

On the Extraordinary Strength of Prince Rupert's Drops

H. Aben¹, J. Anton¹, M. Öis¹, K. Viswanathan², S. Chandrasekar², M. M. Chaudhri^{3*}

¹Institute of Cybernetics, Tallinn University of Technology, 19086 Tallinn, Estonia

²Center for Materials Processing and Tribology, Purdue University, West Lafayette, IN 47907, USA

³Department of Physics, Cavendish Laboratory, University of Cambridge, Cambridge, CB3 0HE, UK

*corresponding author, mmc11@cam.ac.uk

Abstract

Prince Rupert's drops (PRDs), also known as Batavian tears, have been in existence since the early 17th century. They are made of a silicate glass of a high thermal expansion coefficient and have the shape of a tadpole. Typically, the diameter of the head of a PRD is in the range 5 – 15 mm and that of the tail is 0.5 to 3.0 mm. PRDs have exceptional strength properties: the head of a PRD can withstand impact with a small hammer, or compression between tungsten carbide platens to high loads of ~ 15, 000 N, but the tail can be broken with just finger pressure leading to catastrophic disintegration of the PRD. We show here that the high strength of a PRD comes from large surface compressive stresses in the range 400 – 700 MPa, determined using techniques of integrated photoelasticity. The surface compressive stresses can suppress Hertzian cone cracking during impact with a small hammer or compression between platens. Finally, it is argued that when the compressive force on a PRD is very high, plasticity in the PRD occurs which leads to its eventual destruction with increasing load.

Keywords: Prince Rupert's drops; integrated photoelasticity; stress measurements; Hertzian fracture; plasticity; residual stress; mechanical strength

Prince Rupert's drops (PRDs), tadpole-shaped glass drops also known as Batavian tears, have been in existence since before 1625¹. Prince Rupert brought five PRDs, *two with liquor in them and other three solid*², from Germany to England and presented them to King Charles II, who became interested in their extraordinary properties and communicated them to His Society at Gresham College (later to be known as The Royal Society) for investigations. PRDs are made by dropping red hot blobs of molten glass into cold water at $\sim 20^\circ$ C from a height of 15–20 cm. On cooling down and solidifying, the drops are of a tadpole shape. They have a bulbous head $\sim 5 - 15$ mm in diameter and a tail with diameter in the range of 0.5 to 3 mm. The head of a PRD is so strong that it can withstand impact with a small hammer or compression between hard platens to ~ 15000 N. On the other hand, the tail of a PRD can be broken with just finger pressure and within a few microseconds the entire PRD shatters into fine powder with an accompanying sharp noise. Since the seventeenth century, famous scientists and natural philosophers^{3, 4, 5, 6} have been trying to understand the exceptional properties of these drops. However, a plausible explanation of the disintegration was provided by Chandrasekar and Chaudhri⁷, who used high-speed framing photography at 0.5×10^6 frames. s^{-1} to follow this process. Nevertheless, one aspect of the conundrum still remains: why is the head of a PRD so strong? Here we provide an answer by using techniques of integrated photoelasticity^{8, 9}, which shows that the exceptional strength of the PRDs comes from very high surface compressive stresses present in the head.

Some of the previous workers, namely Hooke⁴, a Committee of the Royal Society (see Moray³), de Luynes⁶ and Peligot⁵ had realised that the method of preparation of a PRD left it in a state of unstable equilibrium, which was disturbed by the breakage of its tail from which the disintegration of the entire PRD ensued. However, both Hooke⁴ and de Luynes⁶ suggested that the surface of a PRD was in a state of tension; this is an incorrect notion, and, unfortunately, continues to be endorsed to this day^{10, 11, 12}. Contrary to this notion,

Chandrasekar and Chaudhri⁷ proposed that the surface of a PRD was in compression and the interior of the PRD in tension. They also believed that the residual stress distribution in a PRD was qualitatively similar to that in thermally tempered plate glass, but did not measure this distribution. Thus it is the main purpose of the present work to determine experimentally the residual stress distribution in PRDs and to use this information to explain the extraordinary strength properties of PRDs.

PRDs were made of soda-lime glass of composition SiO₂, 72.5 %; Na₂O, 13%; K₂O, 0.3 %; Al₂O₃, 1.5 %; CaO, 9.3 %; MgO, 3%; Fe₂O₃, 0.1 %. An end of a 10 – 15 mm diameter soda-lime glass rod was heated with a natural gas-oxygen flame so that the glass became red hot and melted. By continuously spinning the rod about its axis, a molten glass blob was formed on the heated end. The blob was then allowed to drop into a 500 ml borosilicate glass beaker filled with cold water at ~ 18-20 °C from a small height. During the drop, the glass blob remained attached to the parent glass rod with a tail of decreasing diameter. The tail was detached from the parent rod using the gas-oxygen flame. The lengths of the PRDs (head + tail) were typically 100 mm. On cooling down, a PRD having the shape of a tadpole was formed. Some of the PRDs formed in this manner had one or more 2-3 mm diameter bubbles in their heads. Such bubbles would disappear on annealing the PRDs at 700 °C for about two hours and are thought to be vacuous. The residual stress distribution in PRDs was experimentally determined using integrated photoelasticity^{8, 9}; this technique is suitable for axisymmetric, three dimensional and optically transparent objects. A schematic diagram of the experimental set up is shown in Fig. 1.

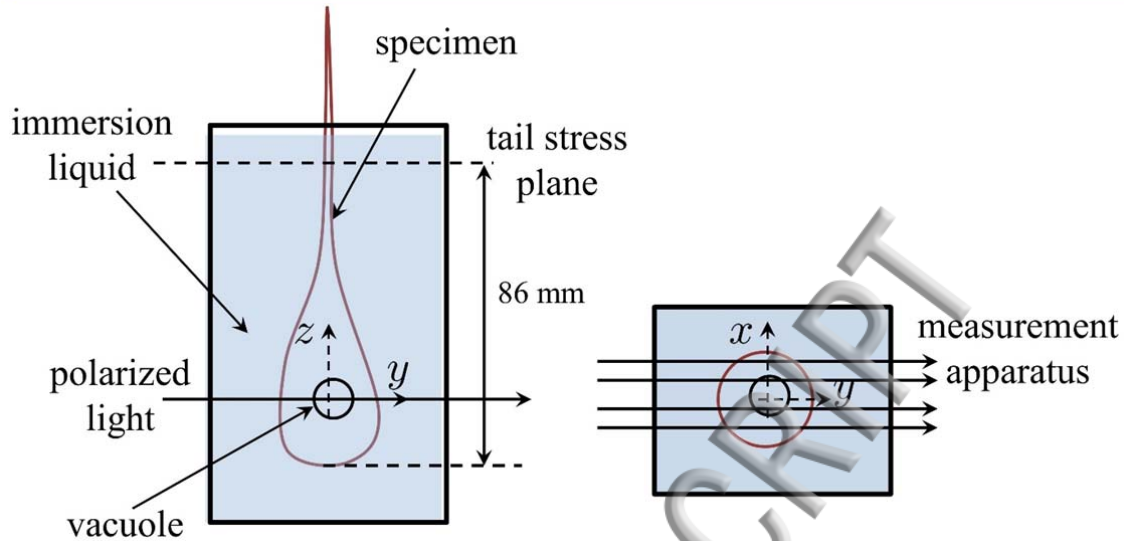


Figure 1 Schematic diagram of the optical arrangement used for measuring the axisymmetric residual stress distribution in PRDs. The stresses in the head are truncated at the edge of the vacuole (black circle). In the drop PRD1, the stress in the head was measured at a plane located at a distance of 4.9 mm from the top of the head, the widest location in the head. The origin of the (x, y) plane is at the center of this measurement plane, with radial coordinate $r = \sqrt{x^2 + y^2}$. The plane in which the tail stresses was measured is also marked.

The test PRD with its axis vertical is suspended in a small rectangular parallelepiped transparent chamber filled with a clear liquid of refractive index matching that of soda-lime glass (1.521). A parallel beam of circularly polarised monochromatic light of wavelength $\lambda = 627$ nm from a red light emitting diode was then incident on the drop at an angle of 90° to the vertical, as shown. A ray of the light beam going through the PRD suffers optical retardation, which is measured with a computer controlled transmission polariscope. The spatial resolution of the polariscope was $12 \mu\text{m}$ (equivalent to 83 pixels per mm). From the measured optical retardation and knowledge of the birefringence of soda-lime glass, and using a scheme known as the Abel inversion, the distribution of the residual stress in the PRD was obtained⁸,⁹ (see supplementary material). The uncertainty in the stress measurement is less than 5 %.

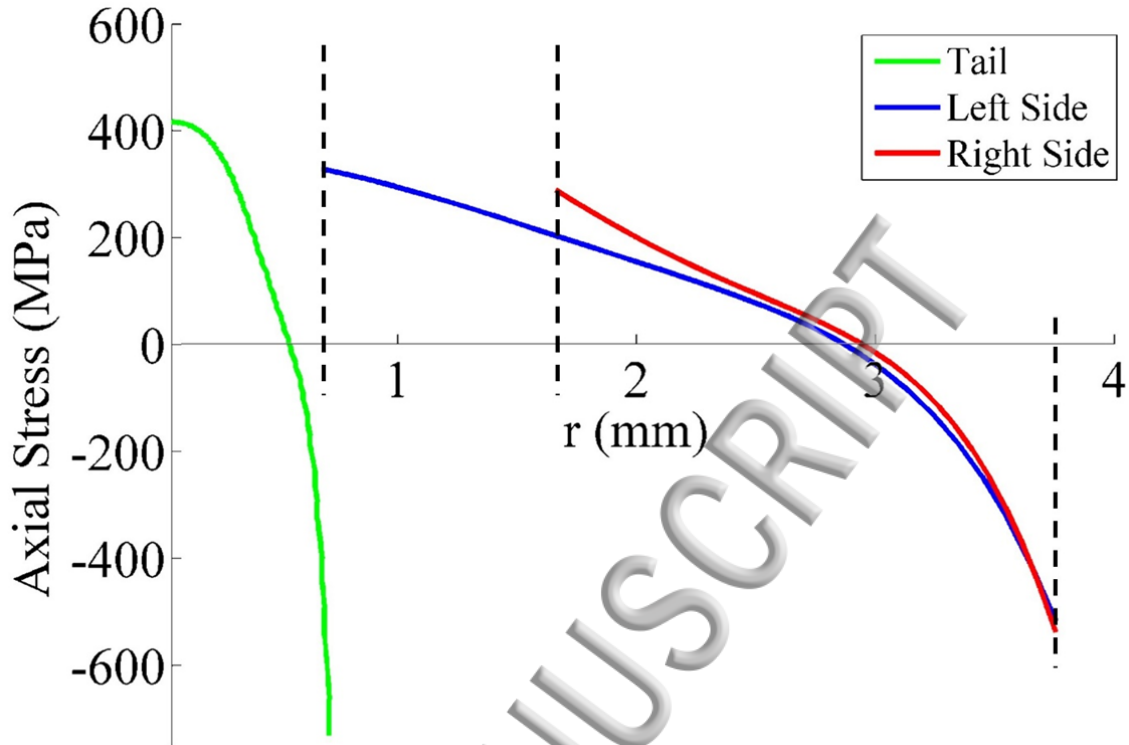


Figure 2 Measured axial residual stresses in the head and tail of PRD1 of head diameter of 7.4 mm. The two residual stress curves (left, right) for the head are truncated at the vacuole boundary, which is denoted by black dashed lines. The vacuole is located asymmetrically in the drop head. The right hand side dashed line marks the outer surface of the drop. Note that the circumferential stress at a point on the drop surface is equal to the axial stress therein, based on the sum rule¹³.

We note from Fig. 2 that the surface of the PRD is in compression (negative stress) and the interior of the drop in tension (positive stress), both in the head and tail regions. The maximum surface compressive stress is as high as ~ 525 MPa in the head and ~ 700 MPa in its tail, whereas the maximum tensile stress both in the head of the PRD and in its tail is 300 ~ 400 MPa. Similar values were obtained from three other PRDs. Table 1 shows the residual stress data from the head region for all the four PRDs.

Table 1. Collective data from residual stress measurements of all the four PRDs' heads

Drop No.	Drop's head diameter (mm)	Center stress (MPa) (tensile)	Surface stress (MPa) (compressive)	Ratio of maximum compressive stress to maximum tensile stress	Thickness of surface compressive layer (mm)	Ratio of compressive layer thickness to head diameter
PRD1	7.4	400	-525	1.30	0.75	0.10
PRD2	6.5	225	-700	3.1	0.50	0.08
PRD3	6.5	360	-450	1.25	0.75	0.12
PRD4	6.5	230	-400	1.80	0.85	0.13

We note from Table 1 that the surface compressive stress in the heads of the PRDs is in the range 400 – 700 MPa, which is considerably higher than our previously reported value of 90 – 170 MPa determined using an indentation technique⁷. However, the surface compressive stress values given in Table 1 are closer to the value of 750 MPa calculated by Narayanaswamy and Gardon¹⁴ in the case of solid spheres having properties similar to those of soda-lime glass. These spheres were 6 – 10 mm in diameter and were quenched from initial temperatures in the range 650 to 1000 °C.

We did not examine the fracture loads of the four PRDs shown in Table 1 because all of them had been mounted in the apparatus. However, we have examined the fracture behaviour of similar size PRDs when they were compressed between tungsten carbide platens to high loads. The observations obtained are shown in Table 2.

Table 2. The response of four PRDs when compressed between tungsten carbide platens to high loads.

PRD No.	PRD's head diameter / mm	Maximum compressive load / N	Remarks
5	7.83	11, 000	Broke at the maximum load
6	7.35	19, 000	Broke at the maximum load
7	7.20	10, 000	Did not break
8	~ 7.5	18, 000	Broke during unloading

We note from Table 2 that some of the PRDs can sustain compressive loads of up to 18, 000 N, although one, PRD 5, broke at 11, 000 N and another one, PRD 8, broke during unloading.

As regards the thickness of the surface compressive layer in the heads of the four PRDs (Table 1), it was determined to be in the range of 0.50 to 0.85 mm. The residual stresses were present throughout the drop, including the region very close to the vacuole, as will be seen in Fig. 3. The stresses varied smoothly and monotonically with distance along the radius, without any singularity at the vacuole surface (Figs. 2 and 3). It may also be noted that the speed of the fracture front is not noticeably affected when it crosses over a vacuole¹⁵.



Figure 3 Isochromatics through PRD1 obtained using white light. In the centre of the head of the PRD 1 (head diameter 7.4 mm) is a vacuuous bubble and the colour fringes exist throughout the PRD. Fringes exist very close to the vacuuous bubble, indicating the existence of residual stresses in all parts of the drop.

From knowledge of the compressive residual stress value in a PRD, we will now explain how the head of a PRD can withstand being compressed to loads as high as ~ 15000 N between two tungsten carbide platens without breaking. Some relevant mechanical properties of tungsten carbide and soda-lime glass are shown in Table 3.

Table 3. Relevant physical properties of tungsten carbide and soda-lime silica glass

Material	Elastic modulus / GPa	Poisson's ratio	Indentation hardness /GPa
Tungsten carbide	650	0.26	12
Soda-lime glass	69	0.25	5.4*

*Note that thermal tempering of soda-lime glass does not affect its indentation hardness¹⁶.

In order to simplify our theoretical consideration, we assume that the head of a PRD is spherical and loaded against a flat WC platen. At low applied loads, the deformation of the PRD will be elastic and the Hertzian equations¹⁷ will give the contact stress field. Moreover, the contact area between a tungsten carbide flat and the PRD head will be a circle, whose diameter will increase with increasing compressive load. Outside the circle of contact the surface radial stress will be tensile. Using the Hertzian equations, it can be shown that the mean normal contact pressure p_m for an applied normal contact load P is given by

$$p_m = 2.747 \times 10^8 P^{1/3}, \quad (1)$$

where the units of p_m and P are Pa and N, respectively.

By taking p_m equal to the indentation hardness of the glass, which is 5.4×10^9 Pa, we find that the threshold compressive load required to cause plasticity in the sphere is 7590 N. At this threshold load, the radius of the circle of contact is calculated to be 0.668 mm. The maximum tensile radial stress σ_m at the circle of contact is then given by

$$\sigma_m = \frac{P(1-2\nu)}{2\pi a^2}, \quad (2)$$

where ν is Poisson's ratio of the soda-lime glass and a is the radius of contact. Taking $\nu = 0.25$, and $a = 0.668 \times 10^{-3}$ m, the value of σ_m at the contact circle is 1.35×10^9 Pa. In stress-free plate glass, a tensile stress of this magnitude at the contact circle of this size will usually

create a ring crack in the plate glass, but this stress is not sufficient in the case of a PRD to cause such a crack since the fracture strength of pristine soda-lime glass fibers is in the range 3 – 4 GPa.¹⁸

In order for a PRD to disintegrate catastrophically, it is necessary for any cracks, induced by the compression process, to enter the tension zone in the head of the PRD. In the elastic regime, a Hertzian ring and associated cone crack can form, but because of the very high surface compressive stress, any cone cracks will grow almost parallel to the PRD's surface, as has been shown experimentally by Chaudhri and Phillips¹⁶ in the case of thermally tempered plate glass containing residual surface compressive stress of 100 MPa. Moreover, the very high surface compressive stress in a PRD will discourage the formation of any ring and cone cracks. In fact, we have not seen a ring crack in a PRD which was compressed to 10000 N between tungsten carbide flats.

At compression loads higher than 7590 N, the test PRD will deform plastically and the stress field at the circle of contact and outside it will be different from that in the elastic case. There are no analytical solutions to the problem of elastic-plastic deformation of a solid sphere by a hard flat. However, Kienzler and Schmitt¹⁹ have carried out a numerical analysis of the problem and have stated that due to plasticity of the sphere the radial tensile stress at the contact circle between the sphere and the flat will be reduced (the degree of reduction has not been given), whereas the circumferential stress at and outside the contact circle will become tensile. Furthermore, Kienzler and Schmitt¹⁹ stated that the magnitude of the tensile circumferential stress will increase with increasing plastic deformation of the PRD. When the tensile circumferential stress at the contact circle becomes greater than the residual surface compressive stress in the PRD, meridional cracks will form²⁰ which will extend into the tension zone of the PRD leading to its total disintegration. If, on the other hand, the PRD has not deformed plastically to a sufficiently high value during loading, then median cracks can

form in the interior of the PRD during unloading, as has been shown in the case of plastically deformed steel ball bearings²¹; thus, catastrophic disintegration will then follow.

To summarise, our residual stress measurements have shown that very high surface compressive stresses of up to 700 MPa exist in the heads of soda-lime PRDs. These high compressive stresses protect the PRDs against impact with a small hammer or compression between hard platens to very high loads of ~15000 N. Thus, combined with our previous investigations⁷ into the mechanism of the propagation of catastrophic disintegration of PRDs, this work completes a solution of the conundrum which appeared in the middle of the seventeenth century.

Note – See supplementary material for description of experimental details, methods and materials.

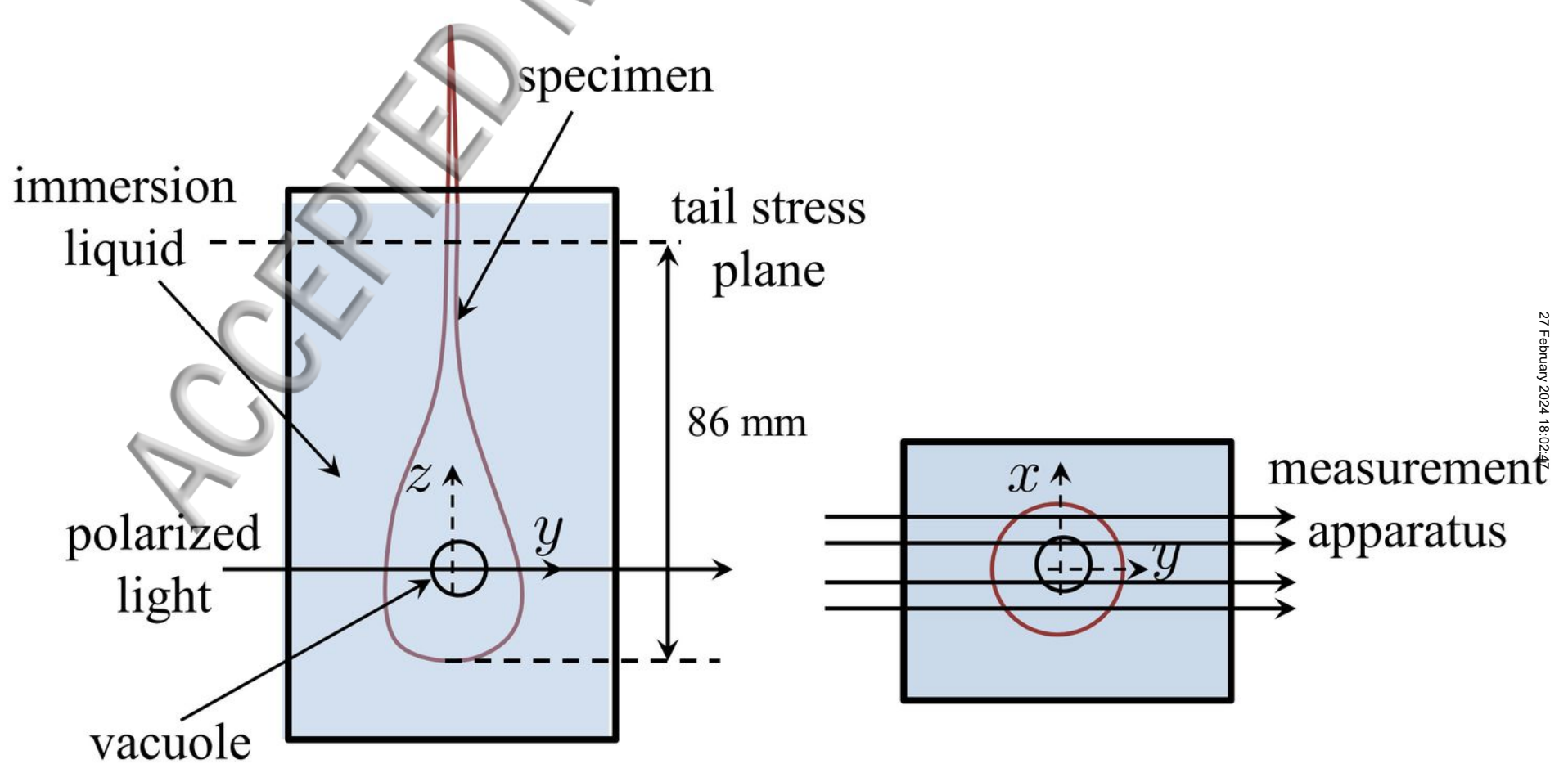
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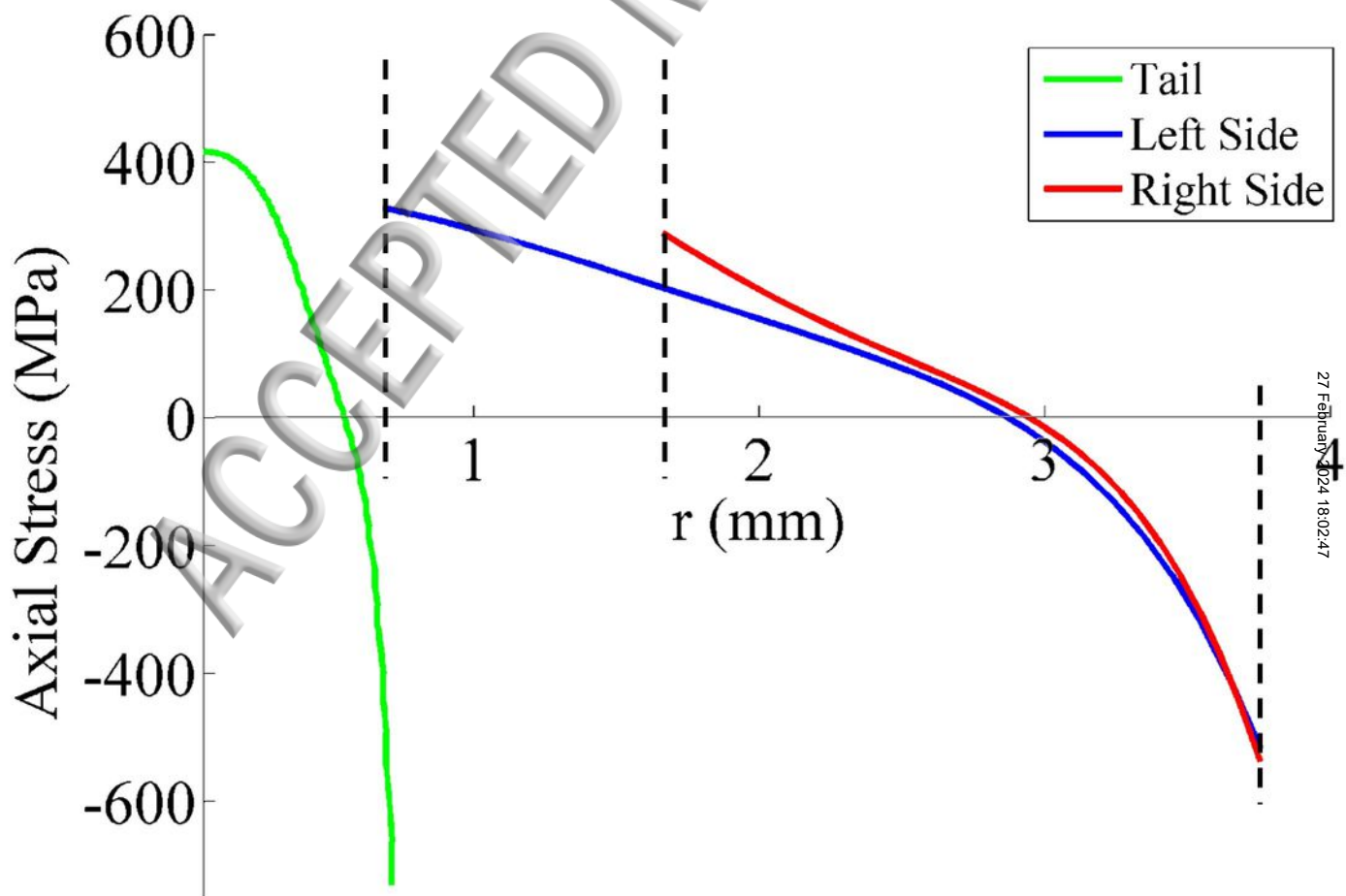
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