needed. This is so because setting $x = 0$ or $y = 0$ in equation (34) gives an integrand of the same form as the integrand in equation (11). The integration proceeds as for equation (37) and easily leads to the conclusion that v_j^Q is continuous over the planes $x = 0$ and $y = 0$.

For a more complete list of references include: Thompson, W. T., and Kobori, T., "Dynamic Compliance of Rectangular Foundations on an Elastic Half Space," Journal of Applied Mechanics, Vol. 30, Trans. ASME, Vol. 80, Series E, 1963, pp. 579–584.

Misprints in the paper are as follows:

1 The left side of equation (7*a*) should read $\tilde{f}(x, y, z, p)$.

2 In the definition of $s_3(x, z, t)$ appearing below Fig. 3, $|tx|$ should be replaced by $t|x|$.

3 The left side of equation (28) should read $v_{z\alpha}{}^l(x, z, t)$.

Approximate Roots of Flügge's Characteristic Equation for the Closed Cylindrical Shell[1]

J. H. WILLIAMS, JR.[2] In a recent Note by Colbourne [1][3] the frequently discussed [1–5] accuracy of the Donnell equations was examined with some useful insight but unfortunately an incorrect conclusion was drawn. The purpose of these comments is to clarify the apparent discrepancies between Professor Colbourne's inquiry and the earlier studies of Hoff [2] and Morley [3] in the case of a closed cylinder subject to applied loads at edges where x = const.

In cases where low harmonic deformation predominates, Professor Colbourne's approximate analysis contributes usefully to the previous discussions. However, contrary to one of his conclusions, the roots of the Donnell equations do behave asymptotically as those of the Flügge equations as the harmonic number m increases. Professor Colbourne has apparently used the word "result" to mean the "imaginary parts of the roots" instead of the "roots" themselves. It seems useful to observe the relative magnitudes as well as the relative errors of the α_{Dj} and the β_{Dj} for increasing m; see Table 1.

Table 1 Approximate magnitudes of ratios (α_{Dj}/β_{Dj}) of Donnell equations

a/h \ m	50	100	150	200
20	17	35	52	70
50	11	22	33	44
100	8	16	24	31

So, in spite of the fact that the percentage error in the β_{Dj} generally increases with increasing m, the increasing magnitude of the (α_{Dj}/β_{Dj}) ratios and the decreasing percentage error in the α_{Dj} could more than compensate. This is, in fact, what occurs and it results in increasingly accurate roots.

References

1 Colbourne, J. R., "Approximate Roots of Flügge's Characteristic Equation for the Closed Cylindrical Shell," Journal of Applied Mechanics, Vol. 36, No. 2, Trans. ASME, Vol. 91, Series E, 1969, pp. 352–355.

2 Hoff, N. J., "The Accuracy of Donnell's Equations," Journal of Applied Mechanics, Vol. 22, No. 3, Trans. ASME, Vol. 77, 1955, pp. 329–334.

3 Morley, L. S. D., "An Improvement of Donnell's Approximation for Thin-Walled Circular Cylinders," *Quarterly Journal of Mechanics and Applied Mathematics*, Vol. 12, 1959, p. 89.

4 Rish, R. F., "The Roots of Flügge's Cylindrical Shell Equation," Journal of Applied Mechanics, Vol. 35, Trans. ASME, Vol. 90, Series E, 1968, pp. 172–173.

5 Kempner, J., "Remarks on Donnell's Equations," Journal of Applied Mechanics, Trans. ASME, Vol. 77, 1955, pp. 117–118.

Author's Closure

The author would like to thank J. H. Williams for his comments on the relative size of the real and imaginary parts of the roots of the cylindrical shell characteristic equation. It is quite correct that the magnitude of the real part is several times that of the imaginary part, and that their ratio grows with increasing harmonic number m. This does not mean, however, that the accuracy of the imaginary part is less important than that of the real part. The value of any displacement at points along a generator is given by a damped sine wave, in which the damping factor depends on the real part of the root, and the wavelength on the imaginary part. Since the two parts of the roots have quite distinct physical interpretations, the accuracy of the shell analysis depends on that of both the real and the imaginary parts of the characteristic equation roots.

Analysis of Heterogeneous Anisotropic Plates[1]

W. Y. CHUNG.[2] The authors developed a general theory of anisotropic laminated plates, in which they made a conclusion for authors' reference [1] by stating that "that work firmly established that a previous treatment of the problem by authors' reference [2], in which he concluded that the laminate behaved as a homogeneous orthotropic plate, was incorrect." In view of the authors' reference [2], in which Professor Smith treated a two-ply plate as a classical theory of plane stress under bending, and based on such formulation, it is understood that the middle plane of the plate doesn't undergo any deformation after loading. Thus there is no stretching (or $u^0\ (x, y) = v^0\ (x, y) = 0$). It seems that the correctness of the solution depends on how properly and consistently the problem has been solved, under given assumptions, rather than the order of approximation used in the formulation of the problem (or system of equations). For instance, the classical flexural vibration theory of beams cannot be said to be incorrect because of the existence of Timoshenko's beam theory. The latter theory indeed gives a better result in some applications. The authors' large deflection theory can serve a similar purpose if the problem is solved properly.

In the authors' section, "Linear Analysis of Coupled Plates," the authors tried to solve the displacement components of the plate from a system of linear differential equations (20)–(22) or

[1] By J. R. Colbourne, published in the June, 1969, issue of the Journal of Applied Mechanics, Vol. 36, Trans. ASME, Vol. 91, Series E, pp. 352–355.

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[3] Numbers in brackets designate References at end of Discussion.

[1] By J. M. Whitney and A. W. Leissa, published in the June, 1969, issue of the Journal of Applied Mechanics, Vol. 36, Trans. ASME, Vol. 91, Series E, pp. 261–266.

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