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4 The result you obtain is clearly wrong. Only in rare circumstances where there are no reflections is the phase velocity independent of frequency.

You have made the mistake of ignoring the next higher-order term. Include that term and substitute the expansion for v_p and do the algebra necessary for collecting like order terms. You will obtain my results for v_0, v_1, and v_2 from setting to zero the coefficients of ω^2, ω^3, and ω^4.

Elastic Dispersion, Homogeneous Dispersive Media and an Application to Periodic Elastic Media[1]

G. A. Bécus.[2] After a general discussion of homogeneous dispersive media, author considers the very particular case of an isotropic periodic elastic medium with constant density and transverse sound speed but with periodically varying longitudinal sound speed. By expressing the displacement field in terms of scalar and vector potentials, the problem is reduced to two wave equations, only one of which, that governing the dilatational motion, having periodically varying speed. This allows author to use previously obtained results [1].

Notwithstanding this discusser's reservations about some results in [1] (cf. the discussion of that paper by this discusser) some remarks about the foregoing paper are in order.

1 It is the very particular and very artificial nature of the inhomogeneities (affecting only longitudinal sound speed) which allows the successful use of potential methods. If the transverse sound speed and/or the density are also varying with position then the equation of motion cannot be separated into two equations, one for each potential.

[1] By G. N. Balanis, and published in the June, 1978, issue of the ASME JOURNAL OF APPLIED MECHANICS, Vol. 45, No. 2, pp. 337–342.

[2] Department of Engineering Science, University of Cincinnati, Cincinnati, Ohio 45221.

2 The statement to the effect that wave propagation in a three-dimensional periodic inhomogeneous medium has not been solved yet needs some qualification. Indeed Kohn's work [2] provides a low frequency approximation for such a problem. Furthermore, another approximation can be obtained using the methods and concepts of homogenization [3, 4 and the references therein].

References

1 Balanis, G. N., "Low Frequency Bloch Waves for Wave Equations Whose Speed is a Deterministic, or Randomlike, Periodic Function," ASME JOURNAL OF APPLIED MECHANICS, Vol. 45, 1978, pp. 331–336.

2 Kohn, W., "Propagation of Low Frequency Elastic Disturbances in a Three-Dimensional Composite Material," ASME JOURNAL OF APPLIED MECHANICS, Vol. 42, 1975, pp. 159–164.

3 Babuška, I., "Homogenization and Its Application. Mathematical and Computational Problems," *Numerical Solution of Partial Differential Equations—III, Synspade 1975*, Hubbard, B., ed., Academic Press, Inc., New York, 1976, pp. 89–116.

4 Lions, J. L., *Sur Quelques Questions d'Analyse, de Mécanique et de Contrôle Optimal*, Presses de l'U. de Montréal, Montréal, 1976, Chapter 3.

Author's Closure

Again, the author would like to thank Professor Bècus for his interest in the author's work. The following are the author's comments.

1 The sentence "If the transverse sound speed and/or . . . , for each potential" requires proof. The fact that we believe that homogeneous and isotropic elastic media represent a long wavelength model to some periodic media indicates a proof may be impossible. Since in the isotropic elastic medium the longitudinal and shear potentials are uncoupled, why shouldn't there exist some media whose three material constants have three-dimensional periodicities for which my method would work at the long wavelength limit? A medium whose unit cell is a cube inside which is centered another cube filled with different material may be such a structure. An exact analytical solution to this problem could be useful.

2 I am not aware of any exact analytical solutions to wave propagation in an elastic medium for which all three material constants have three-dimensional periodic inhomogeneities.

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