information to another, given a set of data representing that mapping (Garrett 1994). The advantage of ANN over traditional equations is that the exact function between a set of variables need not be known, this being a particular advantage to engineers where the underlying science of problems is not as yet determined and where data may be incomplete or noisy. A neural network 'learns', from a set of training data, a method of manipulating the input data in order to achieve the given result. Once a neural network has been trained in this way an additional set of data previously unseen can be presented to the network and the performance of the trained network assessed. Examples of the use of neural networks in civil engineering can be found in the *ASCE Journal of Computing in Civil Engineering* vol. 8 no. 2, which was devoted solely to this topic, and Rao (2000), which considers the use of artificial neural networks in hydrology.

Some use of artificial neural networks has been made in sediment transport. Trent *et al*. (1993) applied ANN to scouring at bridge piers and it was found that using 515 field measurements of scouring split into 387 training observations and 128 testing observations that ANN's improved upon the predictions of the empirical equations.

ANN design

A feed-forward artificial neural network trained by back-propagation was used in this study. The model consists of one hidden layer and one output node in each case. Using back propagation the network learns through an iterative procedure involving two steps performed many times. First the network is shown examples of the training data which pass forward to the output layer with the error in the output being computed. The second step works backward through the network. The errors at the output layer are propagated backwards through the network and the weights allocated to each nodal connection are adjusted to minimise the error in the output data. Using this technique it is possible for the network to become trapped in a local minima and for this reason a supervised training method was used. After the training data has been presented to the network a pre-determined number of times a test data set is presented. The result from the previous presentation of

Table 1 | Experimental data used in ANN

Researchers	Culvert shape	Vertical jet size	Sediment uniformity	F_0	Submergence ratio	No. of results
Abida & Townsend (1991)	Rectangular	76 mm	Uniform and graded	0.44–11.44	0.14–1.55	20
Ade & Rajaratnam (1998)	Circular	5 mm, 19 mm, 25.4 mm	Uniform	6.29–88.2	0.75, 25, 124	12
Aderibigbe & Rajaratnam (1998)	Circular	5 mm, 14 mm, 25.4 mm	Uniform and graded	2.79–29.46	12, 21, 60	31
Ali & Lim (1986)	Square	51 mm	Uniform	4.17	0.49	4
Bohan (1970)	Circular	68 mm, 101 mm, 305 mm	Uniform	0.45–3.16	0.5–1.0	33
Lim (1995)	Circular	15 mm, 26 mm	Uniform	1.91–24.6	0.47	16
Liriano (1999)	Circular	20 mm, 52 mm, 146 mm, 311 mm	Uniform	2.2–9.0	0.5–3.0	50
Opie (1967)	Circular	309 mm, 442 mm, 914 mm	Uniform	0.38–3.8	0.29–1.1	21
Rajaratnam (1981)	Circular	3.6 mm, 6.6 mm, 24.9 mm	Uniform	4.4–14.1	15, 58, 107	14
Rajaratnam & Berry (1977)	Circular	25.4 mm	Uniform	8.6–12.4	24	4
Rajaratnam & MacDougall (1983)	Rectangular	6.4 mm, 12.7 mm, 23.3 mm, 33.3 mm	Uniform	4.3–18.3	1.0	12
Ruff *et al.* (1982)	Circular and square	102 mm	Uniform and graded	5.5–33.7	0, 0.25, 0.45	55

the test dataset is compared and if there is an improvement training continues. When no improvement is observed the cycle of presenting training and testing data continues until no improvement has been noted with the test dataset for 30 consecutive attempts. At this point the training is terminated and the weights set to those from which the best result was obtained with the test data. This prevents the network over-learning the training data by checking the performance with the test data and reduces the likelihood of the network reaching a local minimum as opposed to the global minimum by continuing training for 30 cycles after a minimum error has been achieved.

Once the network has been trained a further validating set of data not used during training is presented to the ANN and the output compared with the known output in order to assess the predictive capabilities of the network.

Model I

Scour hole dimensions are dependant on many variables that can be grouped in general to describe the physical properties of the outlet and receiving channel, the bed materials and the flow characteristics. These can listed as

$$d_{se} = f(\rho, \mu_0, u_0, d_0, H, W, W_0, g, \rho_s, d_{50}, \sigma_g, \text{culvert shape}) \quad (1)$$

where d_{se} is the maximum depth of scour, ρ is the density of water, μ_0 is the dynamic viscosity of water, u_0 is the mean velocity at the outlet, d_0 is the pipe diameter for circular outlets and the outlet height for non-circular outlets, H is the depth of water in the downstream receiving channel (tailwater depth), W is the width of the receiving channel, W_0 is the width of the outlet, g is the acceleration due to gravity, ρ_s is the density of the sediment bed material, d_{50} is the median sediment size, and σ_g is the geometric standard deviation of the sediment bed material and describes how graded the material is and shape is the shape of the outlet.

This resulted in 12 variables being entered as inputs to the neural network. The optimal number of nodes in the hidden layer has been determined by trial and error, in this case using 8 nodes in the hidden layer was found to give

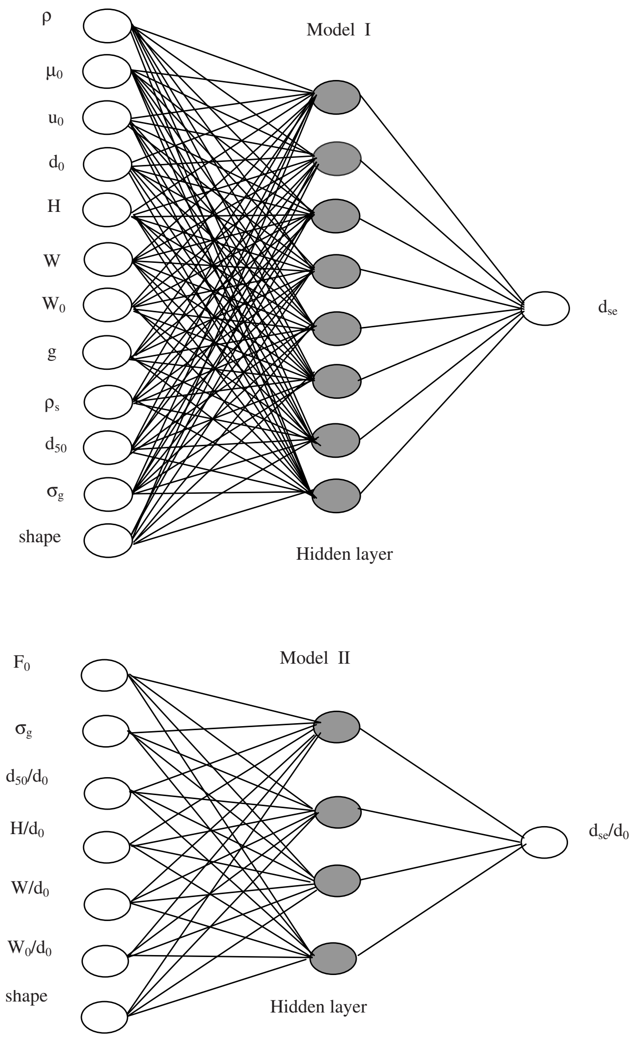

Figure 1 | Network architecture for models I and II.

the best result. The neural network architecture is shown in Figure 1(a).

Model II

Given that the flow is turbulent in each case and assuming that the viscous effect is not important a dimensional analysis of (1) gives

$$\frac{d_{se}}{d_0} = f\left(F_0, \frac{H}{d_0}, \frac{W}{d_0}, \frac{W_0}{d_0}, \frac{d_{50}}{d_0}, \sigma_g, \text{shape}\right) \quad (2)$$

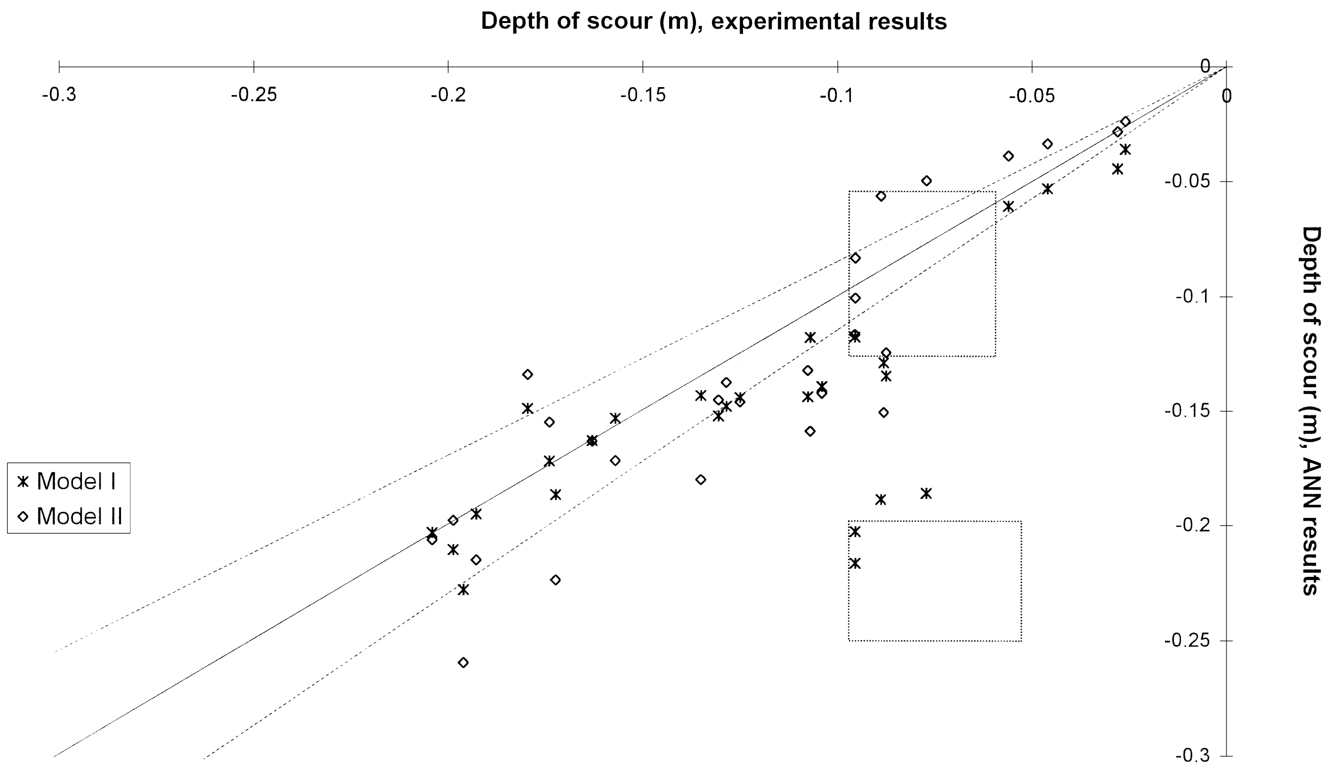

Figure 2 | Comparison of neural network models for predicting scour depth.

where $F_0 = u_0/[(S-1)gd_{50}]^{0.5}$ is the densimetric Froude number and $S = \rho_s/\rho$ is the specific gravity of the sediment. The parameters on the right hand side of Equation (2) are the inputs to model II with d_{se}/d_0 as the output value. Physically, F_0 represents the ratio of the tractive force acting on the individual sediment grain to the submerged weight retaining the grain in place. Empirical equations are typically a function of densimetric Froude number only and are applicable over a limited range of parameters only.

For model II there are 7 input variables and a network with 4 nodes in the hidden layer was found to give the optimal result.

RESULTS OF TRAINING

In order to examine the performance of the neural network models a validation data set was presented to the neural networks and the results obtained are compared with the known results. The validation data set has not been used by the neural network models during training and includes data from a range of experimental conditions as shown in Table 2.

The results obtained from the neural networks using the validation data are compared with the experimental values in Figure 2. The solid black line shown on the graph in Figure 2 is the line of perfect agreement with the dashed lines showing $\pm 15\%$ variation. The results show that Model II is a slight improvement on Model I but with both networks closely predicting the experimental results. The largest errors in the predicted data are seen for the box outlets with Model I, the data from the box outlets indicated with a dashed rectangle in Figure 2. Both models give a better fit to the data with the pipe outlets than from the box outlets, which is likely to be a result of insufficient data for the box outlets since only 15% of the data is from box outlets.

Table 2 | Range of validation data

Variable	Range of data
Outlet shape	Circular and box
Outlet diameter, d_0 (m)	0.0254–0.146
Sediment size, d_{50}/d_0	0.016–0.28
Tailwater depth, H/d_0	0.5–25
Exit velocity, u_0 (m/s)	0.747–11.176

DISCUSSION OF RESULTS

The results show that a neural network with 1 hidden layer, trained with either the raw data or following a grouping of the raw data into dimensionless terms, can predict scour depth downstream of the outlets. Model II, trained with the dimensionless groups, gives a slightly better prediction of the experimental results. Additionally once this pre-processing of data has been conducted the training of the network is also quicker since there are less input variables and fewer nodes in the hidden layer.

With both neural network models it was found that the results for the pipe outlets could be predicted with greater accuracy than those with the box outlet and it is likely that this is due to the lack of data available on the box outlets.

Figure 3 shows a comparison of the results obtained with the neural networks and those obtained using the empirical equations available from published studies. The models used are from Abt *et al*. (1984), Lim (1995), Chiew & Lim (1996), and Liriano (1999). In each case the model is only applied to the range of the validation data which it is stated as applicable, as indicated in Table 3. The ANN models are applied to all the validation data as the training set covers the complete range of data used. This immediately demonstrates the benefit of the neural

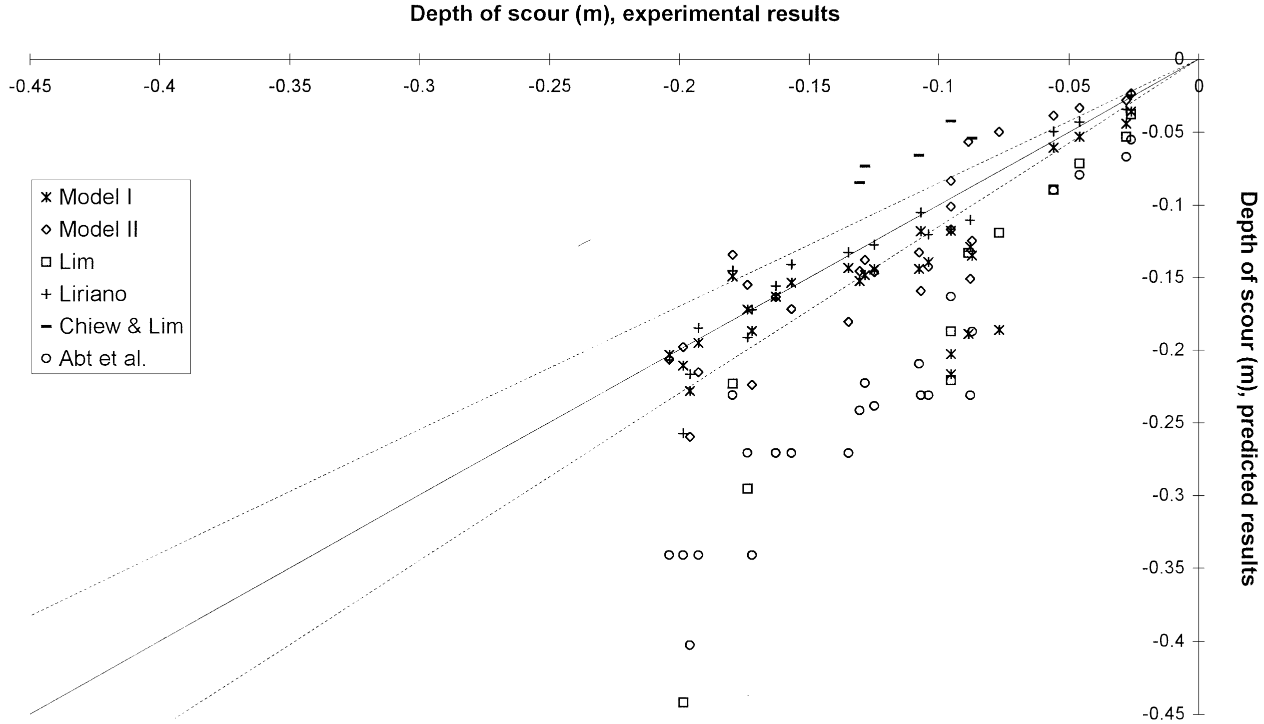

Figure 3 | Comparison of neural network models and traditional scour predictors.

Table 3 | Equations used to compare performance of the ANN with existing models and the limitations of the existing models

Author	Equation	Limitations
Lim (1995)	$d_{se}/d_0 = 0.45F_0$	$1 < F_0 < 10$, $H < 1.0d_0$, $\sigma_g < 2.0$
	$d_{se}/d_0 = 0.45$	$F_0 > 10$, $H < 1.0d_0$, $\sigma_g < 2.0$,
Liriano (1999)	$d_{se}/d_0 = a\ \ln(F_0) + b$	$3 < F_0 < 9$, $5d_0 < H < 2.0d_0$, $\sigma_g < 2.0$,
	where $a = 0.877(H/d_0)^{-0.37}$ and $b = 0.20\ \ln(H/d_0) - 0.24$	circular outlets
Chiew & Lim (1996)	$d_{se}/d_0 = 0.21F_0$	$H \geqslant 1$, $4.8 < F_0 < 85.3$, $H \geqslant 1.0d_0$, circular outlets
Abt *et al.* (1984)	$d_{se}/d_0 = -3.67(F_0^{0.57} d_{50}^{0.4} \sigma_g^{-0.4})$	Circular outlets

network models where no changes need to be made for the different conditions encountered.

The graph in Figure 3 shows that the neural network models give a prediction of scour depth that is as good, and in most cases better, than the empirical models for all conditions. The neural network models are the only predictors that can be used for the whole range of conditions represented by the validation data and therefore offer a greater flexibility than the traditional scour prediction models. This would be of significant benefit to engineers where only one method for calculating scour depth is required for the complete range of potential operating conditions.

CONCLUSIONS

The aim of this research was to determine whether a neural network could accurately predict scour depth downstream of culvert outlets for a range of hydraulic and physical conditions. It has been demonstrated that it is possible with a network using 1 hidden layer and two models have been compared. Pre-processing the data into non-dimensionless groups enables the training time to be reduced due to a smaller number of nodes in the input layer and a smaller umber of nodes in the hidden layer was found to give the optimum result. Both neural network models predicted scour downstream of pipe outlets more accurately than box outlets and further work is being undertaken to explore this. The application of artificial neural networks to scour prediction shows the potential to lead to a flexible tool for engineers. Artificial neural networks have the advantage of being applicable to a wider range of hydraulic conditions than traditional empirical models removing the requirement of the designer to choose the appropriate equation for the anticipated hydraulic conditions. Further work is required to provide a complete data set to train the network and validate its usefulness.

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