



# AN INTERACTIVE INEXACT-FUZZY APPROACH FOR MULTIOBJECTIVE PLANNING OF WATER RESOURCE SYSTEMS

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## ABSTRACT

In this study, an inexact fuzzy multiobjective programming (IFMOP) method is developed and applied to a case study of water pollution control planning in the Lake Erhai Basin. The IFMOP improves upon the existing multiobjective programming methods with advantages in data availability, solution algorithm, computational requirement and result interpretation. The case study project was supported by the United Nations Environment Programme (UNEP). The results indicate that desired schemes for a number of system activities in different subareas/periods were obtained. Inheriting uncertain natures of the model inputs, the majority of solutions present as inexact values which provide decision-makers with a flexible decision space. Generally, the modeling results would provide scientific bases for the formulation of policies/strategies regarding regional socio-economic development and environmental protection.

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## KEYWORDS

Inexact-fuzzy programming, interactive approach, multiobjective, planning, uncertainty, water pollution.

## INTRODUCTION

Regional water quality management is often associated with multiple activities and objectives with complicated and dynamic interrelationships between each other. Also, uncertainties exist in many system components and may affect processes of data investigation, modeling computation and results presentation when the water management system is analyzed through simulation/optimization techniques. Multiobjective programming (MOP) under uncertainty has gained great interest in the past decade, with a number of fuzzy/stochastic MOP methods being proposed to address the multiobjective and uncertain features (Slowinski and Teghem, 1990; Lai and Hwang, 1994). However, very few applications of them to water management were reported, in contrast to a number of previous studies for applying deterministic MOP to water resources problems (Hipel, 1992). In fact, the fuzzy/stochastic methods were applicable to problems with known possibilistic and/or probabilistic information. However, this type of information may not be available for many practical problems in which only fluctuate intervals are known for a number of system factors. In this study, a hybrid inexact-fuzzy multiobjective programming (IFMOP) method is proposed and applied to a multiobjective water management problem. The IFMOP allows uncertainties presented as fluctuation intervals and/or possibilistic distributions to be directly communicated into the modeling

processes and resulting solutions. Feasible decision alternatives can then be generated through interpretation of the IFMOP solutions. The proposed IFMOP solution approach does not lead to complicated intermediate submodels, and thus has reasonable computational requirements. The IFMOP is applied to a research project supported by the United Nations Environment Programme (UNEP) for integrated water pollution control planning in the Lake Erhai Basin, China. Complexities of the study system with multiobjective, interactive, uncertain and dynamic natures are effectively reflected through development of an IFMOP model.

### INEXACT-FUZZY MULTIOBJECTIVE LINEAR PROGRAMMING

A linear MOP problem with inexact parameters can be formulated as follows:

$$\begin{aligned}
 \min \quad & f_k^\pm = C_k^\pm X^\pm, & k = 1, 2, \dots, p, & (1.a) \\
 \max \quad & f_l^\pm = C_l^\pm X^\pm, & l = p+1, p+2, \dots, q, & (1.b) \\
 \text{s.t.} \quad & A_i^\pm X^\pm \leq b_i^\pm, & i = 1, 2, \dots, m, & (1.c) \\
 & A_j^\pm X^\pm \geq b_j^\pm, & j = m+1, m+2, \dots, n, & (1.d) \\
 & X^\pm \geq 0, & & (1.e)
 \end{aligned}$$

where  $X^\pm \in \{\mathcal{R}^\pm\}^{1 \times n}$ ,  $C_k^\pm \in \{\mathcal{R}^\pm\}^{1 \times n}$ ,  $C_l^\pm \in \{\mathcal{R}^\pm\}^{1 \times n}$ ,  $A_i^\pm \in \{\mathcal{R}^\pm\}^{1 \times n}$ ,  $A_j^\pm \in \{\mathcal{R}^\pm\}^{1 \times n}$ , and  $\mathcal{R}^\pm$  denotes a set of inexact numbers. An inexact number  $x^\pm$  is defined as an interval with known upper and lower bounds but unknown distribution information:

$$x^\pm = [x^-, x^+] = \{t \in x \mid x^- \leq t \leq x^+\} \quad (2)$$

where  $x^-$  and  $x^+$  are the lower and upper bounds of  $x^\pm$ , respectively. When  $x^- = x^+$ ,  $x^\pm$  becomes a deterministic number. When all parameters in model (1) are known as intervals without distribution information, this is an inexact MOP (IMOP) problem. When some of the parameters are assigned with membership functions, the model becomes a hybrid inexact fuzzy MOP (IFMOP) problem. In this study, the inexact fuzzy linear programming (IFLP) algorithm proposed by Huang *et al.* (1993) is used for dealing with uncertainties in the IFMOP. Thus, coefficients in the objective functions and the constraints' left-hand sides are handled as inexact intervals, while linear membership function are assigned to fuzzy goals for the system objectives and the constraints' right-hand sides.

### FLP Transformation and Fuzzy Goals

A fuzzy goal can be established by specifying "aspiration level" and "inferior limit" for each objective function or constraint. With 'min' operator  $\lambda^\pm$  (Zimmermann, 1978), model (1) can be transformed to:

$$\begin{aligned}
 \max \quad & \lambda^\pm & (3.a) \\
 \text{s.t.} \quad & f_k^\pm(X^\pm) \leq f_k^+ - \lambda^\pm(f_k^+ - f_k^-), & k = 1, 2, \dots, p, & (3.b) \\
 & f_l^\pm(X^\pm) \geq f_l^- + \lambda^\pm(f_l^+ - f_l^-), & l = p+1, p+2, \dots, q, & (3.c) \\
 & A_i^\pm X^\pm \leq b_i^+ - \lambda^\pm(b_i^+ - b_i^-), & i = 1, 2, \dots, m, & (3.d) \\
 & A_j^\pm X^\pm \geq b_j^- + \lambda^\pm(b_j^+ - b_j^-), & j = m+1, m+2, \dots, n, & (3.e) \\
 & X^\pm \geq 0, & & (3.f) \\
 & 0 \leq \lambda^\pm \leq 1. & & (3.g)
 \end{aligned}$$

The concepts of "individual optima" and "worst justifiable solution" can be applied to determine fuzzy goals ("aspiration level" ( $f_k^+$  and  $f_l^-$ ) and "inferior limit" ( $f_k^-$  and  $f_l^+$ )) for objective functions in model (3) before interacting with decision makers. Thus, further modifications of the goals would be within their two bounds, unless any system parameters are significantly changed. A convenient way for obtaining the "individual optima" and "worst justifiable solution" is to construct a payoff table (Cohen, 1978). Each individual problem could be solved by the inexact linear programming (ILP) method (Huang *et al.*, 1992) with one set of solution being inexact numbers. The  $b_i^+$ ,  $b_j^+$ ,  $b_i^-$  and  $b_j^-$  values are the upper and lower bounds of "tolerance intervals" for constraints (3.d) and (3.e).

**ILP Transformation**

Due to the multiobjective feature of the study problem, interactive relationships between model parameters and decision variables might become much more complicated compared with single objective problems. This would bring about difficulties in transforming model (3) to its deterministic submodels.

For a specific bound of  $\lambda^\pm$ , it may not function consistently with all objective functions and constraints. For example,  $\lambda^+$  may correspond to both  $f_k^+(\mathbf{X}^+)$  in (3.b) and  $f_l^+(\mathbf{X}^+)$  in (3.c) while  $f_k^-(\mathbf{X}^+)$  and  $f_l^-(\mathbf{X}^+)$  correspond to different constraint structures (Huang, 1996). An approach to mitigate this problem is to introduce two individual operators  $\lambda_1^\pm$  and  $\lambda_2^\pm$ , where  $\lambda_1^\pm$  is for (3.b) and (3.d) with “ $\leq$ ” constraints while  $\lambda_2^\pm$  for (3.b) and (3.d) with “ $\geq$ ” constraints. Thus, we have:

$$\max \quad \lambda_1^\pm + \lambda_2^\pm \tag{4.a}$$

$$\text{s.t.} \quad f_k^\pm(\mathbf{X}^\pm) \leq f_k^\pm - \lambda_1^\pm(f_k^+ - f_k^-), \quad k = 1, 2, \dots, p, \tag{4.b}$$

$$f_l^\pm(\mathbf{X}^\pm) \geq f_l^\pm + \lambda_2^\pm(f_l^+ - f_l^-), \quad l = p+1, 2, \dots, q, \tag{4.c}$$

$$A_i^\pm \mathbf{X}^\pm \leq b_i^\pm - \lambda_1^\pm(b_i^+ - b_i^-), \quad i = 1, 2, \dots, m, \tag{4.d}$$

$$A_j^\pm \mathbf{X}^\pm \geq b_j^\pm + \lambda_2^\pm(b_j^+ - b_j^-), \quad j = m+1, m+2, \dots, n, \tag{4.e}$$

$$\mathbf{X}^\pm \geq 0 \tag{4.f}$$

$$0 \leq \lambda_1^\pm \leq 1, \tag{4.g}$$

$$0 \leq \lambda_2^\pm \leq 1. \tag{4.h}$$

When all  $f_k^+(\mathbf{X}^+)$  and  $f_l^-(\mathbf{X}^-)$  [or  $f_k^-(\mathbf{X}^+)$  and  $f_l^+(\mathbf{X}^-)$ ] correspond to a consistent bound of  $\lambda^\pm$ , only one operator is needed. This will happen only when all objective functions are to be either maximized or minimized and all objective coefficients have the same sign (positive or negative).

**IFMOP Submodels**

For an IFLP problem with single objective, the distribution of bounds (upper or lower) for its constraint’s left-hand side coefficients corresponds to the signs of coefficients in the system objective. This algorithm is applicable to multiobjective problems only when all objective functions have the same sign distribution for their coefficients, which may seldom happen in practice. Consequently, a sign decomposition (SID) method is proposed for solving the above problem. For an objective function (max or min) with both positive and negative coefficients, it can be transformed into two decomposed sub-objectives, with one of them being maximized and the other minimized. Thus, all coefficients in the decomposed sub-objective functions become positive, enabling application of the ILP algorithm.

Assume  $u$  of  $p$  minimized objective functions in (1.a) and  $v$  of  $q-p$  maximized ones in (1.b) need to be decomposed. Thus, there would be totally  $q'$  ( $= q + v$ ) minimized objective functions and  $p' - q'$  ( $= p - q + u$ ) maximized ones obtained after decomposition. Two submodels for solving the IFMOP problem defined in model (1) can be finally obtained by applying the SID as follows:

$$\max \quad \lambda_1^+ + \lambda_2^+, \tag{5.a}$$

$$\text{s.t.} \quad \sum_{s=1}^t c_{k's}^+ x_s^+ \leq f_{k'}^+ - \lambda_1^+(f_{k'}^+ - f_{k'}^-), \quad k' = 1, 2, \dots, q', \tag{5.b}$$

$$\sum_{s=1}^t c_{l's}^+ x_s^+ \geq f_{l'}^+ + \lambda_2^+(f_{l'}^+ - f_{l'}^-), \quad l' = q'+1, q'+2, \dots, p', \tag{5.c}$$

$$\sum_{s=1}^t |a_{is}| \text{Sign}(a_{is}^\pm) x_s^+ \leq b_i^+ - \lambda_1^+(b_i^+ - b_i^-), \quad i = 1, 2, \dots, m, \tag{5.d}$$

$$\sum_{s=1}^t |a_{js}| \text{Sign}(a_{js}^\pm) x_s^+ \geq b_j^+ + \lambda_2^+(b_j^+ - b_j^-), \quad j = m+1, m+2, \dots, n, \tag{5.e}$$

$$x_s^+ \geq 0, \quad s = 1, 2, \dots, t, \tag{5.f}$$

$$0 \leq \lambda_1^+ \leq 1, \tag{5.g}$$

$$\begin{aligned}
 & 0 \leq \lambda_2^+ \leq 1. & (5.h) \\
 \text{and} & & \\
 \text{max} & \lambda_1^+ + \lambda_2^+, & (6.a) \\
 \text{s.t.} & \sum_{s=1}^I c_{k_s^-} x_s^- \leq f_k^+ - \lambda_1^+(f_k^+ - f_k^-), & \forall k', & (6.b) \\
 & \sum_{s=1}^I c_{l_s^-} x_s^- \geq f_l^- + \lambda_2^-(f_l^+ - f_l^-), & \forall l', & (6.c) \\
 & \sum_{s=1}^I |a_{is}^+| \text{Sign}(a_{is}^+) x_s^- \leq b_i^+ - \lambda_1^-(b_i^+ - b_i^-), & \forall i, & (6.d) \\
 & \sum_{s=1}^I |a_{js}^+| \text{Sign}(a_{js}^+) x_s^- \geq b_j^- + \lambda_2^-(b_j^+ - b_j^-), & \forall j, & (6.e) \\
 & x_s^- \geq 0, & \forall s, & (6.f) \\
 & x_s^+ \leq x_{s, \text{opt}}^+, & \forall s, & (6.g) \\
 & 0 \leq \lambda_1^+ \leq 1, & & (6.h) \\
 & 0 \leq \lambda_2^- \leq 1. & & (6.i)
 \end{aligned}$$

Submodel (6) can also be first solved in the solution process. The detailed sequence can be determined by integrated analysis and comparison of relative priorities for different system objectives. Fuzzy goals for the decomposed sub-objectives in submodels (5) and (6) can be specified by using decision variable values at the “individual optima” and “worst justifiable solution” obtained from model (1). This method can help ensure that solutions from the submodels correspond to the objectives defined in model (1). With the above submodels, solutions for all decision variables ( $x_s^{\pm}$ ) can be obtained, while the objective function solutions ( $f_k^{\pm}$  and  $f_l^{\pm}$ ) can be obtained by using model (1) and the generated  $x_{s, \text{opt}}^{\pm}$  values.

**Interactive Process**

The IFMOP is an interactive approach for solving real-world multiobjective problems. Solution from each iterative computation should be presented to decision-makers for their feedback. The following aspects would be emphasized when evaluating the IFMOP results: (i) satisfaction to system objectives and trade-offs between them; (ii) satisfaction to model constraints related to possible failure of the system; (iii) uncertain level of the solutions (highly uncertain solutions may be of limited use for decision-making). Based on decision-makers’ satisfaction degree to the results, modification of the IFMOP can be undertaken through further interactions and communications.

APPLICATION TO WATER QUALITY MANAGEMENT PLANNING

**Overview of the Study System**

Lake Erhai is located in Yunnan Plateau of Southwestern China with an area of 250 km<sup>2</sup>. This freshwater lake plays a vital role in local economic development with its resources for water supply, agricultural irrigation, fishery, tourism, and navigation. The study area, Lake Erhai Basin, has a total area of 8,763 km<sup>2</sup>. There is a variety of economic activities in the area, including agricultural/industrial production, net-cage fish culture, forestry, tourism, and lime/brick production. Currently, many environmental problems, such as water pollution, soil erosion and ecological deterioration, exist in the system. Among them, the most pressing one is the deterioration of lake water quality.

As required by the UNEP, the socio-economic activities in the basin need to be comprehensively studied and designed in order to minimize environmental impacts over the planning time span (15 years). In this modeling study, a number of physical, biochemical and socio-economic factors were considered. Issues of biodiversity, water supply/demand, wastewater management, water quality, industrial production,

infrastructure development, and tourism activities were emphasized. Thus, an IFMOP model was formulated for the study problem. In the IFMOP modeling system, the study basin was divided into seven subareas with different environmental, economic and resource characteristics. Two time periods (1995 to 2000 and 2001 to 2010) are considered within the planning horizon. The model contains several objectives related to different socio-economic and environmental concerns. The constraints include relationships between decision variables and related environmental/resources conditions. Since water quality management is related to a number of socio-economic and ecological factors which are uncertain in their natures, application of the IFMOP seems desirable for effectively incorporating the uncertainties within the systems analysis framework.

### Modeling Formulation

Based on detailed analysis of the study system, three aspects of objectives were included in the modeling formulation, including (i) economic return, (ii) water quality and soil loss protection, and (iii) forest coverage. The water quality objective consists of three sub-objectives — minimization of nitrogen, phosphorous, and COD losses, respectively. Thus, the IFMOP contains six objectives with over 250 constraints and 200 decision variables as follows:

(i) *Economic Return Objective:*

$$\begin{aligned} \max f_1 = & \sum_{i=1}^3 \sum_{j=1}^7 \sum_{k=1}^2 (AB_{ijk}^{\pm}) AG_{ijk}^{\pm} + \sum_{i=1}^7 \sum_{j=1}^7 \sum_{k=1}^2 IN_{ijk}^{\pm} + \sum_{j=1}^7 \sum_{k=1}^2 (NB_k^{\pm}) NT_{jk}^{\pm} \\ & + \sum_{j=1}^7 \sum_{k=1}^2 (TB_k^{\pm}) TR_{jk}^{\pm} + \sum_{j=1}^7 \sum_{k=1}^2 (BB_k^{\pm}) BR_{jk}^{\pm} + \sum_{j=1}^7 \sum_{k=1}^2 (LB_k^{\pm}) LM_{jk}^{\pm} \\ & - \sum_{j=1}^7 \sum_{k=1}^2 (FC_k^{\pm}) FR_{jk}^{\pm} - \sum_{j=1}^7 (FE^{\pm}) FR_{j2}^{\pm} \end{aligned} \quad (7.a)$$

where  $AG_{ijk}^{\pm}$ ,  $IN_{ijk}^{\pm}$ ,  $NT_{jk}^{\pm}$ ,  $TR_{jk}^{\pm}$ ,  $BR_{jk}^{\pm}$ ,  $LM_{jk}^{\pm}$  and  $FR_{jk}^{\pm}$  represent agricultural land area, industrial production level, net-cage fish culture size, tourist flow, and brick and lime production levels, respectively;  $AB_{ijk}^{\pm}$ ,  $NB_k^{\pm}$ ,  $TB_k^{\pm}$ ,  $BB_k^{\pm}$ , and  $LB_k^{\pm}$  are net benefits from agriculture, net-cage fish culture, tourism, and brick and lime productions, respectively;  $FC_k^{\pm}$  and  $FE^{\pm}$  are maintenance and expansion costs for forests, respectively; symbol  $i$  is for different agricultural and industrial activities, where  $i = 1, 2, 3$  for agriculture,  $i = 1, 2, \dots, 7$  for industries; symbol  $j$  is for different sub-areas,  $j = 1, 2, \dots, 7$ ; and  $k$  is for planning periods,  $k = 1, 2$ .

(ii) *Water Quality Objectives:*

Nitrogen loss sub-objective:

$$\min f_2 = \sum_{i=1}^3 \sum_{j=1}^7 \sum_{k=1}^2 (RF_j^{\pm}) AG_{ijk}^{\pm} + \sum_{j=1}^7 \sum_{k=1}^2 (NN^{\pm}) NT_{jk}^{\pm} + \sum_{j=1}^7 \sum_{k=1}^2 (TN^{\pm}) TR_{jk}^{\pm} \quad (7.b)$$

where  $RF_j^{\pm}$ ,  $NN^{\pm}$  and  $TN^{\pm}$  are nitrogen losses from agricultural land, net-cage fish culture and tourist activities, respectively.

Phosphorous loss sub-objective:

$$\min f_3 = \sum_{i=1}^3 \sum_{j=1}^7 \sum_{k=1}^2 (RF_j^{\pm}) AG_{ijk}^{\pm} + \sum_{j=1}^7 \sum_{k=1}^2 (NP^{\pm}) NT_{jk}^{\pm} + \sum_{j=1}^7 \sum_{k=1}^2 (TP^{\pm}) TR_{jk}^{\pm} \quad (7.c)$$

where  $RF_j^{\pm}$ ,  $NP^{\pm}$  and  $TP^{\pm}$  are phosphorous losses from agricultural land, net-cage fish culture and tourist activities, respectively.

COD loss sub-objective:

$$\min f_4 = \sum_{i=1}^7 \sum_{j=1}^7 \sum_{k=1}^2 (IC_{ik}^{\pm}) IN_{ijk}^{\pm} + \sum_{j=1}^7 \sum_{k=1}^2 (TC^{\pm}) TR_{jk}^{\pm} \quad (7.d)$$

where  $IC_{ik}^{\pm}$  and  $TC^{\pm}$  are COD losses from industrial and tourist activities, respectively.

Soil loss sub-objective:

$$\begin{aligned} \min f_5 = & \sum_{i=1}^3 \sum_{j=1}^7 \sum_{k=1}^2 (AS_{ijk}^{\pm}) AG_{ijk}^{\pm} + \sum_{j=1}^7 \sum_{k=1}^2 (FS_k^{\pm}) FR_{jk}^{\pm} + \sum_{j=1}^7 \sum_{k=1}^2 BR_{jk}^{\pm} \\ & + \sum_{j=1}^7 \sum_{k=1}^2 (LS_k^{\pm}) LM_{jk}^{\pm} \end{aligned} \quad (7.e)$$

where  $AS_{ijk}^{\pm}$ ,  $FS_k^{\pm}$ ,  $BS_k^{\pm}$ , and  $LS_k^{\pm}$  are soil losses from agricultural land, forest land, brick production, and lime kiln, respectively.

(iii) *Forest Coverage Objective:*

$$\max f_6 = \sum_{j=1}^7 \sum_{k=1}^2 FR_{jk}^{\pm} \quad (7.f)$$

where  $FR_{jk}^{\pm}$  is forest cover in subarea  $j$  during period  $k$ .

In the IFMOP model, there are a number of constraints related to soil and pollutant losses, environmental regulations, resources availability, forest cover, and many other environmental and ecological concerns. The following is one of the constraints related to water resources availability:

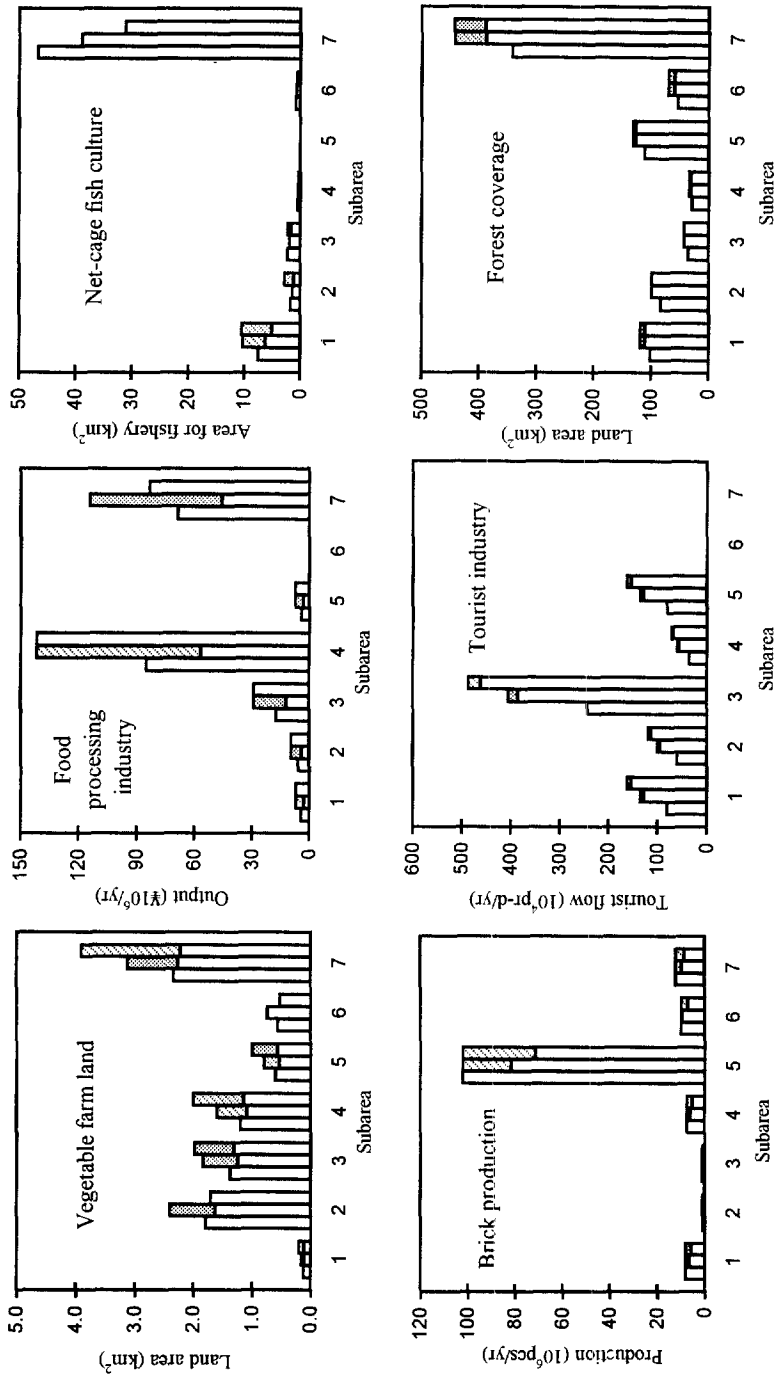
$$\begin{aligned} \sum_{i=1}^3 \sum_{j=1}^7 (WA_i^{\pm}) AG_{ijk}^{\pm} + \sum_{i=1}^7 \sum_{j=1}^7 (WI_{ik}^{\pm}) IN_{ijk}^{\pm} + \sum_{j=1}^7 (WT_k^{\pm}) TR_{jk}^{\pm} \\ + \sum_{j=1}^7 (WB^{\pm}) BR_{jk}^{\pm} \leq CWS_k^{\pm} \quad k=1, 2 \end{aligned} \quad (7.g)$$

where  $WA_i^{\pm}$ ,  $WB^{\pm}$ ,  $WI_{ik}^{\pm}$  and  $WT_k^{\pm}$  represent water demands for agricultural, industrial, tourist and brick-production activities, respectively; and  $CWS_k^{\pm}$  is the amount of water resources available in period  $k$ . The detailed description of all model constraints were provided in Huang and Wu (1996).

## Results and Discussion

The proposed interactive IFMOP solution method proceeded with frequent interactions with local authorities in the study area. Several scenarios were considered which correspond to different trade-offs between the conflicting objectives. In this paper, a scenario emphasizing water quality objectives is provided and interpreted. Table 1 shows solutions to the six objective functions under this scenario, with their initial fuzzy goals obtained from payoff computation. It is indicated that the conflict between environmental and economic objectives is obvious (i.e. water quality (in terms of COD) in the lake has a strong conflict with industrial production which contributes significantly to the local economy). Thus, an improvement in water quality can be achieved through a reduction of industrial production.

Solutions to decision variables under this scenario are graphically depicted in Figure 1. They provide desired schemes for a number of system activities in different subareas/periods. Inheriting uncertain natures of the model inputs, the majority of solutions present as inexact values which provide decision-makers with a flexible decision space. Generally, the modeling results would provide scientific bases for the formulation of policies/strategies regarding regional socio-economic development and environmental protection. For example, the tourist industry should keep growing due to its high economic efficiency while net-cage fish culture should be controlled due to its significant contribution to lake water pollution. Also, to guarantee



1) From left to right, the three columns for each subarea are for existing pattern, solutions for period 1 and 2, respectively.  
 2) The top of shaded area corresponds to the upper bound solution, and the bottom to the lower bound.

Figure 1. IFMOP solutions for selected activities in the study system

environmental objectives are met, a number of other economic activities, such as leather industry and brick/lime production, should be limited. Agriculture will keep being developed since it is the major economic sector in the area. However, the pattern and distribution for three types of activities (i.e. paddy land, dry soil, and vegetable land) need to be adjusted according to the IFMOP outputs. The forest cover should be maintained or increased from environmental and ecological points of view.

Table 1. Solutions to objective functions

Objective function	Inexact solution	Fuzzy goal*
Economic benefit, ¥10,000	[3,418,614 - 4,816,598]	[1,676,694 - 5,072,856]
Nitrogen loss, tonne	[7,839 - 10,067]	[5,867 - 11,106]
Phosphorous loss, tonne	[1,292 - 1,658]	[966 - 1,830]
COD loss, tonne	[251,129 - 348,815]	[141,800 - 598,871]
Forest coverage, km <sup>2</sup>	[1,712 - 1,875]	[1,508 - 1,905]
Soil loss, tonne	[12,311,270 - 13,377,654]	[9,278,538 - 14,391,181]

\* The fuzzy goals are from "aspiration levels" and "inferior limits" of the objective functions as described in model (3).

#### CONCLUDING REMARKS

In this paper, a hybrid inexact-fuzzy approach was proposed for solving multiobjective environmental decision-making problems under uncertainty. The method is a significant advance based on single objective inexact programming. It also improves upon the previous MOP methods with advantages in data availability, solution algorithm, and results interpretation. Multiobjective, uncertain and interactive features of a variety of system components are tackled jointly within an integrated optimization framework. Application of the IFMOP to the case study in the Lake Erhai Basin indicated that the method inherits advantages of the inexact programming methods. It allows uncertainties and decision-makers' aspirations to be effectively communicated into the modeling process. The proposed interactive solution approach can assure desired compromises are obtained. The interface for obtaining feedback from decision-makers is straightforward and explicit. The approach also has reasonable computational requirements due to the simplicity of its intermediate submodels. The generated inexact solutions and the relevant alternatives are favored by decision-makers due to their increased flexibility and applicability in determining the final decision schemes. The successful application of the IFMOP demonstrates that the method is an effective tool for solving real-world decision-making problems.

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