and the conditions imposed are very simple. However, the domains of asymptotic stability obtained through applications of this simple function are somewhat disappointing. For example, for the oscillator of van der Pol, equation (11) of the paper, one obtains from the Liapunov function of Leighton the domain of stability $x_1^2 + x_2^2 < 1$. A comparison of this result with the one given in the paper, equation (19) and Fig. 2, shows that this domain is entirely contained within the one obtained by the more complicated Liapunov function generated in the paper. Furthermore, the domain of stability obtained in the paper is sensitive to the parameter E, a very desirable quality. For each of the examples shown in the paper the Liapunov function of Leighton gives results which are inferior to those presented. This is not surprising, since Leighton uses one Liapunov function, and in this paper a Liapunov function is generated for each specific differential equation. This yields an improvement in the results obtained at the expense of generality, as would be expected.

Dr. Szëgo points out that most of the methods of Liapunov function generation are mathematically very similar in that they are all based on the search for a vector $a(x)$ that satisfies equations (11) and (12) of the Discussion. It is at this point that the methods diverge. In this paper, the authors tried to generate this vector from the differential equations themselves. It is for this reason that emphasis is placed on the auxiliary system of differential equations and on the cross-product. That this viewpoint is a very rewarding one was shown recently by Walker and Clark,[10] who extended these ideas to higher-order systems. They have been able to reproduce in a very simple and elegant manner the results of Barbasin and Simanov, as well as those of Cartwright.[11]

As with all methods of Liapunov function generation, the present one is far from an ideal one. It is particularly adaptable to the study of systems of differential equations which are not regular in the sense of Leighton and are exemplified by the problems presented in the paper.

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[10] J. A. Walker and L. G. Clark, "An Integral Method of Liapunov Function Generation for Nonlinear Autonomous Systems," ASME Paper No. 64—WA/APM-42.

[11] W. Hahn, *Theory and Application of Liapunov's Direct Method*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1963.

Sandwich Shells of Arbitrary Shape[1]

J. L. BAYLOR.[2] Through a sequence of approximations, the author obtains equations which are applicable to very thin shallow shells. Let us examine the most stringent approximations.

The equilibrium equations (10*a*, *b*, *c*) are the equations for a flat plate. Consequently, the result of integrating these equations is the well-known description of a weak core in a sandwich plate.

When simplifying the kinematic equations (4) and equilibrium equations (5), the author neglects certain derivatives (7) of the metric tensor. Consequently, the subsequent equations embody the Donnell-type approximations usually associated with shallow-shell analysis; for example, the term V/r_2 is absent from (8*e*). Similar omissions occur in equations (14), (15), (16) and again in (31*d*, *f*). The equilibrium equations (9) and (20) have the same lack of precision.

Since the analysis is limited to shells with equal facings, the writer wonders why the author has bothered to refer to the middle surface of the facings (such reference has merit if the facings are dissimilar). Had he referred to the interfaces and employed interface displacements, he would have no need to concern himself with continuity of displacements at the interfaces. The relative displacements of two particles on the normal, one at each interface, can be obtained directly from the strain field of (11) and (12) by means of a definite integral (such relative displacements can be replaced easily by the components $\bar{u}$, $\bar{v}$, $\bar{w}$ at any later time if desired). This procedure has the additional advantage that no unidentified "functions of integration" need arise; all functions of the surface coordinates are subject to immediate physical interpretations.

In view of the shallow-shell approximations used, the author might have compared his equations with those of Grigolyuk[3] and Fulton.[4] Also, it would be worthwhile to specialize these equations for membrane facings and to compare with a more precise theory (for membrane facings) given by Reissner[5] and Wang.[6]

In view of what has been said, the writer fails to see the significance of the paper.

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[1] By Robert Schmidt, published in the June, 1964, issue of the Journal of Applied Mechanics, vol. 31, Trans. ASME, vol. 86, Series E, pp. 239–244.

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[3] E. I. Grigolyuk, "Equations of Three Layer Sandwich Shells with Light Packing," Izvestia Akademii Nauk USSR, *Otdelenize Teckhnicheskikh Nauk*, no. 1, 1957, pp. 77–84.

[4] R. E. Fulton, "Non-linear Equations for a Shallow Unsymmetrical Sandwich Shells of Double Curvature," *Proceedings of the 7th Midwestern Mechanics Conference*, Plenum Press, New York, N. Y., 1961, p. 365.

[5] E. Reissner, "Small Bending and Stretching of Sandwich-type Shells," NACA Report 975, 1950.

[6] C. T. Wang, "Principle and Application of Complementary Energy Method for Thin Homogeneous and Sandwich Plates and Shells With Finite Deflections," NACA Technical Note 2620, February, 1952.

Author's Closure

Dogmatic statements are not necessarily true. That the present theory of sandwich shells could be made more precise is beyond any doubt. However, most of the possible improvements are likely to result in lengthier equations, which are rather complicated even in their present state.

The choice of the middle surface of the core as the reference surface was determined by common usage. Choosing the interfaces as reference surfaces simplifies certain steps in the derivations and complicates others. There are pros and cons in either approach.

In the case of cylindrical shells, the author's equations coincide with those derived by Kurshin, who compared his equations with those given by Grigolyuk, Reissner, and Wang and found that his equations are not only not inferior but, in some cases, superior to the aforementioned ones. Thus, the discusser's assertion about the greater precision of those theories seems to be without any foundation. Only the most recent theory published by Grigolyuk and Kiryukhin[7] appears to be less restrictive than the one in the present paper, in certain applications.

The author has found that the process of derivation of equations employed in the paper under discussion is very convenient in developing the governing equations for some rather unusual sandwich plates[8, 9, 10] and shells.

In the author's opinion, the discusser's right to doubt the significance of any paper whatsoever should never be denied.

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[7] E. I. Grigolyuk and Yu. P. Kiryukhin, "Linear Theory of Three-Layered Shells with a Stiff Core," *AIAA Journal*, vol. 1, no. 10, October, 1963, pp. 2438–2445.

[8] Sandor Silverman and Robert Schmidt, "Flexure of Sandwich Plates with Linearly Tapered Cores," Engineering Research Laboratories Report, University of Arizona, Tucson, Ariz., August, 1963.

[9] Robert Schmidt, "Flexure of Five-Ply Sandwich Plates," Engineering Research Laboratories Report, University of Arizona, Tucson, Ariz., July, 1963.

[10] Robert Schmidt, "Flexure of Linearly Tapeied Sandwich Plates," Engineering Research Laboratories Report, University of Arizona, Tucson, Ariz., May, 1963.

