Table 1 Magnitudes of Different Parameters

No.	Description of equations	Parameter	Chamber pressure = 1 atm, Injection pressures: 100 atm	125 atm	150 atm	175 atm	200 atm
1	$\bar{u}/u_m = e^{-a_1(r/x)^{1.5}}$	a_1:	273.0	250.0	230.0	210.0	200.0
2	$\bar{f}/f_m = e^{-a_2(r/x)^{1.5}}$	a_2:	220.0	187.5	167.0	147.0	137.5
3	$\beta = a_1/a_2$	β:	1.24	1.34	1.38	1.425	1.450
4	$r_{1/2u} = \lambda_1 x$	λ_1:	0.01875	0.0197	0.0208	0.0222	0.0230
5	$r_{1/2f} = \lambda_2 x$	λ_2:	0.0216	0.0239	0.0256	0.0280	0.0293
6	$\bar{u}_m/u_0 = C_1 D/x$	C_1:	20.0	20.0	20.0	20.0	20.0
7	$\bar{f}_m/f_0 = C_2 D/x$	C_2:	1.4	1.4	1.4	1.4	1.4

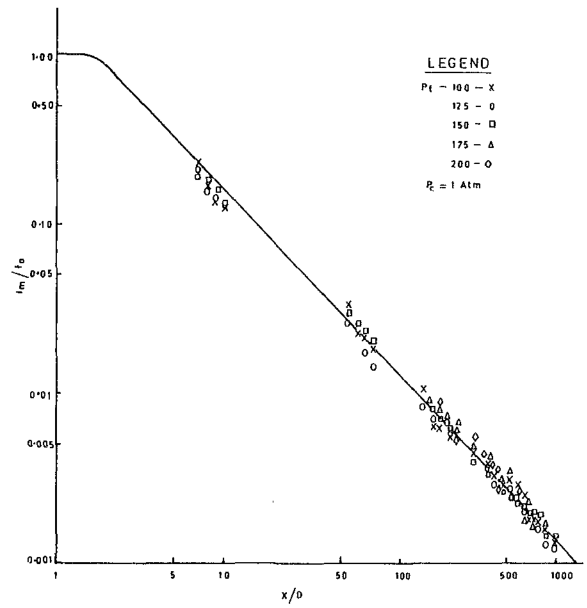

Fig. 19 Log-log plot of decay of concentration of mass of fuel on the axis of the spray

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DISCUSSION

R. A. Moreau[2]

This paper addresses one approach which can be pursued in an effort to analytically describe the spacial development of mass flow and velocity within a fuel spray. That approach is to represent the spray as a turbulent plane jet. The authors chose here an analytical representation which is prevalent in work previously done on jets of air flowing into air. Specifically, the equations given for momentum and mass transport are reduced to analogous forms by the simplifying assumptions chosen. The equations are further reduced by approximating the turbulent transport phenomenon with the Boussenesq substitution (constant ratio for the coefficient of the turbulent velocity exchange). This, in turn, enabled derivation of a ratio of the diffusivities of mass to momentum exactly as predicted by Van Der Hegge Zijnen [23].[3] However, the question arises as to the validity of such a simple approach to the physics of turbulence. Might the authors consider another, more sophisticated expression to approximate the turbulent exchange? Schlichting [24] presents two such expressions which model the physics of turbulence to a greater degree than that presented here. This analytical evaluation serves merely as a means to calculate the Schmidt number for fuel spraying into ambient air, a parameter which is a consequence of the data rather than the purpose of the experiment.

In my opinion, the major effort of this paper is clearly experimental. The data taken are used for empirical descriptions of the mass and momentum profiles. The parameters describing those profiles were determined by the authors from curve fits of data. But the authors have not identified their curve fitting techniques. It appears that several of the curves do *not* fit the data! (Figs. 7 and 9.) Of course, changing any parametric values by changing curve fits alters the re-

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[3] Numbers in brackets designate Additional References at end of discussion.

sults. The question then arises, why did the authors choose the values of $a_1 = 200$ and $a_2 = 137.5$ as shown in Figs. 7 and 9, respectively? There appear to be other more reasonable curve fits which would change the results given.

This work does enable a technique whereby simple analytical expressions are found to approximate the velocity and mass profiles of a fuel spray. Some generality is lost by the fact that those descriptive parameters are direct functions of both supply and receiver pressure.

The actual diesel injection process is a widely varying pressure event.

If it is to be modeled by this technique, further experimental determination of the parameters would have to be made. However, the authors have done a good job in extending experimental turbulent jet analysis of similar fluids to that including the effects of large density ratios between the fluids. I feel they have successfully expressed a first approximation to a very difficult problem.

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Authors' Closure

The authors are priveleged to have the discussion of Moreau. This paper deals with the basic idea of representing the spray as a round turbulent jet. This trend has already been established by the works of Abramovich [8], Lyshevskiy [9], Sitkei [10], and Hakki [11]. We wanted to further probe into the problem and see to what extent the similarity profiles of mass and momentum of turbulent jets are to be modified from the point of view of the complicated nature of the fuel spray.

Moreau raised a doubt whether the Boussenesq substitution for approximating the turbulent transport phenomenon is sufficient to explain the physics of turbulence in this case. We feel that though there are more sophisticated expressions to approximate the turbulent exchange phenomenon, there is no need to complicate the problem at this stage, as the one employed by us is capable of giving reasonably good results. The Schmidt number, evaluated from the experimental data, will serve as a useful parameter for bringing out the comparison between the fuel spray and that of turbulent jet with negligible density variations.

He raised another point regarding the validity of curve fitting especially in the Figs. 7 and 9. We would like to stress here that no sophisticated curve fitting techniques are employed as the trend of the experimental points evidently shows the type of curve that could be fitted in. Once again it is to be brought out here that our main purpose is not to obtain complicated expressions for the profiles by employing sophisticated curve fitting techniques, but to see the extent to which the similarity profiles of turbulent jets are to be modified. Hence the structure of the models describing the profiles of mass and momentum is not disturbed. With this aim in background, average values of $a_1 = 200$ and $a_2 = 137.5$ are arrived at from the experimental points.

Moreau raised another very pertinent point that the descriptive parameters are direct functions of supply and receiver pressures. It is true that in the experiments reported by us the receiver pressure was at ambient conditions all through. We would like to mention that further experimentation was carried out by us later, varying the receiver pressure as well, and this work is being reported in our next paper as a follow- up work shortly.

We thank Moreau for his compliments on the work done by us and we appreciate the keen interest he has taken in our paper.