all other cases, for both Problems 1 and 2 in the paper, can be found.

In the case $V = c_b$, one will find the discontinuity of $w_\tau$ and discontinuity of $\psi_\xi$ at $x = x_b$ by expansion of the inverse integral (18) immediately after the bending wave front. In this case, $\Delta w_\tau$ is constant while $\Delta\psi_\xi$ increases indefinitely at $x = x_b$ as the force travels along the beam.

## Author's Closure

The author would like to thank Dr. Sing-chih Tang for his comments.

The author agrees that the discontinuities of velocity (or of other quantities of interest) are readily obtainable. Term-by-term integration of the expansions in $(1/p)$ of the appropriate Laplace transforms yields, for $p$ large, the behavior of the velocity at the shear wave front and force. In particular, the first term in each expansion provides the discontinuity in the velocity. In problem 1, with the force moving supersonically, the discontinuities are both equal to $\Delta w_\tau = \kappa(1 + \eta)$. In problem 2, they are, respectively, $\Delta w_\tau = \kappa(1 + \eta)(\beta/\gamma^{1/2})$ and $\Delta w_\tau = \kappa(1 + \eta)$.

# Newton's Method Applied to Problems in Nonlinear Mechanics[1]

**R. R. ARCHER.**[2] The author has given a very clear exposition of Newton's method as applied to nonlinear boundary-value problems in mechanics. The examples are particularly well chosen to point up the definite advantages which this technique has over some alternate schemes.

In the matter of the convergence of the Newtonian iteration to various branches of the multivalued solution curves, which is discussed here in connection with the shallow spherical shell buckling problem, a paper[3] by Keller and Wolfe is of considerable interest. In this paper, an exhaustive examination of the remarkable variety of solution branches for the snap-through buckling problem is given. Without a good "physical understanding" of the phenomenon described by a nonlinear boundary-value problem, there is a danger that indiscriminate use of Newton's method might produce misleading solutions.

Finally, the discusser would like to add that some recent work on the nonlinear dynamic behavior of shallow shells[4] which he has carried out with Lange has indicated the importance of Newton's method when coupled with implicit finite-difference schemes for nonlinear partial differential equations of the parabolic type.

## Author's Closure

The author would like to thank Professor Archer for his comment and additional references.

Another recent paper[5] by Bueckner, Johnson, and Moore shows how Newton's method can be modified to obtain convergence where the homogeneous linear "variational" equations have a nontrivial solution.

It is true that "a good 'physical understanding' of the phenomenon described by a nonlinear boundary-value problem" is necessary. However, multiplicity of solutions is a property of nonlinear equations and not a disadvantage caused by using Newton's method.

[1] By G. A. Thurston, published in the June, 1965, issue of the JOURNAL OF APPLIED MECHANICS, vol. 32, TRANS. ASME, vol. 87, Series E, pp. 383–388.

Visiting Professor, Department of Civil Engineering, Indian Institute of Technology, Kanpur, India.

[3] Keller and Wolfe, "On the Nonunique Equilibrium States and Buckling Mechanism of Spherical Shells," to appear in *Society for Industrial and Applied Mathematics Journal.*

[4] R. R. Archer and C. G. Lange, "On the Nonlinear Dynamic Behavior of Shallow Spherical Shells," to appear in *AIAA Journal.*

[5] H. F. Bueckner, M. W. Johnson, Jr., R. H. Moore, "The Calculation of Equilibrium States of Elastic Bodies by Newton's Method." Paper presented at the 9th Midwestern Mechanics Conference, Madison, Wis., August 16–18, 1965.

# On the Almost Sure Stability of Linear Dynamic Systems With Stochastic Coefficients[1]

**FRANK KOZIN.**[2] We wish to commend the authors on their reported results concerning almost sure stability of linear stochastic systems. It is gratifying to know that the assumptions and methods proposed by this writer in [1][3] (Ref. [9] in original paper) have been made to yield the sharper sufficient condition obtained by the authors. There are, however, two minor points that this writer wishes to discuss concerning the paper.

First, the authors state, referring to [1], "These results are, however, too conservative and predict that the standard deviation of the parametric term should be zero when the system is critically damped . . . ." If the authors will look again at equation (4.16) of [1], they will find an inequality for $z = 1$. The graph is a vertical line, at $z = 1$, that is present in Fig. 1 of [1] but does not appear in the authors' reproduction of the graph.

The second but most important point concerns the authors' stated results concerning the almost sure stability of the class of nonlinear systems they treat. They have *not* established almost sure Lyapunov stability for these systems. They have established a "quasi-stability" defined by (13), which implies Lyapunov stability for the linear homogeneous system (1) but not for systems in general. (See [2], p. 96, for discussion and further references; see also [3].) A condition guaranteeing almost sure Lyapunov stability of the system (56) would be to place an almost sure bound on the collection of sample functions of the $f(t)$ process to guarantee with probability one that the right side of the inequality (69) is negative.

Until this point is further clarified, this writer feels that the results on the class of nonlinear systems treated by the authors remain incomplete. There are a number of subtleties involved in the almost sure properties of the solutions of stochastic differential equations. Some of these points are taken up in [4].

Except for the foregoing few comments, this writer wishes to reiterate his compliments to the authors for a worthy contribution to the field of stochastic systems.

### References

1 Frank Kozin, "On Almost Sure Stability of Linear Systems with Random Coefficients," *Jour. of Math. and Physics*, vol. 42, March, 1963, pp. 59–67.

2 L. Cesari, "Asymptotic Behavior and Stability Problems in Ordinary Differential Equations," Springer-Verlag, Berlin, 1959.

3 J. L. Massera, "Contributions to Stability Theory," *Ann. of Math.*, vol. 64, 1956, pp. 182–206.

4 F. Kozin, "On Relations Between Moment Properties and Almost Sure Lyapunov Stability for Linear Stochastic Systems," to appear in *Jour. for Math. and Its Applications.*

**C. B. MEHR and P. K. C. WANG.**[4] The authors have presented an interesting paper on the almost sure stability of a class of sto-

[1] By T. K. Caughey and A. H. Gray, Jr., published in the June, 1965, issue of the JOURNAL OF APPLIED MECHANICS, vol. 32, Trans. ASME, vol. 87, Series E, 1965, pp. 365–372.

[2] Professor of Aeronautical and Engineering Sciences, Purdue University, Lafayette, Ind.

[3] Numbers in brackets indicate References at end of this Discussion.

[4] Formerly, Members of Research Staff, IBM Research Laboratory, San Jose, Calif. Now with Department of Mathematics, Ohio University, Athens, Ohio, and Department of Electrical Engineering, University of Southern California, Los Angeles, respectively.