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A new theoretical analysis on organizing principles of water supply networks

L. H. Chai and H. B. Li

ABSTRACT

Structural features of water supply networks remain unresolved despite decades of research. An efficient statistical mechanics method was applied for exploring the water consumption elements interactions, and the self-organized emergence of evolving water supply networks was revealed. Basing upon this new framework, the paper discussed hierarchical structures of fractal growth of water supply networks and their fractal dimensions. The paper made a renewed effort to deepen the theoretical understanding of water network systems, and also has the potential to give important scientific guides to the engineering practice of water supply networks.

Key words | fractal growth, self-organization, water supply network

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INTRODUCTION

Water supply systems are the lifelines of urban and rural communities. The water distribution networks are typically the most expensive components of water supply systems (Kleiner *et al.* 2001). A proper spatial water distribution system will help to distribute domestic water to all places with proper pressure. It will also avoid the probable risk of failure in the complete distribution system and can make the system cost effective (Eusuff & Lansey 2003). In fact water distribution networks belong to network-structured systems, or networks for short, distributing "products" such as water, gas or electricity. These networks typically consist of conduit components (pipes and cables) and control components (pumps, valves, transformers and switchgear), and they link a few producers to numerous consumers (Scarf & Martin 2001). In fact networks are ubiquitous in many aspects of everyday life, both in nature and in society, from the watershed where the river water is collected, to the veins and lymphatic channels which distribute blood and nutrition in animals and plants (McMahont & Bonner 1983; Banavar *et al.* 1999).

The development of the models and algorithms for water supply networks is a natural road since engineers are striving to reflect the physical characteristics of water distribution systems and then provide scientific supports for describing and constructing the water distribution system. In the available literature, the physical behavior of a water supply network was described by a set of nonlinear equations. The analysis was then based on the mathematical description of the equations and their performances. Over the past few decades the development of powerful hardware and numerical methods facilitated the simulation of the hydraulic behavior of large size networks, and considerable effort has been devoted to the development of optimization algorithms and models for the design of water distribution networks, i.e. the simulation of system operation under various conditions and the search for the operational conditions most favorable for maximum objectives. Nevertheless, a water distribution system is a collection of enormous hydraulic control elements jointly connected to convey quantities of water from source to consumer (Ostfeld 2005), being a complex system and a multi-level network. NP (Non-deterministic Polynomial)-hard problems are frequently encountered in available avenues. Although more recently, the stochastic optimization such as genetic algorithms and simulated annealing algorithms try to overcome some drawbacks, the optimal design of water supply networks remain difficult (Cunha & Sousa 1999; Van

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Zyl *et al.* 2004). In addition, it should not be ignored that many problems in sophisticated and complex networks can be partially solved through the recent development and application of complex network theories such as scale-free network and small-world network, which clearly demonstrate many interesting and useful organizing qualities in complex networks, e.g. cellular networks, ecological networks, internet, movie actor web, science collaboration graph, citation networks, phone call networks, and networks in linguistics etc (Albert & Barabasi 2002). It may provide the water engineers with quite different views and insights into the solutions of water supply network distribution. However, for interplay between local dynamics and global optimization it is far from being fully understood in this framework, as this perspective still lives in its infancy for water engineers today.

A new perspective is highly needed for the study of water supply networks. The water network is an interactive system adapting to the wider urban system, being the germ cells of development for evolving city development systems (Jeffrey *et al.* 1997). It is well known that natural systems adapt to achieve better performance or ensure their survival by evolving their intricate networked structures. The scale-free structure of veins distributes the blood so efficiently that every cell is reached on a reasonably short path with the minimum possible structure (Caldarelli *et al.* 2000). It is exactly this scaling property that allows animals to survive with a quantity of blood much smaller than the solid volume occupied by their bodies. Though the emergence of water distribution networks embodies some of the expertise of the water engineers and adds flexibility, the potential optimized structure will be the same as some complex networks in nature and science. Water supply networks must trust in the inherent principles of self-organization. Through self-organization, water supply networks can become a responsible unit able to make decisions which will, in the end, automatically adjust to the criteria of optimization and sustainable development. However, comparing optimization techniques with natural adaptations, formation of networked structures of water distribution can be seen as an extremely complex optimization problem for engineers. Finding an optimization technique to provide a robust and an efficient way to search the best complex water distribution structure, maintenance and preplanning of the current network system is a great dream for water engineers.

In this study, basing on available investigations, an original non-equilibrium statistical dynamic analysis on complex interactions among elements constituting water distribution networks is offered. It is then shown that the hierarchies of water supply networks result from the distribution of flux at the crunodes of transition, and the natural emergences of the fractal structures are theoretically derived, addressing the issue of the characterization and the design of a rational and optimal web for a water supply network. It is our strong brief that our new preliminary analyses might convey a new avenue for understanding complex water systems through the evolution of the networks behind them and should provide some promising instructions for the confident design of water supply systems in the future.

STATISTICAL DYNAMIC DESCRIPTIONS ON ELEMENTS INTERACTIONS OF WATER SUPPLY SYSTEMS

A distinctive feature of a complex system is the emergent order or macroscopic structure resulting from their many interacting elements (Gong & Van Leeuwen 2003). Many complex networks in nature and society can be described in terms of networks capturing the intricate web of connections among the units which they are made of. A key question is how to interpret the global organization of such networks as the coexistence of their structural subunits associated with each other as highly interconnected parts (Palla *et al.* 2005). The network structure is tightly related to the nonlinear interactions between elements, and an accurate description of the coupling architecture and a characterization of the structural properties of the networks must be based on fundamental understanding of the interacting dynamics between elements (Lutz 2001).

To adapt to a changing environment, a system needs a lot of states to react to all possible perturbations. The most adequate states are selected according to their fitness to environments, or during the early stage of evolution, by subsystems' adaptation to early environments. So a basic mechanism underlying self-organization is the system's entry to attractor that is fit to environment, and lots of independent freedom of components will be restricted. This is equivalent to the increase of order that directly represents the process of self-organization. The water distribution network is the spatial distribution of water supply. The emergence of the patterns in water supply networks is not the result of sole planning but of a highly complex process of subunit interactions, which should be interpreted as the process of self-organized pattern formation. Then let us discuss how a water supply network is established. Physically, the water system represents the multitude of resource, water transport and distribution unit and so on. The transmission system is a line carrier of water, through which the distribution system brings the water to each element of the territory (Portugali & Benenson 1997). Our new approach is illustrated by considering the fundamental problem of distributing a supply of water as uniformly as possible over a given and growing territory. It would be useful to keep in mind that the skeleton of a water distribution system is a tree-shaped network. Though the actual networks in engineering are looped networks, whose emergence is only for reliability and safety. After deducing the best optimized tree-shaped networks, adding pipes or connections at certain points will naturally provide the looped networks. In this connection and for simplicity of analysis, it is assumed that the consumers are supplied by a single water resource to many users. It is here emphasized again that the water distribution network has expanded continuously, which is a process of water flow network growth. They change in time, evolve and grow in order to meet increasing demands in water load. So here the best optimized hierarchical structure is being searched, having access from the beginning to the infinity of structures that exist and is at a relative stable state. In order to know the emergence of the network, we first focus on the interacting actors in water supply systems. In the framework of statistical mechanics, particle-like elements must be first determined. Therefore, to conduct statistical dynamic analyses on water supply systems, here the elements of the constituting water supply system are generally defined as actors, which may represent the elements of the water supply network at any levels, i.e. from small pipes to large water factories. In principle, water consumption variables of each actor can be expressed as $x_1, x_2, \cdots x_n$, which are the driving forces and the cause of receiving flux from the water resource. They can be lumped as a vector $\mathbf{x} = (x_1, x_2, \cdots x_n)$, and the driving forces here indicate competence and adaptability demand difference between the water demand actors. Similar to classical statistical theory, here we can consider that all possible microstates compose a continuous range in the Γ space. $d\mathbf{x} = dx_1 dx_2 \cdots dx_n$ is a volumetric unit in Γ space. The probability for the state of the system existing within the volumetric unit $d\mathbf{x}$ at time t is

$$\rho(\mathbf{x}, t) d\mathbf{x} \tag{1}$$

$\rho(\mathbf{x}, t)$is distribution function of ensemble, which satisfies the normalization condition

$$\int \rho(x, t) dx = 1 \tag{2}$$

Defining that the water flux is J when the state of the system exists within the volumetric unit $d\mathbf{x}$ at time t, the averaged flux over all possible microstates is

$$\bar{J} = \int \rho(x, t) J(\rho) dx \tag{3}$$

The flux can be seen as generated from coupled interactions among actors. These actors are complex, for each actor consumption demands may result from the effect of socio-economic factors such as city population, household income, commercial and industrial establishment, and water price and also the seasonal variations (Zhou *et al.* 2000). More specifically, these actors are largely affected by factors such as innovations, natural resources capacity, the living habits, policies, the cost, infrastructure size, and demand variance etc. (Tillman *et al.* 1999). Thus, including these factors, J can be completely written as

$$J = \eta + \sum_i \gamma_i x_i + \sum_{ij} \gamma_{ij} x_i x_j + \sum_{ijk} \gamma_{ijk} x_i x_j x_k + \sum_{ijkl} \gamma_{ijkl} x_i x_j x_k x_l + \cdots \tag{4}$$

By use of Lagrange multiplier, let us maximize the Equation (3) under the following constraints (i.e. given prices), which stands for mass or energy conservations among actors

$$\langle \mathrm{x_i} \rangle = f_1, \langle x_i x_j \rangle = f_2, \langle x_i x_j x_k \rangle = f_3, \langle x_i x_j x_k x_l \rangle = f_4 \tag{5}$$

It is obtained that

$$\rho = \frac{c}{J - \alpha - \sum_i \beta_i x_i - \sum_{ij} \beta_{ij} x_i x_j - \sum_{ijk} \beta_{ijk} x_i x_j x_k - \sum_{ijkl} \beta_{ijkl} x_i x_j x_k x_l} \quad (6)$$

Substituting Equation (4) into Equation (7) yields

As well known that

$$\rho = \frac{c}{\eta - \alpha + \sum_i (\gamma_i - \beta_i) x_i - \sum_{ij} (\gamma_{ij} - \beta_{ij}) x_i x_j - \sum_{ijk} (\gamma_{ijk} - \beta_{ijk}) x_i x_j x_k - \sum_{ijkl} (\gamma_{ijkl} - \beta_{ijkl}) x_i x_j x_k x_l + \cdots} \quad (7)$$

$$e^x = 1 + x + \frac{1}{2!} x^2 + \frac{1}{3!} x^3 + \cdots \quad (8)$$

It is further obtained that (Haken 2000; Chai & Shoji 2002)

$$\rho = e^{\zeta + \sum_i \sigma_i x_i + \sum_{ij} \sigma_{ij} x_i x_j + \sum_{ijk} \sigma_{ijk} x_i x_j x_k + \sum_{ijkl} \sigma_{ijkl} x_i x_j x_k x_l + \cdots} \quad (9)$$

Defining the exponential term of Equation (9) as potential function (Haken 2000; Chai & Shoji 2002)

$$\Phi(\sigma, x) = \zeta + \sum_i \sigma_i x_i + \sum_{ij} \sigma_{ij} x_i x_j + \sum_{ijk} \sigma_{ijk} x_i x_j x_k + \sum_{ijkl} \sigma_{ijkl} x_i x_j x_k x_l + \cdots \quad (10)$$

σ in left term represents vector, and σ in right term represents scalar. μ and σ are both parameters produced by Lagrange optimization, which are determined by parameters β and γ in Equation (7). The potential function regulates all dynamic behaviors of interacting actors systems.

Accordingly, by transformation of

$$x_i = \sum_k \psi_{ki} \xi_k \text{ or } \xi_i = \sum_k \omega_{ki} x_k \quad (11)$$

Equation (11) can be changed as (Haken 2000; Chai & Shoji 2002)

$$\bar{\Phi}(\lambda, \xi) = \zeta + \sum_k \lambda_k \xi_k^2 + \cdots \quad (12)$$

By transformation of Equation (11), it actually means taking ξ to describe the all possible large-scaled patterns or configurations of water supply structure because ξ is a linear combination of all kinds of actors expressed by x_k. Figure 1 uses an elastic spring to clearly mimic this relationship: the wave form ξ_i is the manifestation of the x_i-actors interaction. ξ_i is the representation of space-time pattern and function of all possible water supply configurations. For water supply systems, ξ represents usually water demand difference between outside environment and the interior bulk. Of course, ξ is a dynamic variable for the configurations of water supply systems which keep evolving during the whole constructing processes.

Inactive patterns ($\lambda_k < 0$) represent that the water network configurations will be eliminated and cannot survive to form, and the active patterns ($\lambda_k > 0$) stand for survival and formation of water network configuration. In other words, water network configuration is largely dependent on parameter λ. From the above analysis, it is clear that λ in Equation (12) or σ in Equation (10) and ω in Equation (11) are decided by the parameter β and γ in Equation (7). While β and γ have a tight relationship with operational parameters, at least in principle. In another word, the above novel theoretical framework bridges the gap between operational conditions and the dynamic evolution of water supply network structures. The functional variational

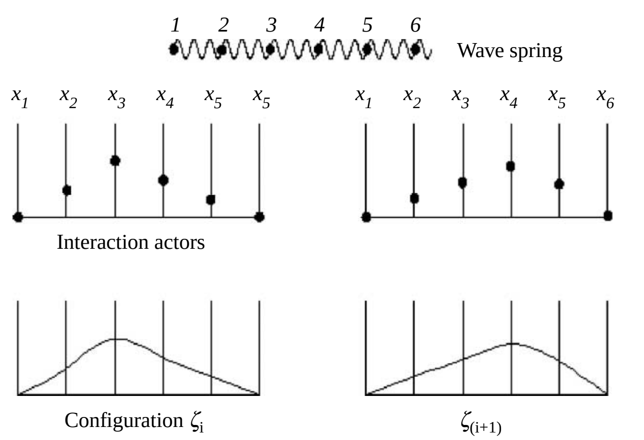

Figure 1 | Phenomenological relationships between actors (such as pipes and/or corresponding other facilities) and configurations of water supply networks.

technique with Lagrange multiplier is an essential avenue in the research of natural and engineering problems. The above newly established variational method makes the constraint conditions that once rather difficult to be tackled for the nonlinear interactions among them be easier to be dealt with (Chai & Wen 2004).

In order to further reveal the evolving dynamics of water supply network structures, the following two groups of Langevin equations can be obtained from Equation (12)

$$\dot{\xi}_u = \lambda_u \xi_u + S_u(\xi_u, \xi_s) + F_u(t), \quad \lambda_u > 0 \tag{13}$$

$$\dot{\xi}_s = \lambda_s \xi_s + T_s(\xi_u) + F_s(t), \quad \lambda_s < 0 \tag{14}$$

According to the above Langevin equations, it is clear that parameter λ is actually a kind of damping coefficient. In principle, a very large number of possible configurations have to be contemplated. Figure 2 roughly shows the two types of possible alternative configurations. In practice, only one or a few configurations ξ_i with $\lambda_u > 0$ (small damping coefficient) optimize their performance and become the selecting or evolving stage whose structures lead to more promising configurations satisfying the demand requirements of the water supply nodes, simultaneously enslave water supply systems, and ultimately lead to better optimized design (Bejan & Lorente 2001). It would be useful for the designers to contemplate all possible configurations, and then choose the best. Our analyses facilitate the mechanistic process to find the optimal configurations. As shown in Equations (13) and (14), selection of active configurations is dependent on dynamic parameters in Equations (13) and (14).

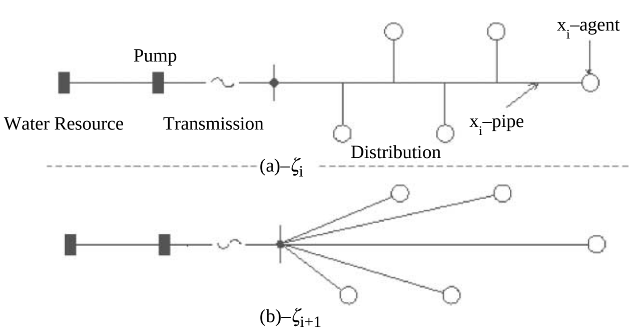

Figure 2 | Two types of possible alternative configurations of water supply networks.

Through a series of analyses (Haken 2000; Chai & Shoji 2002), it is found that the flux change close to the instability point is governed by that of the patterns with $\lambda_u > 0$ (small damping coefficient) alone, i.e.

$$\bar{J}(\alpha_1) - \bar{J}(\alpha_2) \approx J_u(\alpha_1) - J_u(\alpha_2) \tag{15}$$

Equation (15) provides a vivid physical dynamic picture for water supply systems: only active patterns of water supply network structures with stronger ability can get more flux to grow. Though within open complex water supply systems, many potential patterns may exist, only some of them can utilize the input of flux better. In other words, flux is concentrated in one or a few patterns, which may dominate the macroscopic behavior of the whole system. Water supply network structures formation is a self-organized dynamic process, which means that water supply network systems is a typical ordered dissipative structure. The self-organized analysis provides abundant information in the understanding of the formation of this kind of ordered dissipative structure. Equation (15) inevitably constructs a tree-like hierarchical network and provides a clear physical picture. When the demands exceed the capacity of the network to deliver the water, and/or a reduced ability of the system to carry flow due to increases in the carrying capacity of pipes, and/or increases in the demands imposed on a network (Xu & Goulter 1998), the remaining water distribution states (the present configuration $\xi_{u(l)}$) are now a starting point for generating new states (the configuration $\xi_{u(l+1)}$ in a higher level) in the transition to the next stage. The flux is increased. Equation (15) tells us how the water flux is increased and distributed. Apparently macroscopic behavior of the water supply system is that pipes (and/or corresponding other facilities) are added to adapt the increased water flux. It is not a sole further development of the current system by means of adding new pipes, but rather it is a transition to a new system state (Wallner 1999). Each stage represents an addition of new users and reconstructs the water supply networks. Furthermore, the process of adaptation and evolving is described as a process towards increasing complexity of the system (Holland 1995). The water flux is in a higher level.

Our analyses illustrate that the heterogeneous water supply network structures can occur spontaneously (without any supposition on influence of the actors directly). Thus, the self-organized formation of water supply network structures can be rationalized in a sound way. In the following parts, the heterogeneous water supply network structures will be further analyzed.

FRACTAL-ASSEMBLED WATER SUPPLY NETWORK STRUCTURES

Fractal-assembled structures and fractal dimension

How to assemble the information contained in the system to the structure is a challenging problem to understand the emergence of structures in the complex systems. Tremendous information is saved in each actor of the water supply networks and water flux is transmitted between actors. To consider spatial correlations it is significant to understand the information transmission process from one scale to a higher scale and to discuss water flux distributions among different actors in all kind of spatial scales. Actually, the water supply's life comes from its connectivity and transmission. A physical path must facilitate the transmission of information, otherwise the actors cannot work. The transitions of water supply systems from one state to the next new state are affected by many factors, such as the service pressure available for the transmission utility, the water consumption of the devices owned by the actors and the standards of water consumption utilities etc. Above results show the water system growth is dominated by the ξ_u with the lowest damping coefficient λ. Information of one scale or state $\xi_{u(l)}$ is inherited to next new scale or state $\xi_{u(l+1)}$. This principle is repeated in the optimization of increasingly larger scales, where each new scale state is an assembly of previously optimized smaller sizes (Wechsatol *et al.* 2002). The information of the water supply system is compressed into the evolving equation of parameter ξ_u. By this way, a delicate avenue is provided to tackle the multiple free degree problem of the system and bridge the gap between influencing factors of the system and the structure.

Here it is emphasized how the information in the water flux is transmitted on topology of water supply systems. In other words, we tend to discuss the forming process of water supply networks through studying the water flow information across different scales. Biologists are the first to formulate empirical principles of systems self-organization that are expressed on all levels, which means the systems of lower level are elements of higher-level systems (Engelhardt & Zhini 1985). To describe the hierarchical structures quantitatively, let us visualize a physical picture on the growth of a typical micro-volume of water supply system structure, as shown in Figure 3: the structure continues to spread through the interactions (such as absorption) with other actors with going time and increasing scale. A structure of scale l shown in Figure 3 is considered. The term dl represents the infinite scale increment from scale l to scale $l + dl$. The adjacent actors diffuse from the bulk into the formed structure through a boundary layer, within which the actors are continuously adsorbed and converted to the part of the formed structures. Therefore, water supply structures will grow in scale and the boundary layer will thicken accordingly. In Figure 3, vertical axis represents the direction of structures growing, and the curve represents the development of the boundary layer. The terms u and ξ represent the velocity distribution and the concentration distribution of flux induced by water supply structures. Taking account of the mass conservation between the two adjacent scales, as shown in Figure 3, the following series of equations is obtained.

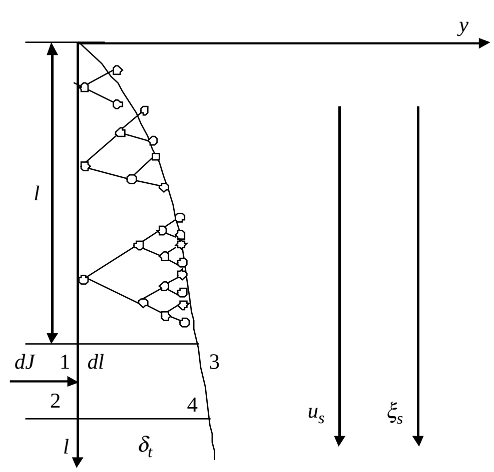

Figure 3 | Physical model for growing water supply networks.

The flux input per time unit from bulk to the growing water supply structure through interface 3–4 is

$$dl\frac{d}{dl}\int_0^l \xi_s u dy \tag{16}$$

The flux output per time unit through interface 1–2 is

$$-adl\left(\frac{\partial\xi}{\partial y}\right)\Big|_w \tag{17}$$

The flux increase per time unit along scale increment through interface 1–3 and 2–4 is

$$dl\frac{d}{dl}\int_0^l \xi u dy \tag{18}$$

Thus, the cascade flux balance gives

$$dl\frac{d}{dl}\int_0^l \xi_s u dy - dl\frac{d}{dl}\int_0^l \xi u dy - adl\left(\frac{\partial\xi}{\partial y}\right)\Big|_w = 0 \tag{19}$$

Therefore, the flux conservation equation for growing water supply structures can be rewritten as

$$\xi_S\frac{d}{dl}\int_0^l u dy - a\left(\frac{\partial\xi}{\partial y}\right)\Big|_w = \frac{d}{dl}\int_0^l \xi u dy \tag{20}$$

Equation (20) conveys and inherits the water flux information across different scales.

The above analyses show that water flux is concentrated in one or a few configurations, though a lot of possible configurations may exist simultaneously. In this way, the expression is satisfied as

$$\bar{J}_n(\alpha_1) - \bar{J}_n(\alpha_2) = J_{un}(\alpha_1) - J_{un}(\alpha_2) \sim h_n l^2 \xi_n \tag{21}$$

h_n is general water flux density transfer coefficient. For arbitrary u and ξ distribution, equation (20) usually yields

$$h_n \sim l^{-p} \tag{22}$$

Depending on actual distributions of u and ξ, as a specific parameter, p usually varies between 0 to 1.

Considering that driving forces are chosen as the demand difference between different water utility systems, we can derive the fractal structure (Note: fractal structure is a kind of self-similar structure, and fractal geometry was developed by Mandelbrot to describe the structure with fractal dimension (Mandelbrot 1982)) describing the distribution of water supply networks in the following way

$$\begin{aligned}\frac{\Omega_n \Delta J}{\Omega_{(n-1)}\Delta J} = \frac{\Omega_n}{\Omega_{(n-1)}} &\sim \frac{J_{un}(\alpha_1) - J_{un}(\alpha_2)}{J_{u(n-1)}(\alpha_1) - J_{u(n-1)}(\alpha_2)} \\ &\sim \frac{h_n l_n^2 \xi_n}{h_{(n-1)} l_{(n-1)}^2 \xi_{n-1}} \sim \left[\frac{l_n}{l_{(n-1)}}\right]^{(2-p)}\end{aligned} \tag{23}$$

In complex systems, scale connections and hierarchical structures allow the achievement of self-organization through a good structural transport coefficient h_n of the general flux $\bar{J}$ between two scales. Parameter 2-p is fractal dimension, whose value indicates spatial distribution of general water flux distribution, i.e. approximate water supply network. Fractal dimension physically embodies dynamic features of evolutional complex water distribution systems.

Let us here consider the solutions of Equation (20) under some typical situations (Chai & Shoji 2002; Chai 2004; Chen & Chai 2006): if u and ξ have trivial polynomial distribution (c_2), which may correspond to the case of laminar flux transfer, we have $p = 0.5$ and 2-$p = 1.5$; If u and ξ have exponential distribution (c_3), which may correspond to the case of turbulent flux transfer, we have $p = 0.2$ and 2-$p = 1.8$; if u and ξ have trivial linear distribution (c_1), which may correspond to sole solid particles conductive flux transfer, we have $p = 1$ and 2-$p = 1$(it is no surprise that no fractal structures will form in this case); if u and ξ have almost uniform distribution (of course it is an ideal case), which may correspond to extremely turbulent flux transfer, we have $p = 0$ and 2-$p = 2$ (it is natural that solid 2D Euclidean structures will form in this case).

The above analyses are based on some typical u and ξ distributions, which correspond to ideal flux transfer situations. For non-typical cases, the generalized flux can be taken as a hybrid of the two types of ideal flux transfer. It is defined as the flux comprises the fraction β of the flux that is related to the index p_1, and the fraction 1-β of the flux that is related to the index p_2, where p_1 and p_2 are the two extreme points of the two ideal flux transfer situations. Based on the conservation of the generalized flux, we have

$$l^{-p} = \beta l^{-p_1} + (1-\beta) l^{-p_2} \tag{24}$$

where β is related to operational factors of the system, essentially reflecting and regulating the actual distribution of u and ξ. Therefore, β is actually a parameter that can reflect all kinds of the flux pattern and the distributions of u and ξ. Equation (24) can be written as

$$p = -\frac{\ln(\beta l^{-p_1} + (1-\beta)l^{-p_2})}{\ln l} \qquad (25)$$

In the above sections, the factors affecting the water supply networks have been mentioned. Here it is further stated that the detailed mechanistic effects of factors on water supply networks can be described by fractal dimension. Doubtlessly, water supply network structures will vary with shifts of operational conditions. Our forgoing theoretical analyses may give a unified theoretical framework for the explanation of parametric effects on water supply networks structures. Obviously, different u and ξ distributions will result in different p values and different water supply network structures. Logical conclusions could be drawn that during a high flux, flow for water supply system is very intense, more like violent turbulent flow, and a high density (i.e. high fractal dimension) of water supply network structures may then be formed. While during a low flux, flow for water supply system is less intense than violent turbulent, a low density (i.e. low fractal dimension) of water supply networks may accordingly be formed. These results are qualitatively in agreements with real statistical facts. Of course, currently, available statistical data on the structures of the water supply networks are scarce. Indirect certification of our theoretical insight would be helpful. Water supply networks, as the lifeline of city, are tightly dependent on spatial structures of city. The fractal structure and self-organization of a city sheds some light on the distribution of water supply networks (Carvalho & Penn 2004). It is reasonable to lead to a logical conclusion that a city with larger fractal dimension has water supply networks with larger fractal dimension. Different cities with diverse factors obviously have water supply networks with diverse fractal dimensions. For example the city of Beijing (capital of China) and other less developed cities in China have water supply networks with largely different fractal dimensions. Generally, the dimensions of water supply networks in a city with more habitants (i.e. distributions of u and ξ are more like exponential, in term of our theoretical framework) are often larger than ones in a city with sparse habitants (i.e. distributions of u and ξ are more like polynomial). No doubt, these results are logical verifications of our theoretical predictions.

Numerical calculations and connections to experimental measurements

The relationship among fractal dimension, scale and influence factor β can be revealed from Equation (25) where the p_1 and p_2 represent values of p with different flow forms. When β have different values between 0 and 1, all possible combinations of typical flow forms can be obtained. Varied p values and fractal dimensions under various scales can be acquired from Equation (25). Some typical values of p can be obtained: 0, 0.2, 0.5, and 1, which divide p value into 3 sections (Chai 2004; Chen & Chai 2006): [0, 0.2], [0.2, 0.5], [0.5, 1]. Different sections represent different flow forms. For example, values in the section [0.2, 0.5] indicate that the flux form is between turbulent flow and laminar flow, which are more common in nature. Figure 4 represents how β influence 2-p under a certain scale. Figure 5 represents the relationship between 2-p and scale under a certain value of β. These figures indicate there are some sorts of relationships existing between fractal dimensions and scale even for similar β values.

Effects of all operational conditions on water supply networks can be expressed by parameters β and γ in Equation (7), or σ in Equation (10), or ω in Equation (11) or λ in Equation (12), or finally the distributions of u and ξ in

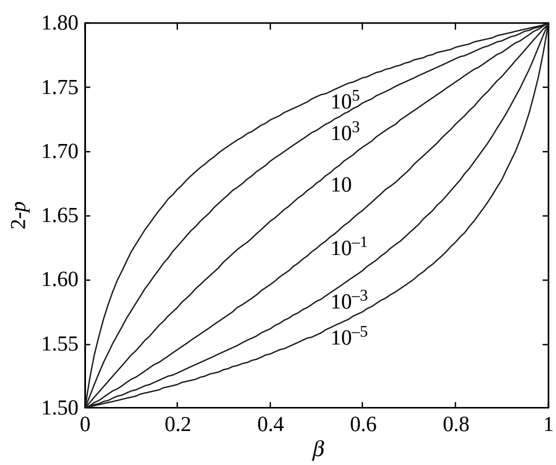

Figure 4 | The influence of β on the fractal dimension 2-p when p is between 0.2 ~ 0.5.

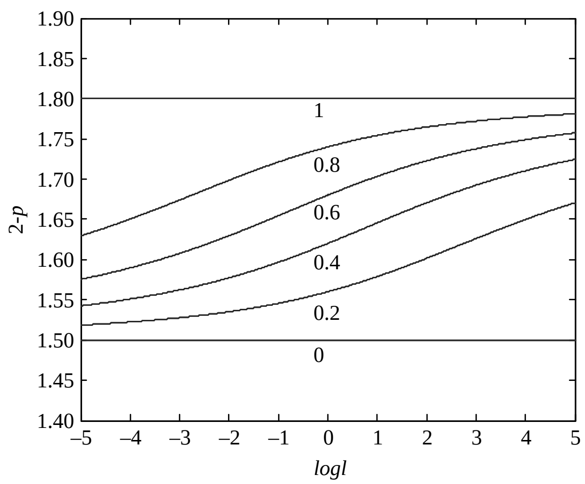

Figure 5 | The influence of the scale *l* on the fractal dimension 2-*p* when *p* is between 0.2 ~ 0.5.

Equation (20). For example, Figure 6 schematically shows the configuration we got by the above approaches and roughly reflects the general structure of water supply networks, whose quantitative features are embodied in Figure 4. Figure 7 is the real topology of the water supply network of Tianjin University, a famous university in China with a history of more than one hundrand years (of course, most real water supply networks hold this kind of similar structure too). According to Figure 4, once we measure the fractal dimension 2-*p* of the water supply network, we can obtain the profile of β to understand the internal dynamics of the system. Conversely, if the dynamic components interaction of the system were known, we can analytically

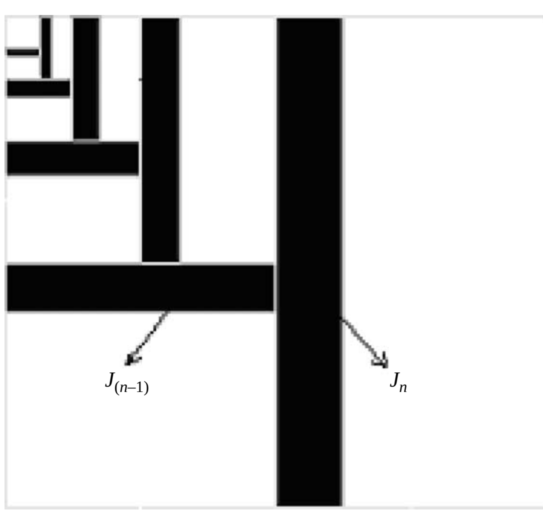

Figure 6 | The theoretical predictive growth of water supply networks.

Figure 7 | A real topology of water supply network in Tianjin university.

derive the fractal dimension 2-*p*. As far as the topology shown in Figure 7, the fractal dimension 2-*p* is measured to be 1.6. Therefore, by choosing a scale $l = 10^3$m, we can get the theoretical values of $\beta = 0.14$ and deduce the local flow forms as $0.14c_3 + 0.86c_2$. The theoretical predictions and real configurations are linked. To state, we have theoretically analyzed the actor's interactions among water supply systems, and fractal features of water supply network structures are analytically revealed. According to our framework, once we have the u and ξ distributions, as shown in Figure 8, we can get the spanning tree of water transportation network that provides a unique route for water flow for each consumer actor,. On the other hand, if

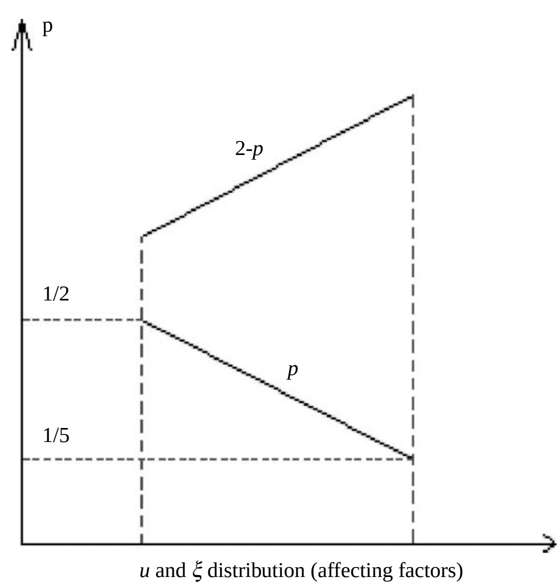

Figure 8 | Variation of fractal dimension with affecting factors.

we know the fractal dimensions of water supply networks, as shown in Figure 8, we can semi-quantitatively deduce the parametric effects of diverse factors in a city on water supply networks, at least on some tending characteristics of water supply systems.

Analyses of the transition between different configurations

It is above shown that water supply network structures (accordingly fractal dimension 2-p) will vary with shifts of operational conditions. In what way does this take place?

The transition to a different configuration involves adding each of the additional pipes (and/or corresponding other facilities) with all possible combinations of rehabilitation alternatives, existing and different selections for connections. According to the theoretical analyses, among all the possible configurations only active configurations with a stronger ability can survive to get flux and develop, whose growth is the optimal one with the least loss of construction of the ordered network. The water flux is concentrated in one or a few active configurations which can use the water flux better, being selected as the optimal sequence of rehabilitating the water supply system, and thus generating the larger-scaled configurations, which may dominate the macroscopic behavior of the whole system (Kleiner *et al.* 2001). The transition process is repeated until all pipes (and/or corresponding other facilities) are considered in the sequences. It is a kind of self-organized growing process. In this way, foregoing framework sets a global criterion at which least cost is used to reconstruct the water supply system, or adjust the optimized transitions between different configurations.

While as self-organization systems, the evolution of water supply networks exhibit a very distinct path: firstly, when there is no change for the complex water supply systems, it will maintain a steady state for a long time, followed by a short period of strong fluctuation, from which the system re-emerges by adding new pipes (and/or corresponding other facilities) or changing the distribution of flux to a new level of steady state with new structures and functions, and so on (as shown in Equations 13 or 14). Figure 9 illustrates the transition process of water networks. Secondly, even in a steady-state, the water supply system might still exhibit deterministic chaotic phenomena in the form of abrupt jumps between the dominated parameters λ_i, for example, between climate changes, water price vs residential etc. It is as if a few unstable chaotic actors are captive within the stable global networks. The water consumption actors have to change their behavior by adjusting water consumption time or saving water to satisfy their own needs. For examples, people will install low-flush cisterns for toilets. Actually, the global structure will not be altered by the few actors' behaviors.

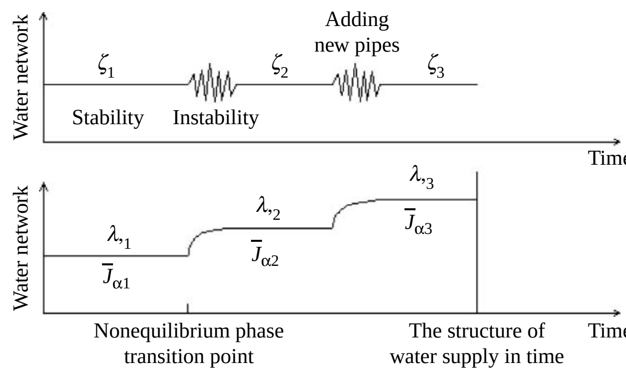

Figure 9 | Transition from one state to another state for water supply networks.

ENGINEERING INSTRUCTIONS

Many optimization models for determining cost-effectiveness and obtaining the least-cost system design for water distribution networks have been developed. This is a complex and difficult task due to variable inflows and a large number of possible design alternatives. The results relied much upon the definition of parameters, the type of algorithms, the simplicity of the real networks, and the research of the network scale etc. Even worse, the efficiency of the enumeration algorithm is not usually an optimal solution when a system combination is too large (Bi *et al.* 1997). Hence a major pending task is on how to reduce the set of alternatives which need to be examined in detail without eliminating the attractive options.

A new flavor for this task may be provided by our framework: the alternative configurations are greatly reduced by the process of competition and adaptability; models on the fractal growth of a water network greatly cut down the massive optimization searching process; the self-organization and fractal process will automatically adjust to the criteria of optimization. In fact, self-organization is always related to adaptation. This gives us instruction that we should seek after the principle of self-organization from the perspective of system adaptation to environment. Perhaps the most attractive application would be to design a complex socio-economic system which relies on self-organization rather than centralized planning and control. Even our present social organization or engineering structure include some effects of self-organization, they are far from optimal.

As consumer demands increase with time and the consumer actors enlarge, the carrying capacity of the current water distribution systems will fail to fulfill their specified function of distributing potable water continuously and within prescribed pressure limits. The water supply system lives in an unstable condition, which will result in a transition to a new stage to make the water supply systems relatively stable. At this time, besides improving the transmission capacity of the network and the production capacity of the infrastructure, there is a need for rehabilitation, replacement and expansion. Choosing the best possible set of network improvements within a limited budget is required of civil engineers. Our framework can play some roles in this aspect: a water supply network is growing in a fractal way, and the fractal growth is a prior and easy optimization method for expansion. It has applicable meanings to recognize the fractal features of the water network for expansion. In the previous section we give detailed methods to calculate p, which greatly relies upon the distributions of ξ and u. Based on the research of affecting factors of a water supply network, it is possible to identify the distributions of ξ and u. Although it is still difficult to determine the distributions of ξ and u today, we wish future work can realize this goal.

In a word, the present analytical results not only give a mechanistic perspective for self-organized emergence of evolving water supply networks, but also may provide good instructions for engineering applications. However, for a water supply system, this is a rather complicated problem and there remain lots of unresolved questions. We hope the present research will act as a catalyst for further mechanistic investigations on water supply systems.

CONCLUSIONS

A new perspective on the fundamental aspects of water distribution networks has been taken to combine optimizations and self-organized evolutions of systems in this paper. A non-equilibrium statistical dynamic method was applied for exploring the organizing emergence of water supply networks. The present studies may not only give more rational and mechanistic descriptions on water supply systems, but have potential for some important industrial implications. The present investigations, although preliminary, made a renewed theoretical effort to understand the underlying mechanisms of water supply systems. Recently, it has come to be appreciated that many complex systems which consist of a large number of interacting subunits obey universal laws such as self-organization and fractal growth, independent of the subunit details. It is hoped that this present method may have many important applications beyond the fields of water supply networks.

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