aries can be clearly identified along which this condition is satisfied. In such cases a solution could be obtained with the modified mixing-length hypothesis, while the k-w model would be incapable of giving a solution at all.

It may well be that although the k-w model is a more physically realistic one, its relative inflexibility will render it difficult to apply in some problems, where the rather crude but simple model may give results of adequate accuracy.

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DISCUSSION

H. J. Herring[2]

This paper presents a body of data which will be of considerable interest to those studying turbulence modelling as well as design engineers. Methods of incorporating the influence of free-stream turbulence on mixing have not received sufficient attention and possibly these results will encourage greater efforts in this important area.

The authors' theoretical predictions of their data represent an interesting step in this direction. However, their choice of the form for the Modified Mixing Length Hypothesis is puzzling. The modification is expressed as products of the relative turbulence levels, $(\sqrt{\overline{u'^2}}/U)_I$, $(\sqrt{\overline{u'^2}}/U)_E$, the jet velocity ratio, U_E/U_I, and the local velocity, U, which are reference frame dependent, both individually and also in the present combinations.

Clearly the turbulence structure of the flow and therefore the effective viscosity must be Galilean invariant. Moreover, assuming that the wake of the dividing wall between the ducts has been removed by suction, the upstream history of the ducts has been eliminated. Therefore, even the relative velocity between the jets and the duct walls is immaterial. The only remaining way the mean velocity could enter the mixing length expression is as a velocity difference between the two free-streams, $(U_I - U_E)$, or better yet, as a local velocity gradient, $\partial U/\partial y$, as in the conventional Boussinesq hypothesis.

A satisfactory form for the Mixing Length Modification can only be determined by comparing predictions with data. One might begin by trying the straightforward assumption,

$$\nu_{eff} = l_m^2 \left|\frac{\partial U}{\partial y}\right| + A l_m \left[(\sqrt{\overline{u'^2}})_I \left(\frac{Y_E - Y}{Y_E - Y_I} \right) + (\sqrt{\overline{u'^2}})_E \left(\frac{Y - Y_I}{Y_E - Y_I} \right) \right] ,$$

where the modification term is independent of the mean velocity field altogether. An alternate form which involves the mean velocity is,

$$\nu_{eff} = l_m^2 \left|\frac{\partial U}{\partial y}\right| + \frac{A}{|\partial U/\partial y|} \left[(\overline{u'^2})_I \left(\frac{Y_E - Y}{Y_E - Y_I} \right) + (\overline{u'^2})_E \left(\frac{Y - Y_I}{Y_E - Y_I} \right) \right]$$

In this case the modification term does not depend explicitly on the length scale. Of course, there are many other possibilities which combine the features of these two expressions.

[2]President, Dynalysis of Princeton, Princeton, N. J.

Authors' Closure

We would like to thank Dr. Herring for his comments and suggestions. We agree with his argument that any modification of the standard Prandtl mixing-length formulation *ought* to be expressed in terms of quantities which are independent of the frame of reference but have so far not found it possible to satisfy this condition and simultaneously achieve satisfactory agreement with experiment.

Dr. Herring's first proposal is, in fact, quite similar to our own version, apart from the factor $(B + U_E/U_I)$ which appears in the latter. This was introduced on entirely empirical grounds. Preliminary calculations have shown that at high values of U_E/U_I the Herring modification underpredicts the effect of free-stream turbulence on growth rate, but gives good agreement with experimental profiles of velocity and turbulence intensity.

The second of Dr. Herring's suggestions is much less satisfactory. The presence of $\partial U/\partial y$ in the denominator leads to unrealistically high values of effective viscosity near the edges of the mixing layer, which in turn produce very cramped velocity profiles.

The subsonic mixing layer provides a convenient vehicle for the study of turbulence. Further work on turbulence models of various levels of sophistication should be encouraged. There is also a need for more experimental work. Our results have shown that free-stream conditions should always be included in the measurements, since they have a significant effect on the development of the flow.

