# Dynamic penalty function as a strategy in solving water resources combinatorial optimization problems with honey-bee mating optimization (HBMO) algorithm

Omid Bozorg Haddad and Miguel A. Mariño

## ABSTRACT

Because of the complexity of some optimization problems, evolutionary and meta-heuristic algorithms are sometimes more applicable than the traditional optimization methods. Some difficulties in solving design-operation problems in the field of engineering are due to the multi-modality of the solution region of these problems. Since the design variables usually are specified as discrete variables and other continuous decision variables have to be set according to the range of the discrete ones, the possibility of trapping the final solution into some local optimum increases. In such cases, the capability of both traditional and evolutionary algorithms decreases. Thus, the development of a strategy to overcome this problem is the subject of this paper. For water utilities, one of the greatest potential areas for energy cost-savings is the effective scheduling of daily pump operations. Optimum design operation of pumping stations is a potential problem in this area that performs a wide background of solutions to this problem with different methods. Computation in all methods is driven by an objective function that includes operating and capital costs subject to various performances and hydraulic constraints. This paper achieves the optimal control and operation of an irrigation pumping station system by one of the latest tools used in optimization problems, which is the honey-bees mating optimization (HBMO) algorithm and is tested with a practical design. The HBMO algorithm with dynamic penalty function is presented and compared with two other well-known optimization tools which are the Lagrange multipliers (LM) method and genetic algorithms (GA) as well as with the previous results of the HBMO algorithm with constant penalty function for the same problem. The LM, GA and HBMO approaches simultaneously determine the least total annual cost of the pumping station and its operation. The solution includes the selection of pump type, capacity and the number of units, as well as scheduling the operation of irrigation pumps that results in minimum design and operating cost for a set of water demand curves. In this paper, the HBMO algorithm is applied and the dynamic penalty function is tested to demonstrate the efficiency of this combination simultaneously. The results are very promising and prove the ability of combining the dynamic penalty function with the HBMO algorithm for solving combinatorial design–operation optimization problems. Application of all these models to a real-world project shows not only considerable savings in cost and energy but also highlights the efficiency and capability of the dynamic penalty function in the HBMO algorithm for solving complex problems of this type.

**Key words** | combinatorial optimization problems, dynamic penalty function, honey-bee mating optimization, optimal design, optimum operation, pumping stations

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doi: 10.2166/hydro.2007.025

## NOTATION

| | |
|---|---|
| $a_i$, $b_i$, $c_i$ | efficiency curve coefficients of $i$th pump |
| $C_E$ | unit energy price |
| $C_i$ | cost of $i$th pump |
| $C'_i$ | equivalent cost of $i$th pump after construction time |
| $D_i$ | delivery pipe diameter of $i$th pump |
| $E_T$ | total annual consumed energy |
| $e$ | efficiency |
| $e_{i,j}$ | efficiency of $i$th pump at $j$th month |
| $g$ | number of current generation |
| $Gen$ | total number of generations |
| $H$ | pumping head |
| $H_{i,j}$ | pumping head of $i$th pump at $j$th month |
| $HS_{i,j}$ | static head of $i$th pump at $j$th month |
| $H_{\max_i}$ | maximum allowable pumping head of $i$th pump |
| $H_{\min_i}$ | minimum allowable pumping head of $i$th pump |
| $i$, $j$ | $i$th pump at $j$th month |
| $LPDF_i$ | penalty allocated to the variable $i$ at generation $g$ |
| $(Q_N)_j$ | total demand at $j$th month |
| $Q_{i,j}$ | discharge of $i$th pump $i$ at $j$th month |
| $Q_{\max_i}$ | maximum allowable discharge of $i$th pump |
| $TC$ | construction length of project |
| $Vio_i$ | violation of variable $i$ from the feasible region |
| $\alpha$ | constant variable |
| $\beta$ | constant variable |
| $\Delta t$ | time step of pumping |

## INTRODUCTION

Evolutionary and meta-heuristic algorithms (EAs) have been extensively used as search and optimization tools in various problem domains. Ease of use and broad applicability are some of the promising and basic reasons for their popularity. Complexity in the nature of engineering optimization problems, such as discretization of the search domain, non-linearity and non-convexity, are among the facts that decrease the capability of traditional methods (linear programming (LP), non-linear programming (NLP) and dynamic programming (DP)), paving the way for the use of evolutionary and meta-heuristic algorithms. Water utilities have begun using a new analysis technique – genetic algorithm (GA) optimization – to help them identify superior, low-cost system expansion and operating alternatives. GA has also been applied in optimization of water supply pumping systems, as in all sub-disciplines within civil engineering. Furthermore GA has been used to improve the design of water distribution systems (Simpson *et al.* 1994; Reis *et al.* 1997; Savic & Walters 1997; Boulos *et al.* 2000; Moradi-Jalal *et al.* 2004).

Evolutionary Algorithms (EAs) are most directly suited to unconstrained optimization. Applying EAs to constrained optimization problems is often a challenging effort. Several methods have been proposed for handling constraints. The most common method to handle constraints is to use penalty functions. They have been applied to a wide range of problems in diverse fields such as engineering, mathematics, operations research, etc. Most of the problems in these fields are stated as constrained optimization problems. Since EAs are directly applicable only to unconstrained optimization, it is necessary to use some additional methods that will keep solutions in the feasible region.

Real-world optimization problems have constraints that must be satisfied by the solution of the problem. A variety of constraint handling methods has been suggested by many researchers. Each method has its own advantages and disadvantages. The most popular constraint-handling method among users is the penalty function method. It is impossible to say which one is the best specific penalty method for every problem. The main problem associated with most methods is the selection of appropriate values of the penalty parameters. Consequently, users have to experiment with different values of penalty parameters. In this paper, we use dynamic penalty function (DPF) and discuss its advantages for solving combinatorial optimization problems compared with the static penalty function approach.

The objective of optimal design and operation of pumping stations, which is a large-scale NLP problem, is to minimize annual design and operational costs over a planning horizon subject to a set of hydraulic constraints, bounding values on the decision variables and constraints reflecting operator preferences and system limitations.

The energy required for operating pumping stations to supply water for irrigation is often significant. The large costs of establishing, maintaining and operating pumping stations, particularly at a time of increasing energy costs, have motivated a search for the optimal design and operation of pumping stations through existing approaches (Ashofteh 1999; Boulos *et al.* 2001; Moradi-Jalal *et al.* 2003).

There have been several recent attempts to develop optimal design and control algorithms to assist in the operation of complex water distribution systems. The various algorithms were oriented towards determining the least-cost pump scheduling policies (typically proper on–off pump operation) and were based on the use of optimization tools including LP, NLP, DP, enumeration techniques, general heuristics and GAs. The success of these procedures has been limited and few have been applied to real water distribution systems. Limited acceptance of optimal control models in engineering practice stems from several possible factors: (1) such techniques are generally quite complex involving a considerable amount of mathematical sophistication (e.g. requiring extensive expertise in systems analysis and careful setting up and fine-tuning of parameters); (2) they are generally highly dependent upon the number of pumps and storage tanks being considered along with the duration of the operating period; (3) they are generally subject to oversimplification of the model and its components along with several simplifying assumptions to accommodate the nonlinear hydraulic constraints that require, for example, demands to be known with certainty; (4) they tend to be extremely time-consuming to run, leading to additional costs and inefficient computer use; and (5) they may be easily trapped at a local optimum and may not lead to a global optimal solution. Another important reason for their lack of acceptance, implied by point (4), is the unavailability of suitable and user-friendly pump optimization packages. As a result, most optimal control models have only been used to support research, and have not been practically used for real system decision-making Bozorg Haddad *et al.* (2007).

Honey-bee mating may also be considered as a typical swarm-based approach to optimization in which the search algorithm is inspired by the process of mating in real honey-bees. Bozorg Haddad *et al.* (2006) demonstrated the efficiency and applicability of the HBMO algorithm by applying it to well-known mathematical optimization problems and compared the final solutions with those from analytical methods and GA. Also, Afshar *et al.* (2007) tested the applicability of the algorithm in the field of water resources. These authors applied the HBMO algorithm to the optimum operation of a single-reservoir problem in a continuous solution domain. In a recent work, Bozorg Haddad *et al.* (2007) applied the HBMO algorithm to solve a design–operation of pumping stations. The obtained results were more cost-effective than those by GA reported by Moradi-Jalal *et al.* (2004) for the same problem.

In this paper, the dynamic penalty function in evolutionary algorithms is presented and tested with the HBMO algorithm in solving a design–operation optimization of a real case pump station to demonstrate the proficiency of this combination in solving combinatorial optimization problems.

## MATHEMATICAL MODEL DEVELOPMENT AND MODEL SIMPLIFICATION

The mathematical model has been completely presented in Bozorg Haddad *et al.* (2007). The goal is to minimize total annual cost which includes both annual energy consumption of each candidate pumping system, based on the increment discharge time of duration curves and the annual depreciation cost of associated capital investments. Thus, the objective function may be expressed as

$$Min(ATC) = \sum_{i=1}^{n} C_{RF}.C_i' + C_E.E_T \quad (1)$$

in which

$$C_i' = \left(1 + \frac{r \times TC}{2}\right) C_i \quad (2)$$

where $ATC$ = annual total cost, $n$ = number of turbines, $C_{RF}$ = capital recovery factor; $C_{i,}$ $C_i'$ = cost of the $i$th pump and equivalent cost of $i$th pump after construction time, respectively, $C_E$ = unit energy cost, $E_T$ = total annual consumed energy, $r$ = rate of interest and $TC$ = length of construction. The project's useful life, rates of interest and depreciation, capital cost and length of construction are all considered in this determination.

The annual consumed energy $E_T$ is determined as

$$E_T = \rho g \sum_{i=1}^{n} \sum_{j=1}^{m} H_{i,j}(Q_{i,j}) \frac{Q_{i,j}}{e_{i,j}(Q_{i,j})} \Delta t_{i,j} \quad i = 1, \ldots, n \text{ and } j = 1, \ldots, m \tag{3}$$

in which $Q_{i,j}$ = discharge from the $i$th pump at $j$th time step, $e_{i,j}$ = efficiency of $i$th pump at $j$th time step, $\Delta t_{i,j}$ = associated time step of pump operation, $\rho$ = density of water and $g$ = gravitational acceleration. Note that pump efficiency is a function of pump discharge, which is related to the total discharge at the $j$th time step.

The objective function (1) and Equation (3) are constrained by

$$0 \leq Q_{i,j} \leq Q_{\max_i} \tag{4}$$

$$\sum_{i=1}^{n} Q_{i,j} = (Q_N)_j \tag{5}$$

$$H_{\min_i} \leq H_{i,j} \leq H_{\max_i} \tag{6}$$

where $(Q_N)_j$ = total demand discharge required to be supplied at the $j$th time step, $Q_{\max_i}$ = maximum allowable discharge of $i$th pump, $H_{i,j}$ = pumping head of $i$th pump at $j$th month, $H_{\min_i}$ = minimum allowable pumping head of $i$th pump and $H_{\max_i}$ = maximum allowable pumping head of $i$th pump. These constraints are valid for all pumps at all times ($i = 1, \ldots, n$ and $j = 1, \ldots, m$). The net pumping head $H_{i,j}(Q_{i,j})$ is also related to the static head and the total head losses. The Darcy–Weisbach equation has been applied in this paper. The fact that the HW coefficient is assumed to be independent of pipe diameter, velocity of flow and viscosity requires extreme caution when applying this formula to the optimization of water distribution systems.

It is assumed that the pump efficiency curve is a function of discharge as follows:

$$e_{i,j}(Q_{i,j}) = a_i Q_{i,j}^2 + b_i Q_{i,j} + c_i \quad i = 1, \ldots, n \text{ and } j = 1, \ldots, m \tag{7}$$

where $a_i$, $b_i$ and $c_i$ are performance coefficients found for the $i$th pump. By substituting Equation (7) into Equation (3), the annual consumed energy reduces to

$$E_T = \rho g \sum_{i=1}^{n} \sum_{j=1}^{m} H_{i,j}(Q_{i,j}) \frac{Q_{i,j}}{\left(a_i Q_{i,j}^2 + b_i Q_{i,j} + c_i\right)} \Delta t_{i,j} \quad i = 1, \ldots, n \text{ and } j = 1, \ldots, m. \tag{8}$$

The final step in the optimal design is to select an appropriate pumping station system based on the minimum cost, number and type of pumps, demand curve characteristics, feasibility and personal preferences based on experience.

## HONEY-BEE MATING OPTIMIZATION (HBMO) ALGORITHM

A detailed mapping between real phenomena and mathematical representation of HBMO algorithm is shown in Table 1. By realizing the natural mating process and biological statements and their translation into algorithmic statements, the optimization algorithm is developed. A detailed flowchart of the proposed algorithm is presented in Figure 1, where the mapping between the real mating process and the computational steps are noted. The mating process itself is translated into a simulated annealing (SA) process which is presented in Figure 2. Although a detailed description of the proposed algorithm can be gathered from Bozorg Haddad *et al.* (2006), Afshar *et al.* (2007) and Bozorg Haddad *et al.* (2007), an overview of the HBMO algorithm is presented here.

The algorithm receives two sets of model input parameters: (a) model structure parameters that are mainly problem-dependent, such as number of decision variables, upper and lower bounds on decision variables, penalty coefficients, etc., and (b) algorithm parameters that may be used as tuning parameters, such as number of mating flights, size of hive and spermatheca, number of solutions in the simulated annealing process, queen's initial speed and energy, as well as type and number of heuristic functions defined by different workers.

The algorithm begins with the random generation of a set of initial solutions. The generated solutions may or may not belong to the feasible region. In fact, most of the

**Table 1** | Mapping between real phenomena and mathematical representation of HBMO algorithm (Bozorg Haddad *et al.* 2007)

**Mapped components**

| Row | Real Honey-Bees | Mathematical representation |
|---|---|---|
| 1 | Gene | Decision variable |
| 2 | Queen | Best solution |
| 3 | Queen goodness | Objective (fitness) function value |
| 4 | Drones | Trial solutions |
| 5 | Broods | New solutions |
| 6 | Workers (nurse bees) | Heuristic functions |
| 7 | Spermatheca (mating pool) | Pool of nominated trial solutions |
| 8 | Hive | Search space containing feasible/non-feasible solutions |
| 9 | Number of mating flights | Number of iterations |
| 10 | Queen's energy | Parameter defining number of generated simulated annealing trial solutions |
| 11 | Queen's speed | Temperature in simulated annealing |
| 12 | Number of drones queen encounters to mate with her | Set of simulated annealing solutions |

**Mapped procedures**

| Row | Real process | Mathematical operators |
|---|---|---|
| 1 | Mating flight | Simulated annealing |
| 2 | Breeding | Generating new solutions (using heuristic crossover operators) |
| 3 | Brood feeding | Improving new solutions (using heuristic mutation operators) |
| 4 | Queen feeding | Improving best solution (using heuristic mutation operators) |
| 5 | Updating workers | Updating heuristic functions (allocating specified domain size to each function) |

generated solutions may be non-feasible. Randomly generated solutions are then ranked using a penalized objective function. The fittest solution is named the queen, whereas the remaining solutions are categorized as drones (i.e. trial solutions). By defining the queen, drones, broods, and workers (predefined functions), the hive is completely formed and mating may now be started.

The queens play the most important role in the mating process in nature as well as in the HBMO algorithm. Each

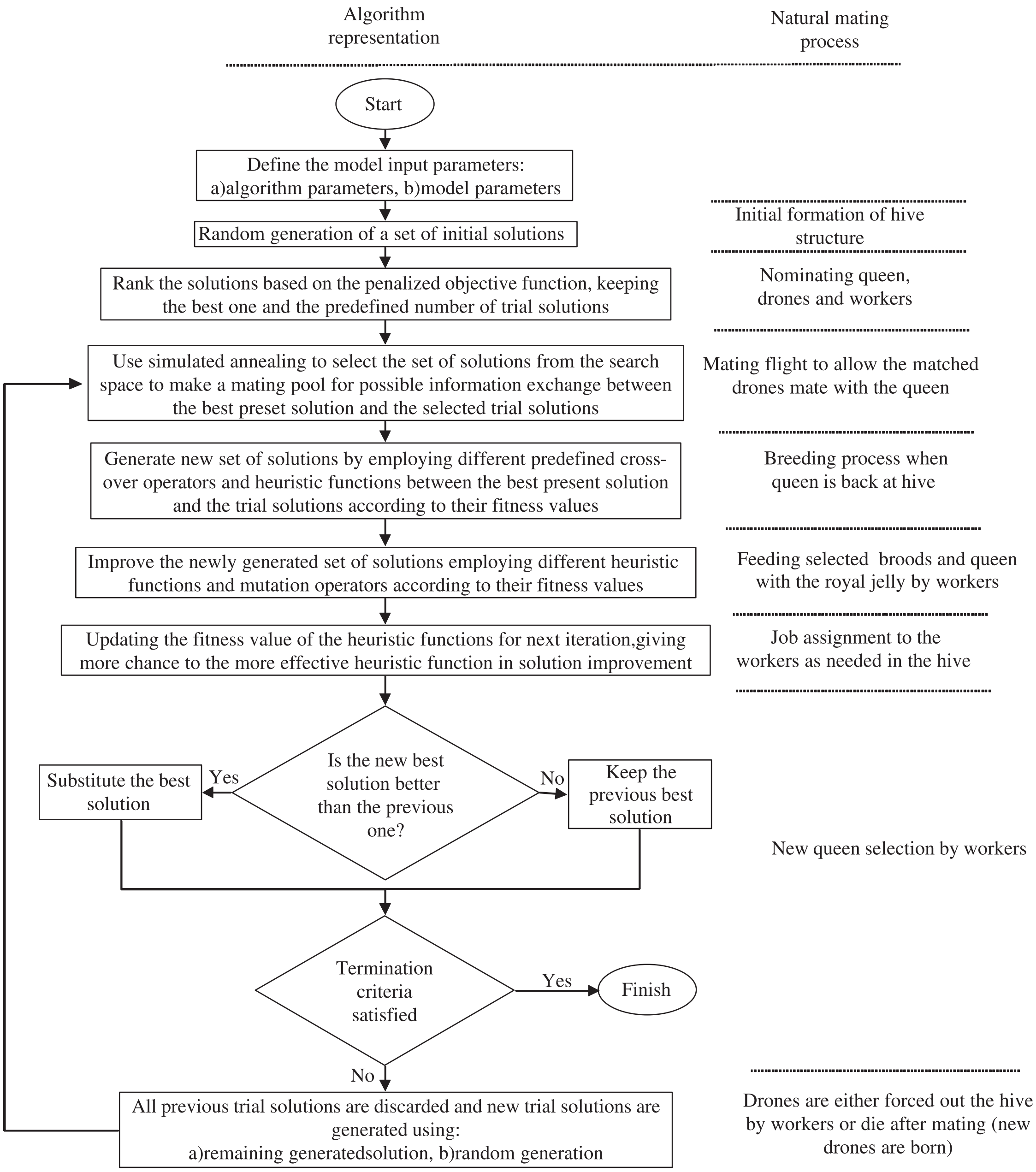


**Figure 1** | Algorithm and computational flowchart with translation of natural processes into algorithmic statements (Bozorg Haddad *et al.* 2006).

queen is characterized with a genotype, speed, energy and a spermatheca with defined capacity.

In the mating flight, drones must be nominated to mate with the queen probabilistically as mentioned earlier. Therefore, the simulated annealing (SA) process is employed to map the real mating flight into a mathematical representation in the algorithm development. Using SA, a set of solutions from the search space is selected to form a mating pool for possible information exchange between the best preset solution and the selected trial solutions (Bozorg Haddad *et al.* 2007)

The mating flight may be considered as a set of transitions in a state-space (the environment) where the queen moves between the different states with some speed

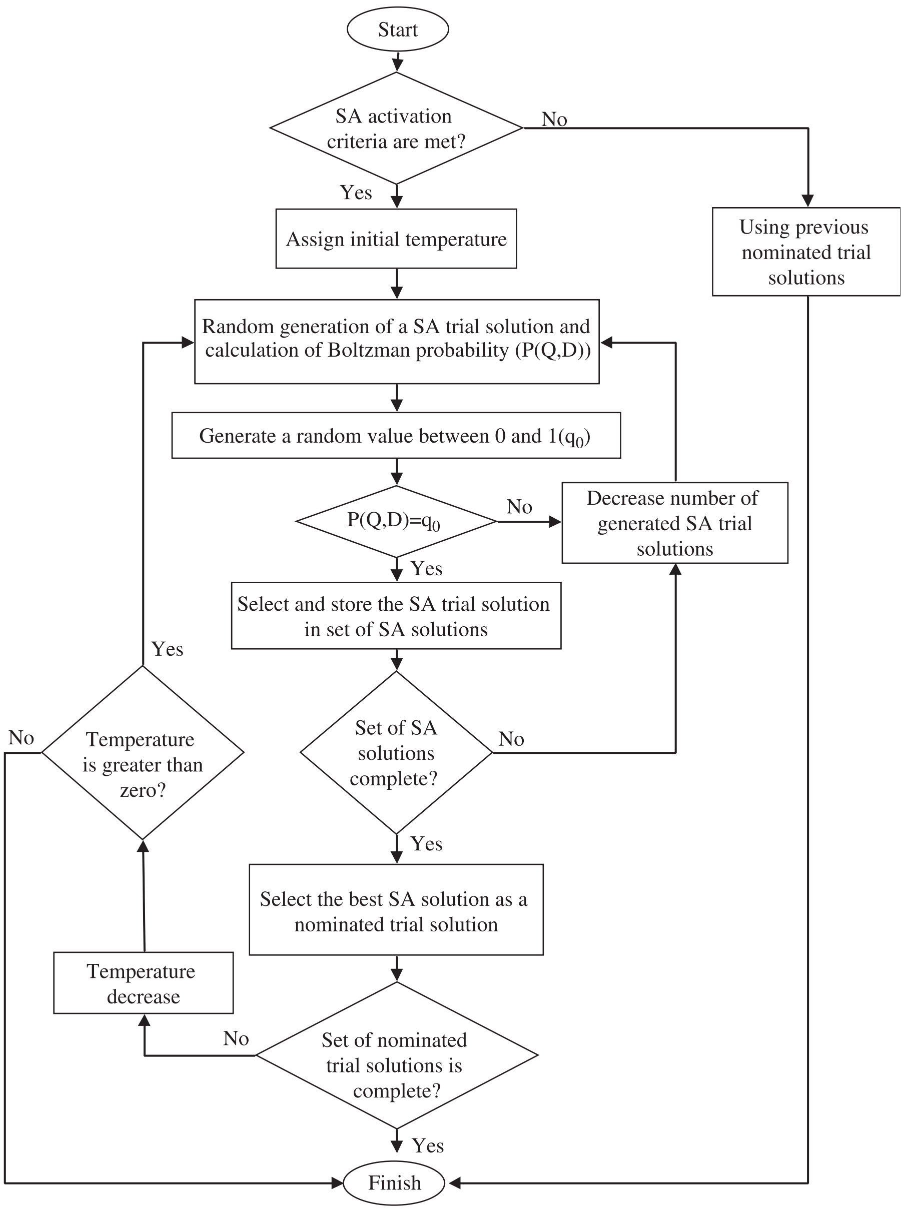


**Figure 2** | Simulated annealing flowchart (Bozorg Haddad *et al.* 2007).

and mates with the drone encountered at each state probabilistically. At the start of the flight, the queen is initialized with some energy content and returns to her nest when the energy is within some threshold from zero or when her spermatheca is full.

A drone mates with a queen probabilistically using an annealing function as follows (Abbass 2001):

$$Prob(Q, D) = \exp[-\Delta(f)/S(t)] \tag{9}$$

where *Prob* (*Q*, *D*) is the probability of adding the sperm of drone *D* to the spermatheca of queen *Q* (that is, the probability of a successful mating), $\Delta(f)$ is the absolute difference between the fitness of *D* (i.e. $f(D)$) and the fitness of *Q* (i.e. $f(Q)$) and $S(t)$ is the speed of the queen at time *t*. A successful mating is when the *Prob* (*Q*, *D*) is greater than a uniform random number $\in$ [0, 1]. It is apparent that this function acts as an annealing function, where the probability of mating is high either when the queen is still at the beginning of her mating flight and therefore her speed is high, or when the fitness of the drone is as good as the queen's. After each successful mating, the queen's speed decreases and after each transition in space, the queen's energy decays according to the following equations:

$$S(t+1) = \alpha(t) \times S(t) \quad (10)$$

$$E(t+1) = E(t) - \gamma \quad (11)$$

$$\alpha(t) = [M - m(t)]/M \quad (12)$$

where $\alpha(t)$ is a factor $\in$ [0,1], $M$ is the spermatheca size, $m(t)$ is the number of drones selected for mating at time $t$ and $\gamma$ is the amount of energy reduction after each transition. In each mating flight some preset allowable number for the queen's transitions have been considered. In each transition, the queen will lose one of her chances for trying new drones. In this process, it is considered that $E(t)$ is equal to the preset allowable number of transitions in space and $\gamma = 1$ (Bozorg Haddad *et al.* 2006).

Real breeding takes place when the queen returns to the hive. This real process is mapped into the developed algorithm to generate a new set of solutions using different predefined crossover operators and heuristic functions between the best current solution and the trial solutions. The rate of contribution of crossover operators and heuristic functions on the information exchange between the solutions is made proportional to their fitness value at the previous cycle. It has been found that the type and number of crossover operators has a significant effect on the quality of the generated new solution (i.e. brood). Therefore, in the present algorithm, four different crossover operators are employed. A fitness value is assigned to each operator which is updated by considering its contribution to solution improvement at each computational step. For example, the fitness value (effectiveness weight) assigned to each crossover operator either increases or decreases at the next generation and eventually its contribution in generating the next generation decreases. In this study, four operators are used in the breeding process (i.e. new solution generation): (1) one-point crossover in which the queen's genotype has been put in the left side of the generated brood's genotype, (2) one-point crossover in which the queen's genotype has been put in the right side of the generated brood's genotype, (3) two-point crossover in which the queen's genotype has been put in the middle of the generated brood's genotype and (4) two-point crossover in which the queen's genotype has been put in both ends of the generated brood's genotype. In general, for further studies, more than four crossover operators can be considered. It will not cause any increase in computational effort, because even in the case that there are so many operators contributing in the breeding and new solution generation, the better functions will almost find a chance to come to the next generation, though the chance of the others will not be eliminated, even without making any improvement.

The feeding process of broods and the queen with royal jelly which is performed by workers, as a very determinant stage in the real honey-bees life cycle, is mapped into the algorithm to improve the new generalized set of solutions. In this stage, by employing different heuristic functions and mutation operators, the best solution is improved. Again, the contribution rate of the operators for solution improvement is made proportional to their fitness value in the previous cycle. For example, in this study two different operators for mutation have been considered: (1) random cut–random value and (2) random cut–random boundary value. In the second case, an assigned value to the genotype will be chosen as a random value towards the feasible boundary region by increasing the number of generations. The ranking process and selection of the best heuristic functions for the next generation is the same as that described for crossover operators. However, in its present form, the algorithm benefits from a combination of four different crossover operators acting as breeding processes as well as two mutation operators (heuristic functions) acting as different feeding performance.

As life in the hive continues, the proposed algorithm continues until the termination criteria (meeting the predefined number of mating flights) are satisfied, and the best solution from the set of current best solutions and improved solutions are selected. If the termination criteria are not satisfied, all trial solutions are discarded and a new set of trial solutions are generated to make the search process more extensive.

Drones are either killed or die after mating is complete. This real process is also mapped into the HBMO algorithm by killing all drones after a cycle and new drones (i.e. trial solutions) are generated. To generate a new set of trial solutions, remaining broods with desirable fitness are partially used along with the random generation of new (trial) solutions needed to fill the spermatheca (mating pool). Usage of remaining broods with desirable fitness as well as the random generation of new solutions is considered in this study. These new drones are constructed by copying some of the queen's genes into the drone genotype and completing the reminder of the genes from a random production process. The percentage of copied genes increases from 0 at the start to 100 at the end of the algorithm.

## DYNAMIC PENALTY FUNCTION

During the past few years, several methods have been proposed for handling constraints by GAs (Michalewicz 1995a; Smith & Coit 1997; Coello 1999, 2002). Most of these methods have serious drawbacks. While some of them may give infeasible solution or require many additional parameters, others are problem-dependent (i.e. an specific algorithm has to be designed for each particular problem). The most popular approach in the EA community to handle constraints is to use penalty functions that penalize infeasible solutions by reducing their fitness values in proportion to their degrees of constraint violation (Michalewicz *et al.* 1996; Smith & Coit 1997).

There are several approaches proposed in EAs to handle constrained optimization problems. These approaches can be grouped into four major categories (Michalewicz & Schouenauer 1996): (1) methods based on penalty functions, (2) methods based on a search of feasible solutions, (3) methods based on preserving feasibility of solutions and (4) hybrid methods.

The penalty method transforms a constrained problem to an unconstrained one using two approaches: additive approach and multiplicative approach. The additive penalty type has received much more attention than the multiplicative type in the EA community.

In classical optimization, two types of penalty function are commonly used: interior and exterior penalty functions. In EAs, exterior penalty functions are used more than interior penalty functions mainly because there is no need to start with a feasible solution in exterior penalty functions. Also, finding a feasible solution in many problems is NP-hard itself. If either the penalty is too large or too small, the problem could be very hard for EAs. A big penalty prevents us from searching unfeasible regions. In this case, EA will converge to a feasible solution very quickly even if it is far from the optimal. A very small penalty will cause it to spend so much time in searching an unfeasible region; thus EA would converge to an infeasible solution (Michalewicz & Fogel 2000).

In static penalty methods, penalty parameters do not depend on the current generation number and a constant penalty is applied to infeasible solutions. Homaifar *et al.* (1994) proposed a static penalty approach in which users describe some levels of violation. The disadvantage of this method is the large number of parameters that must be set. Michalewicz (1995b) showed that the quality of solutions is very sensitive to the values of these parameters. Kuri Morales & Quezada (1998) suggested a static penalty approach that uses information about the number of violated constraints, not the amount of constraint violation.

In dynamic penalty functions, penalty parameters are usually dependent on the current generation number. The dynamic method increases the penalty as the generation grows. The quality of a possible solution is very sensitive to changes of used parameters. Michalewicz (1995b) gave some examples to state that these parameter values cause premature convergence. He also showed that the method converges to an unfeasible solution or a solution that is far from an optimal solution.

In this paper, a linear dynamic penalty function (LDPF) has been considered as below:

$$LDPF_i = (g/Gen)(\alpha)(Vio_i)^{\beta} \qquad (13)$$

where $LDPF_i$ is the penalty allocated to the variable $i$ at generation $g$, $Gen$ is the ultimate number of generations, $Vio_i$ is the violation of variable $i$ from the feasible region and $\alpha = 1 \times 10^6$ and $\beta = 2$ are constant variables. Thus, the objective function will be converted to the form

$$Min(ATC) = \sum_{i=1}^{n} C_{RF}.C_i' + C_E.E_T + \sum_{i=1}^{n} LDPF_i \qquad (14)$$

in which the amount of violation assigned for each infeasible solution increases with a linear pattern as the generation increases. It means that the objective function at first will receive a very small penalty at the start of the algorithm for a certain amount of violation compared to the penalty due to the same violation at later generations. This will let the algorithm test the other areas of the solution region to find a more suitable solution for the problem. During this procedure, which is similar to an annealing process, the algorithm will search more local optima compared with the case using a static penalty function. Although the computational time will increase, there is a chance to find the better solution at the end of the process because of the wide search space domain. One of the potential cases for testing the proficiency of this strategy is in the case of design–operation problems in the field of engineering. In such cases, due to setting the operational decision variables according to design decision variables, there are so many local optima in the problem. Thus, the applicability of the proposed linear dynamic penalty function will be tested in the following design–operation problem.

## CASE STUDY

Iran is located in a semiarid region of the Middle East. Distribution of precipitation is uneven, with an average precipitation of less than one-third of the world average (Alizadeh & Keshavarz 2005). In the year 2000, about 43 billion m$^3$ of surface water resources, including regulated flows, were used by reservoir dams, pumping stations, small-scale water supply projects or traditional stream systems. (Jamab 1999).

As a case study, the main pumping station of Iran's Farabi Agricultural and Industrial Complex is considered. It consists of 20,000 ha agricultural land, which is located in the Khoozestan province in southwestern Iran. Irrigated water in this project is used for sugar cane and other crops. In this station, demanded water is supplied for agricultural use from the Karoon River to the main lot. The Karoon River, 890 km in length with a catchment area of 66,930 km$^2$, is the longest river in the country which flows through many industrial and agricultural areas. Karoon water is also used for the water supply of Ahvaz city, the capital city of Khoozestan province.

A demand duration curve is discretized in monthly segments that must be supplied by the main pumping station. For illustration purposes, a problem with one discharge monthly duration curve and four different types of pumps from 10 proposed unit pumps was considered. These sets consist of four different pump types with their cost and characteristics given in Table 2. Note that in the optimization models 'relative discharge' (which is the ratio of discharge to maximum allowable discharge) is selected so as to simplify the calculations. Coefficients of efficiency–relative discharge curves for specified pump types are listed in Table 3 and are used both in the design example and in the optimized designs.

It is clear that optimum discharges for the pumps were greater than half of their maximum allowable discharge. Thus, in a real case, in order to avoid division by zero in the calculation of Equation (8), two different curves were considered for efficiency–relative discharge curves. The main curve, which is related to $0.5Q_{\max_i} < Q_{ij} < Q_{\max_i}$, is the original curve and the additional curve, which is related to

**Table 2** | Specification of pre-selected pumps (Bozorg Haddad *et al.* 2007)

| Cost ($10^6$ Rial) | $L_{eq}$(m) | Diameter (m) | $H_{max}$ (m) | $Q_{opt}$ (m$^3$/s) | $Q_{max}$ (m$^3$/s) | Pump type |
|---|---|---|---|---|---|---|
| 224.37 | 265.36 | 1.35 | 25 | 5.70 | 7.41 | 1 |
| 89.14 | 214.70 | 0.90 | 20 | 2.26 | 2.94 | 2 |
| 82.93 | 183.80 | 0.80 | 18 | 2.06 | 2.68 | 3 |
| 59.04 | 169.02 | 0.70 | 14 | 1.50 | 1.95 | 4 |

**Table 3** | Efficiency-discharge relations for specified pump types (Bozorg Haddad *et al.* 2007)

| Pump type 4 | | Pump type 3 | | Pump type 2 | | Pump type 1 | |
|---|---|---|---|---|---|---|---|
| *Q* (m$^3$/s) | *e* (%) | *Q* (m$^3$/s) | *e* (%) | *Q* (m$^3$/s) | *e* (%) | *Q* (m$^3$/s) | *e* (%) |
| 1.50 | 86.0 | 2.06 | 86.0 | 2.26 | 86.0 | 5.70 | 86.0 |
| 1.35 | 81.7 | 1.85 | 81.7 | 2.03 | 81.7 | 5.13 | 81.7 |
| 1.65 | 83.2 | 2.27 | 83.2 | 2.49 | 83.2 | 6.27 | 83.2 |
| 1.20 | 75.2 | 1.65 | 75.2 | 1.81 | 75.2 | 4.56 | 75.2 |
| 1.80 | 79.1 | 2.47 | 79.1 | 2.71 | 79.1 | 6.84 | 79.1 |

**Table 4** | Coefficients of efficiency-relative discharge curves for specified pump types (Bozorg Haddad *et al.* 2007)

| Coefficient | Additional Curve $q_i < 0.5$ | Main Curve $q_i \geq 0.5$ |
|---|---|---|
| $a_i$ | 1.84 | −4.870 |
| $b_i$ | −0.06 | 7.603 |
| $c_i$ | 0.05 | −2.107 |

$e(q_i) = a_i q_i^2 + b_i q + c_i$ and $q_i = \frac{Q_i}{Q_{\max_i}}$ for $i = 1, \ldots, n$ Main = $\{q \geq 0.5)$ Additional = $\{q < 0.5\}$.

**Table 5** | Specification of pre-selected sets and optimum sets of pumps

| DPF ** -HBMO | SPF * -HBMO | GA | LM | Third pre-selected set | Second pre-selected set | First pre-selected set | Pump type |
|---|---|---|---|---|---|---|---|
| 3 | 3 | 4 | 3 | 5 | 0 | 0 | 1 |
| 4 | 3 | 2 | 4 | 0 | 14 | 0 | 2 |
| 0 | 3 | 0 | 0 | 0 | 0 | 16 | 3 |
| 3 | 1 | 3 | 3 | 3 | 0 | 0 | 4 |
| 10 | 10 | 9 | 10 | 8 | 14 | 16 | Total |

*Static Penalty Function.
** Dynamic Penalty Function.

$0 < Q_{ij} < 0.5Q_{\max_i}$, is a supplemental curve that is used to prevent the reporting of infeasible and incorrect discharge results. Thus, by applying two curves (the main curve and the additional curve, as illustrated in Table 4), losing the final optimal results and diversion of the algorithm to infeasible results is removed during the computational process.

The results of the optimization model are for an optimum set which consists of: (1) the number of pumps and pump types of the set, (2) a value for output discharge for every time step and for each pump, (3) the initial investment and its depreciation cost, (4) the operational cost and (5) the total annual cost of the optimum set, which is the main parameter of the optimization model.

In the existing design of the Farabi main pumping station, only three different pre-sets are selected and cost analysis is limited to the comparison of the results of these three sets. The final set, which is selected in the practical design, is the first pre-set. It consists of 16 Type 1 pumps, for ease of operation and minimum annual cost among the other presets.

## ALGORITHM APPLICATION

Once the optimization process and the method of solution are identified, a Lagrange Multipliers (LM) method is used to find the Optimal Design of the Irrigation Pumping Station (ODIPS). Further information about LM formulation can be found in Moradi-Jalal *et al.* (2003). The optimum set, which is obtained through the LM method, consists of 10 pumps with different pump types: three pumps of Type 1, four pumps of Type 2 and three pumps of Type 5. By using various pump types, the pumping station's operation flexibility increases and the system can find more suitable pump types to operate more effectively.

**Table 6** | Cost specification of pre-selected and optimum sets of pumps

| DPF ** -HBMO | SPF * -HBMO | GA | LM | Third pre-selected set | Second pre-selected set | First pre-selected set | Cost specification ($10^6$ Rial) |
|---|---|---|---|---|---|---|---|
| 1207 | 1248 | 1253 | 1224 | 1299 | 1248 | 1327 | Initial investment |
| 114.7 | 118.6 | 119.1 | 114.7 | 123.5 | 118.6 | 126.1 | Annual depreciation |
| 72.4 | 71.0 | 71.5 | 67.7 | 105.6 | 114.6 | 102.2 | Annual operation |
| 187.1 | 189.7 | 190.6 | 182.4 | 229.1 | 233.2 | 228.3 | Annual total |
| 102.6 | 104.0 | 104.5 | 100.0 | 125.6 | 127.8 | 125.2 | Optimum (%) |

*Static Penalty Function.
** Dynamic Penalty Function.

**Table 7** | Monthly discharges of HBMO optimum set of pumps

| | | | Monthly discharge in different months (m3/s) | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Pump no.** | **Pump type** | **Qmax** | **Jan.** | **Feb.** | **Mar.** | **Apr.** | **May.** | **Jun.** | **Jul.** | **Aug.** | **Sep.** | **Oct.** | **Nov.** | **Dec.** | **Min.** | **Ave.** | **Max.** | **Sum.** |
| 1 | 1 | 7.41 | 0.00 | 4.76 | 4.76 | 4.98 | 5.25 | 6.94 | 7.40 | 7.40 | 5.77 | 5.99 | 5.78 | 4.76 | 0.00 | 5.32 | 7.40 | 63.79 |
| 2 | 1 | 7.41 | 4.74 | 0.00 | 4.76 | 0.00 | 5.23 | 5.70 | 7.24 | 6.74 | 6.12 | 5.52 | 4.97 | 4.76 | 0.00 | 4.65 | 7.24 | 55.78 |
| 3 | 1 | 7.41 | 0.00 | 0.00 | 0.00 | 5.38 | 5.05 | 6.20 | 7.40 | 7.40 | 6.23 | 5.97 | 0.00 | 0.00 | 0.00 | 3.64 | 7.40 | 43.63 |
| 4 | 2 | 2.94 | 0.00 | 1.88 | 1.88 | 1.96 | 2.12 | 2.49 | 2.91 | 2.77 | 2.47 | 2.66 | 1.98 | 0.00 | 0.00 | 1.93 | 2.91 | 23.12 |
| 5 | 2 | 2.94 | 1.88 | 1.88 | 1.88 | 1.96 | 2.11 | 2.25 | 2.93 | 2.93 | 2.09 | 2.15 | 0.00 | 0.00 | 0.00 | 1.84 | 2.93 | 22.06 |
| 6 | 2 | 2.94 | 1.88 | 0.00 | 0.00 | 1.89 | 1.95 | 2.54 | 2.91 | 2.93 | 2.05 | 2.36 | 2.42 | 0.00 | 0.00 | 1.74 | 2.93 | 20.93 |
| 7 | 2 | 2.94 | 0.00 | 0.00 | 0.00 | 2.00 | 2.04 | 2.46 | 2.93 | 2.93 | 2.31 | 2.36 | 0.00 | 1.88 | 0.00 | 1.58 | 2.93 | 18.91 |
| 8 | 4 | 1.95 | 0.00 | 1.23 | 1.23 | 1.31 | 1.27 | 1.76 | 1.94 | 1.80 | 1.57 | 1.73 | 1.32 | 0.00 | 0.00 | 1.26 | 1.94 | 15.16 |
| 9 | 4 | 1.95 | 1.23 | 0.00 | 1.23 | 1.41 | 0.00 | 1.69 | 1.90 | 1.62 | 1.54 | 1.38 | 1.23 | 0.00 | 0.00 | 1.10 | 1.90 | 13.23 |
| 10 | 4 | 1.95 | 0.00 | 1.23 | 0.00 | 1.41 | 1.38 | 1.77 | 1.94 | 1.88 | 1.55 | 1.48 | 0.00 | 0.00 | 0.00 | 1.05 | 1.94 | 12.64 |
| Min. | | | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.69 | 1.90 | 1.62 | 1.54 | 1.38 | 0.00 | 0.00 | | | | |
| Ave. | | | 0.97 | 1.10 | 1.57 | 2.23 | 2.64 | 3.38 | 3.95 | 3.84 | 3.17 | 3.16 | 1.77 | 1.14 | | | | |
| Max. | | | 4.74 | 4.76 | 4.76 | 5.38 | 5.25 | 6.94 | 7.40 | 7.40 | 6.23 | 5.99 | 5.78 | 4.76 | | | | |
| Sum. | | | 9.73 | 10.98 | 15.74 | 22.30 | 26.40 | 33.80 | 39.50 | 38.40 | 31.70 | 31.60 | 17.70 | 11.40 | | | | |
| Demand | | | 9.3 | 10.5 | 15.2 | 22.3 | 26.4 | 33.8 | 39.5 | 38.4 | 31.7 | 31.6 | 17.7 | 11 | | | | |

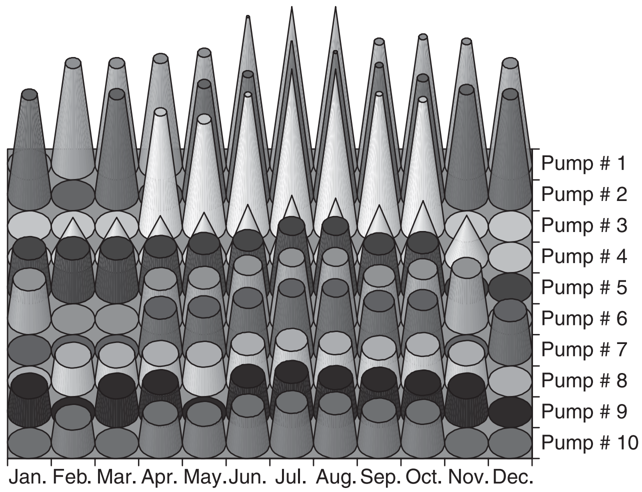


**Figure 3** | Monthly discharge values of the optimum set by the HBMO algorithm and its composition in each monthly demand.

The same mathematical model can be solved by the GA algorithm. The GA approach is a probabilistic global optimization technique based on the mechanics of natural selection and genetics and optimizes the aforementioned model. Numerically the process uses reproduction, crossover and mutation to evolve encoded variables. The algorithm is designed to produce 'populations' of solutions whose 'offspring' display increasing levels of optimality. Using a GA algorithm to optimize the design and operation of a pumping station involves the following steps: (i) randomly generating an initial set of pump combinations for given demand values, (ii) minimizing the total annual cost, which includes operation, maintenance and depreciation costs, by changing the set and discharge of the pumps based on the performance evaluated by the GA process and (iii) achieving the final criterion to stop the optimization process and reporting the number of pumps and pump types, values for output discharge on every time step for the optimum set of pumps, the initial investment and the annual costs of depreciation and operation, and the total costs for the optimum set. Further information about the GA formulation can be found in Moradi-Jalal *et al.* (2004).

Another approach to solve this problem is presented by Bozorg Haddad *et al.* (2007) using the static penalty function (SPF) technique. In this paper, a new approach

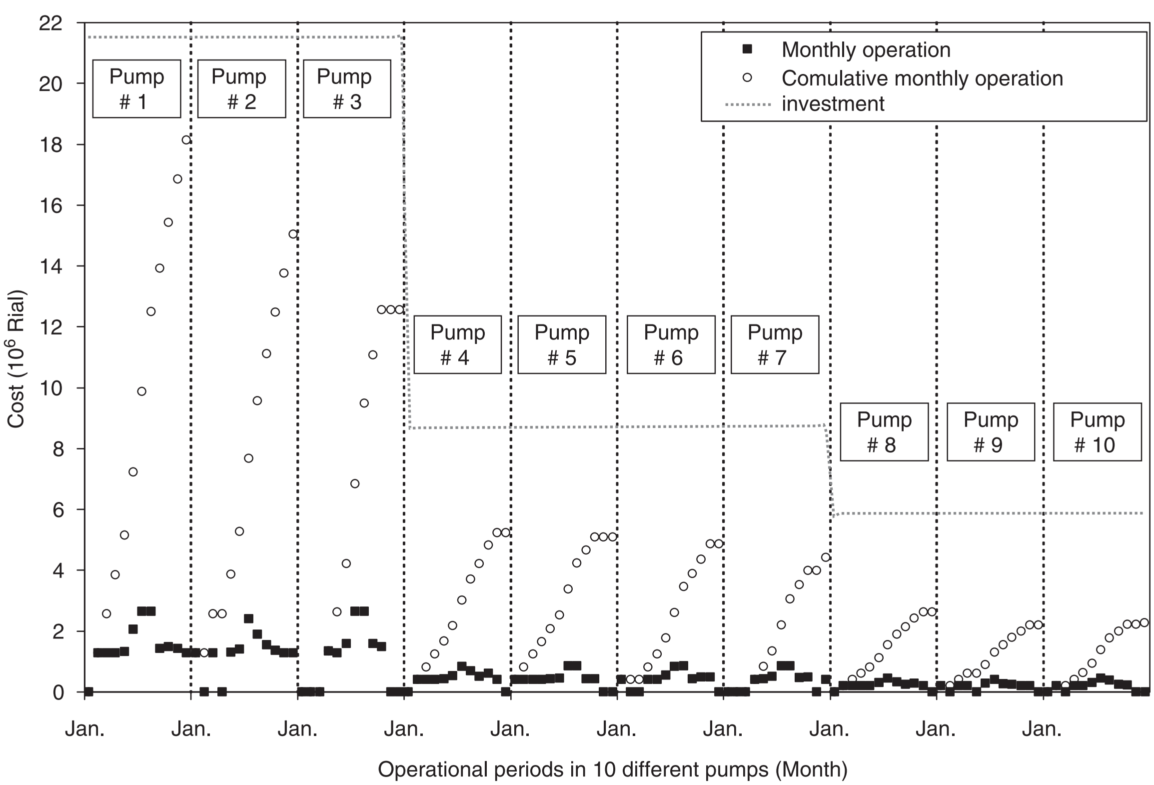


**Figure 4** | Monthly operational and maintenance costs of the optimum HBMO set.

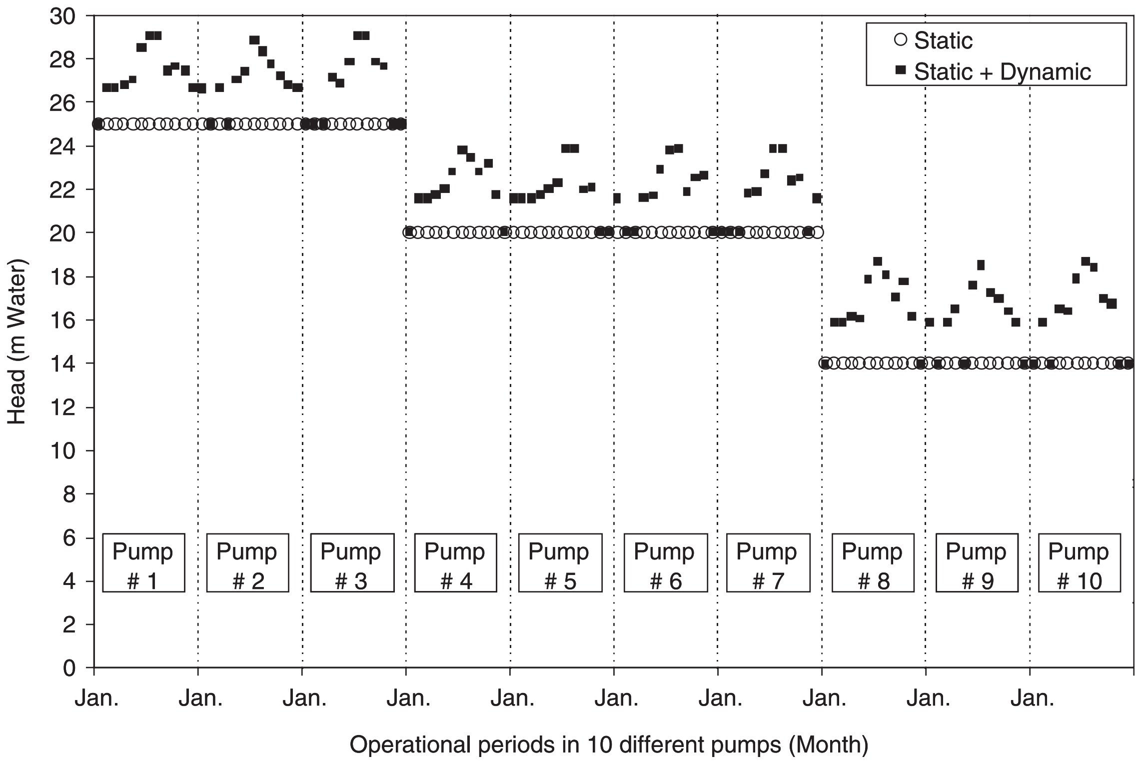


**Figure 5** | Monthly pumping head of selected pumps in the optimum HBMO set.

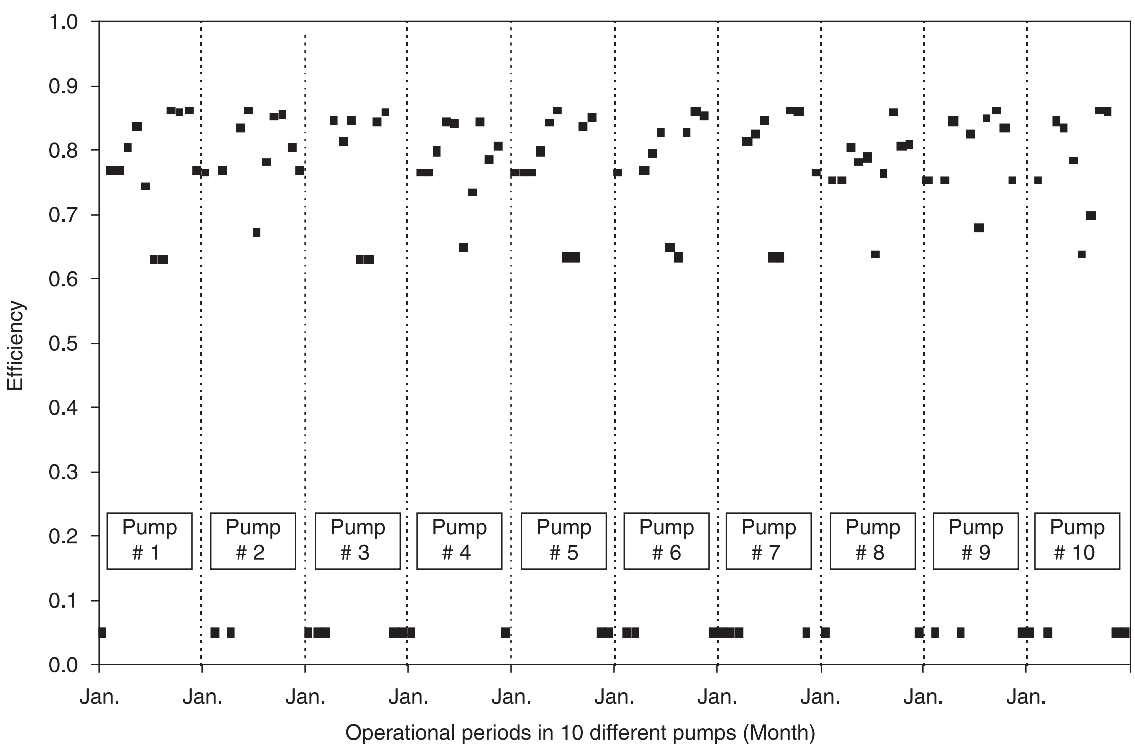


**Figure 6** | Monthly pumping efficiency of selected pumps in the optimum HBMO set.

is considered based on the use of a dynamic penalty function (DPF) in the HBMO algorithm and its results are compared with other previous approaches.

Table 5 shows the specification of three pre-sets in the practical design as well as the mathematically determined sets of other programs (LM, GA, SPF-HBMO and DPF-HBMO optimum sets). Table 6 shows the main output of the optimum set selected by LM, GA, and both SPF and DPF of the HBMO algorithm simultaneously compared with three pre-sets of practical design. As stated earlier, the main purpose of the optimization model is to minimize the total annual cost of feasible sets, which comprises both annual depreciation and operation costs. More precision in Table 6 would help to show that the amount of savings in annual depreciation cost between the optimum set and the pre-sets is quite small. The main savings occurred in the annual operation cost, with nearly 32% savings in energy cost. It is clear that, by using these optimization algorithms, a decrease of about 20% is obtained in annual operating and depreciation costs. The comparison of total cost of the optimization algorithms shows that the least cost ever reported by evolutionary algorithms is that reported by the DPF-HBMO. This cost is a 1.4% improvement over that reported by the SPF-HBMO, though it still has a 2.6% violation from the global result reported by LM. Results show that DPF can be a useful and capable tool to overcome the premature convergence in evolutionary algorithms and especially the HBMO algorithm.

A preliminary operation rule (POR) schedule, which is the optimum discharge distribution of the demand discharge, is derived from the HBMO program and listed in Table 7. The operator must then turn on and off the pumps during the irrigation period according to the POR schedule. Monthly discharge values of the optimum set by the HBMO program and their composition in each monthly demand is shown in Figure 3. While monthly operational and maintenance costs of the optimum DPF-HBMO set is depicted in Figure 4, the monthly pumping head values and the efficiencies of the optimum HBMO set are shown in Figures 5 and 6, respectively.

Finally, at the end of the optimization process, basic information about the DPF-HBMO and SPF-HBMO results, which includes number of program runs, values of objective function for each run and their statistical measures, are

**Table 8** | Objective function value ($10^6$ Rial) and its statistical measures in 10 different runs of the HBMO

| No. of run | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Average | Standard deviation | Coefficient of variation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Static penalty function | 189.7 | 190.8 | 193.0 | 196.0 | 197.1 | 199.4 | 200.1 | 201.0 | 201.2 | 208.9 | 197.7 | 5.70 | 0.029 |
| Dynamic penalty function | 187.1 | 187.2 | 187.2 | 189.1 | 189.2 | 189.4 | 189.5 | 189.6 | 190.3 | 190.7 | 188.9 | 1.33 | 0.007 |

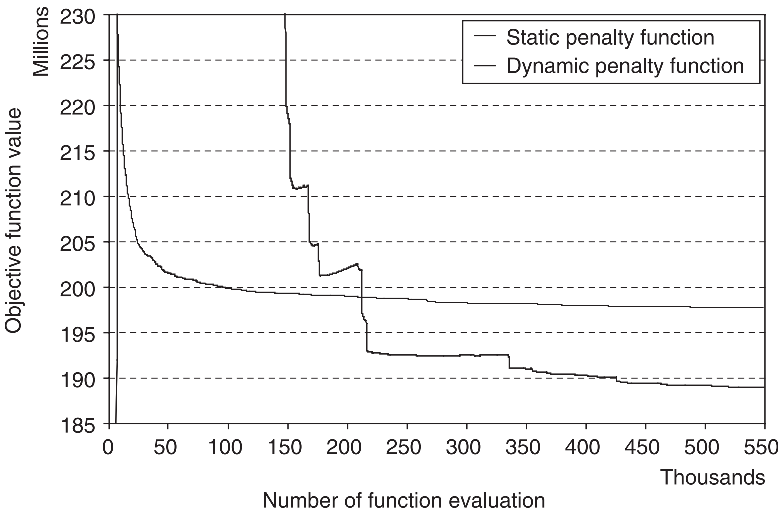


**Figure 7** | Average rate of convergence over 10 different runs for both static and dynamic penalty functions.

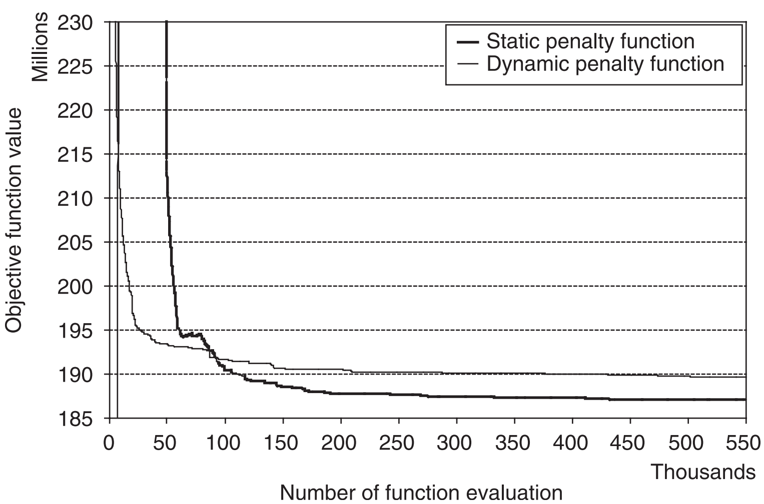


**Figure 8** | Best rate of convergence over 10 different runs for both static and dynamic penalty functions.

obtained and listed in Table 8. This shows that, in almost all of the 10 runs resulting from DPF, the algorithm has converged to better results compared with those of SPF. Convergence curves for the number of function evaluations via the objective function values in the HBMO algorithm for the best and the average of 10 runs in both DPF and SPF are shown in Figures 7 and 8. It is shown that, after a while, the objective function value of SPF converges to a near-optimal value (new evaluations continue without any considerable improvement in the final result) while the convergence of DPF continues toward the better solution.

## CONCLUDING REMARKS

The consumed energy required for operating pumping stations in an irrigation district may be more significant than the energy needed for other water facilities. Thus, serious consideration must be taken to improve the design and operation efficiency of existing or newly developed pumping stations. Optimal design and operation of pumping stations is a large-scale, nonlinear combinatorial optimization problem. Developing a large-scale, discrete and nonlinear optimization model provides the designer, as well as the operator, with the best possible combination of design variables and operational parameters. The computational complexity of determining optimal designs for pumping stations is extremely high. This is true even without considering annual costs or other legitimate objectives.

Minimizing the total annual design and operation cost over a given planning horizon based on a DPF-HBMO approach was taken as the objective of the paper. To test the efficacy and robustness of the proposed strategy, a test problem from the literature has been chosen. This problem has been studied using other optimization methods by several researchers. It was shown that using the proposed algorithm with its attached DPF strategy might significantly reduce the total annual cost. The main portion of the cost reduction resulted from energy savings as a consequence of applying better operational rules. Developing the best operation rule and linking it to the optimum design model showed much promise.

Interactive characteristics of the DPF-HBMO are designed to assist irrigation pumping station operators and the training of new operators in selecting and scheduling efficient and cost-effective pump combinations to plan and operate better systems. The proposed model was tested and verified on an actual large-scale water system. Results indicated that the model can effectively reduce the cost of energy consumed by pumping in a complex water system while maintaining satisfactory service levels. Water utility managers now have a tool to help them produce the best possible pumping schedules with minimum effort and significant cost savings.

Although the HBMO is only applied for research purposes at present, it is not complicated to use and is not mathematically sophisticated, making it capable to be used in real world problems. In expanded network problems, it is hoped that its inherent simplicity will help the HBMO gain acceptance by practitioners familiar with basic network simulation skills. The authors feel strongly that an algorithm such as the HBMO should not be considered as a decision-making tool, but as a technique able to provide alternative solutions from which designers/decision-makers may choose from. Also, the new DPF strategy attached to the HBMO increases its capability in handling combinatorial design–operation optimization problems. The results indicate that, although the best results ever reported for the considered problem is by SPF-HBMO, using the DPF-HBMO presents a better solution, showing a more economical cost of design. So, applying the DPF in such cases is highly recommended to overcome trapping in local optima in the case of design–operation problems.

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