Vibration Frequencies of Tapered Bars and Circular Plates[1]

J. C. GEORGIAN.[2] The authors are to be congratulated on obtaining essentially the eigenvalues for the tapered truncated-cone cantilevers and, by analogy, the axisymmetrical eigenvalues for a circular tapered plate. The very real contribution has been in a computer program to solve the frequency or eigenvalue determinant for the various boundary conditions. It is, therefore, to be regretted that the use of this program has been limited to only the internal use of the IBM Corporation.

For some time, the writer has been interested in the problem of the eigen or characteristic value of truncated, tapered cantilever wedges and cones with a free end, as these simulate the vibration problems of turbine and compressor blades with variable cross sections. Two of his students[3,4] attempted the solution of this problem using the Rayleigh-Ritz method and the counterpart of the Galerkin method.[5]

In addition to the calculations, the eigenvalues of the tapered wedges and cones were determined experimentally up to the fifth mode of vibration. Unfortunately the authors' data have not been put in the usual form and, accordingly, the writer has recomputed their data for the cantilever cone with a free end and tabulated this in Table 1 of this discussion. The usual form of the eigen or characteristic value is given by the equation

$$\lambda = pl^2 \left(\frac{\rho A_b}{EI_b}\right)^{1/2} \tag{1}$$

where λ = eigenvalue; I_b = the cross-section moment inertia at the base of the cantilever cone; A_b = cross-sectional area at the base of the cone; $l = l_1 - l_2$ = the length of the beam; ρ = the mass density of the beam material; p = the circular frequency. The writer prefers to tabulate the characteristic shape of the truncated cone as $(A_t/A_b)^{1/2}$ where A_t = area at tip of cone. Table 2 gives the test results obtained for the cantilever cone, and the results from Tables 1 and 2 are plotted in Fig. 1.

The excellent correlation of the test results and the analytical results of the authors should be noted. The writer considers that the confirmation of the theoretical results solves this problem completely.

In addition to the cone, the tapered cantilever wedge also has been worked out using the counterpart of the Galerkin method and tests of a model cantilever performed by Verma.[4] A six-term deflection curve was used which reduces to a six-by-six eigenvalue matrix. This matrix was solved on the IBM 7090 computer at Washington University, but unfortunately it was discovered that, for the eigenvalue matrices, the computer could carry the solution to only eight significant figures, and this was insufficient to determine the correct eigenvalues. Even spurious values were developed in the answer. Using a superprecision method of sixteen significant figures did not improve the solution. The best that could be expected were accurate values for the first two modes. The 3, 4, 5, 6 modes were in extreme error for the known values for straight cantilevers or completely tapered cantilevers. Table 3 presents the calculated first and second modes, and Table 4 the mode numbers determined experi-

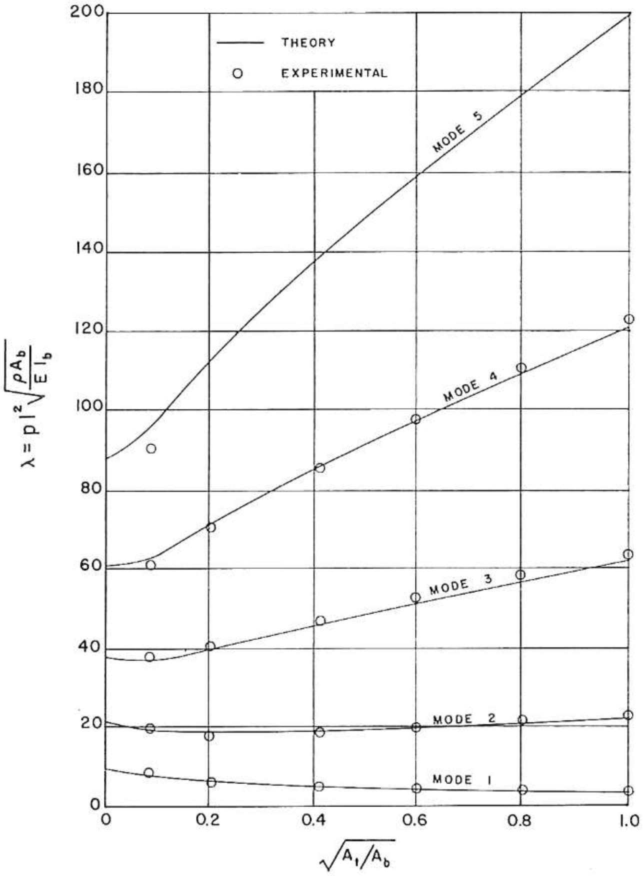

Fig. 1 Theoretical and experimental eigenvalues for frustum of cone cantilever

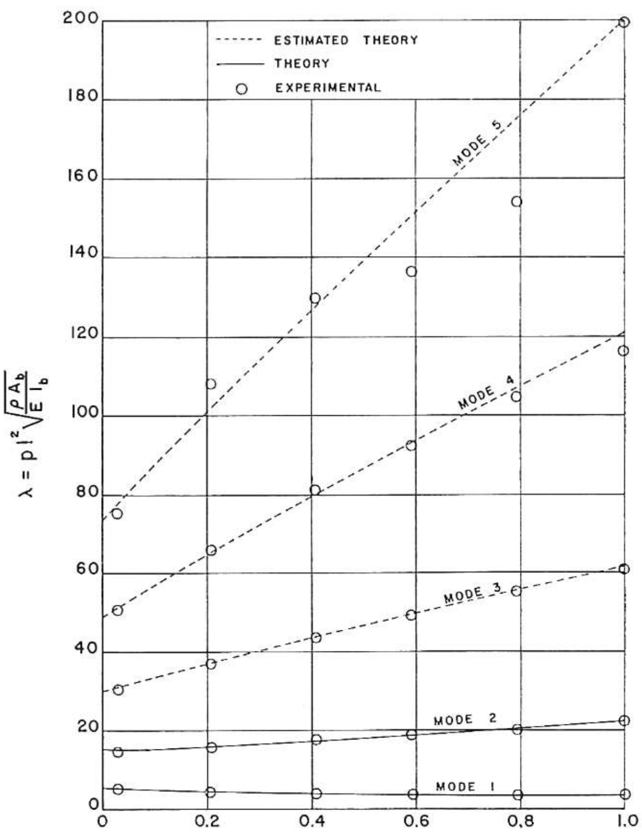

Fig. 2 Theoretical and experimental eigenvalues for frustum and wedge cantilever

[1] By H. D. Conway, E. C. H. Becker, and J. F. Dubil, published in the June, 1964, issue of the JOURNAL OF APPLIED MECHANICS, vol. 31, TRANS. ASME, vol. 86, Series E, pp. 329–331.

[2] Professor of Mechanical Engineering, Washington University, St. Louis, Mo. Mem. ASME.

[3] E. Chicurel, "Natural Frequencies of Vibration of Tapered Truncated Cantilever Bars," Master's thesis, Washington University, St. Louis, Mo., 1957.

[4] S. M. Verma, "Characteristic Values of Tapered Truncated Cantilever Wedges and Cones," Master's thesis, Washington University, St. Louis, Mo., 1960.

[5] C. B. Biezeno and R. Grammel, *Engineering Dynamics*, vol. 3, Blackie and Son Limited, London and Glasgow, 1954, pp. 144–158.

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Table 1 Theoretical eigenvalues for frustum of cone cantilever

$(A_t/A_b)^{1/2}$	Mode number 1	2	3	4	5
1.00	3.516	22.034	61.701	120.91	199.85
0.500	4.625	19.55	48.50	91.75	149.50
0.333	5.289	18.76	43.78	80.89	130.22
0.250	5.850	18.51	41.34	74.81	119.81
0.100	7.201	18.71	37.10	63.50	98.01
0	8.718	21.146	38.45	60.678	87.84

Table 2 Experimental eigenvalues for frustum of cone cantilever

$(A_t/A_b)^{1/2}$	Mode number 1	2	3	4	5
1.000	3.59	22.69	63.35	122.9	
0.803	3.88	21.57	58.45	111.0	
0.601	4.41	19.82	52.60	97.8	
0.411	4.96	18.63	46.40	85.7	
0.207	6.13	18.16	40.70	70.4	
0.085	8.07	19.38	37.40	60.5	90.5

Table 3 Theoretical eigenvalues for frustum of wedge cantilever

A_t/A_b	Mode number 1	2	3	4	5
1.00	3.516	22.034	61.701	120.91	199.85
0.80	3.608	20.616			
0.60	3.737	19.112			
0.40	3.934	17.489			
0.20	4.292	15.743			
0	5.315	15.206	30.030	49.766	74.39

Table 4 Experimental eigenvalues for frustum of wedge cantilever

A_t/A_b	Mode number 1	2	3	4	5
1.000	3.55	22.06	60.85	116.20	199.60
0.797	3.65	20.28	54.80	104.10	153.50
0.592	3.82	18.97	48.97	92.66	136.70
0.407	3.99	17.45	43.70	81.20	129.20
0.206	4.31	15.78	37.08	66.30	107.70
0.030	5.19	14.52	30.50	50.75	75.80

mentally, with both these values plotted in Fig. 2. These are presented in the hope that the authors will perform similar calculations for the tapered cantilever wedge.

Finally, the general differential equation for any tapered beam is given by the expression

$$\frac{d^2}{d^2x}\left[x^m \frac{d^2w}{d^2x}\right] - q^2x^nw = 0 \tag{2}$$

where m and n are constants and the other terms are the same as in the author's paper. For $m = 3$ and $n = 1$, we have the case for the tapered frustum of a cantilever wedge. The writer is inquiring whether the authors have solved any of the cases represented by the foregoing equation (2) other than the conical.

Authors' Closure

Authors are happy when they receive favorable comments on their work, and the present case is no exception. The authors are indebted to Professor Georgian for his discussion, and they believe that his experimental confirmation of their clamped/free results for the cone adds substantially to the investigation. Further results are being worked out for the truncated wedge, and it is hoped to present these before long.

Fixed Tube-Sheet Heat Exchangers[1]

K. A. GARDNER.[2] This paper sets forth in straightforward detail the consequences of combining Yu's method [3][3] of evaluating peripheral restraint of fixed tube sheets with his solution [4] of the deflection equations with appreciable reactive tube bending. Unfortunately, however, an oversight in the writer's original paper on fixed tube-sheet design [1] has been propagated, not only by the authors, but by Yu [3] and others as well. Miller,[4] whose paper on the subject appeared in England simultaneously with the writer's here, apparently fell independently into the same error. Yakovlev [2] did not.

The error consists of overlooking the biaxial state of stress in tubes and shell and its Poisson effect on the elongations of both. Only in 1958, when he finally obtained a copy of Yakovlev's paper and laboriously translated portions of it, did the writer become aware of this shortcoming in his own prior work.

The authors' equations (3), (4), (10), and (11) should read:

$$F_t = \pi t(d - t)\left\{E_t\left(\frac{e - \alpha_t T_t L/2 - w}{L/2}\right) + \nu_t\left[p_t\left(\frac{d - 2t}{2t}\right) - p_s\left(\frac{d}{2t}\right)\right]\right\} \tag{3a}$$

$$p = f_t p_t - f_s p_s + k\left\{(e - \alpha_t T_t L/2 - w) + \frac{\nu_t L}{2E_t}\left[p_t\left(\frac{d - 2t}{2t}\right) - p_s\left(\frac{d}{2t}\right)\right]\right\} \tag{4a}$$

[1] By G. B. Boon and R. A. Walsh, published in the June, 1964, issue of the JOURNAL OF APPLIED MECHANICS, vol. 31, TRANS. ASME, vol. 86, Series E, pp. 175–180.

[2] The M. W. Kellogg Company, New York, N. Y. Mem. ASME.

[3] Numbers in brackets designate References at end of original paper.

[4] K. A. G. Miller, "Design of Tube Plates in Heat Exchangers," *Proceedings of The Institution of Mechanical Engineers*, Series B, vol. 1, 1952, pp. 215–231.

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