DISCUSSION

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Professor Johnson has an interesting idea in this paper where he attempts to define and discuss the concepts of degree of controllability and observability. One might think of terms such as degree of stability and relative stability to get a feel for the idea of Professor Johnson.

Following the suggestion of Kalman, et al., the author has chosen to concentrate on the "controllability matrix," in particular its determinant.

He has worked out in detail 2 examples to illustrate his ideas.

While the formulation of the problem and its solution seem straightforward, the application of these results is disappointing. Perhaps the author should not be criticized for this because he has chosen to provide the theory behind such problems. Hopefully, applications come later.

The discussor will not venture to forecast the amount of application-type papers that Professor Johnson's paper will initiate. Surely, in the academic circles, you will hear extension of these concepts to stochastic versions and so on.

Professor Johnson writes well and has presented his ideas clearly. The only point of improvement may be, don't have 27 footnotes in a paper that is 28 pages long.

Authors' Closure

The author would like to thank Professor Kumar for his interesting comments on this paper. It is true that the present paper is devoted primarily to development of the theory and does not include concrete applications of the theory to practical problems. However, numerous potential areas of application are cited throughout the text (see, for instance, the Introduction, the section on Observability, and the Conclusions). The problem of optimal sensor location is a particularly promising domain of application.

The notion of a quality of complete controllability and observability was apparently first discussed in 1961 by Kalman, et al., in [5]. This idea was later discussed by Kreindler and Sarachik in [8] and, more recently, has been studied by (see Additional References) Brown [36], Monzingo [37], Tomovic [38], Simon and Mitter [39], and Kalman [40], p. 39, Remark (3.11).

In addition to providing guidelines for the actual construction of physical control systems, the results of this paper also have utility in certain control theoretic applications. Consider, for example, the problem of characterizing the condition that a given pair (A, f) is not completely controllable. This condition corresponds to

$$\det H = 0$$

(1c)

and from (20), (53) is characterized by the presence of either (or both) of the following conditions: (i) the matrix A is not cyclic (p < n) or (ii)

$$k \prod (f_j c) = 0$$

(2c)

In other words, (2c) provides a simple proof of the following Criteria for Complete Controllability: Complete controllability of the nonderogatory system (13a) is lost if, and only if, the last element f_j c in at least one subvector of (48) is zero. This important result was first established in 1962 by Y. C. Ho [41] who used a somewhat more complicated method of proof. (See also the recent and more general results of Chen and Desoer [42].) The corresponding n.a.s.c. for loss of complete observability of the system (15), which apparently has not previously been published, is easily obtained from (81) (82) as (assuming p > n)

$$c_j = 0, \text{for some } j \in \{1, 2, \ldots, k\}. \quad (3c)$$

The result (3c) is, of course, simply the dual of the previous controllability criterion (see footnotes 11 and 19).

A question that frequently arises in controllability and observability studies (as well as in pure mathematical questions regarding the generation of A-cyclic subspaces) is the following: If the given n x n matrix A is known to be controllable (observable) for some vector f(c), find a general expression for a vector f* = F(A)c* = c* (A)bc*) which is guaranteed to always make the pair (A, f*) completely controllable [(A, c*) completely observable]. The corresponding question in the theory of A-cyclic subspaces is: Assuming that A is cyclic, find an n-vector b such that the orbit {b, Ab, A^2b, \ldots, A^{n-1}b} is guaranteed to span E. The procedure usually suggested (in mathematics texts) for finding a suitable vector b is, "... choose almost any bc*". The results of the present paper offer some improvement on this procedure since one can choose f* = f(c* = c*) where f* and c* are given by the solution of (55) and (82). Moreover, as demonstrated in the Examples section, explicit general expressions for f* and c* can be obtained for certain classes of matrices A.

The assumption that the matrix A is cyclic (nonderogatory) is absolutely essential for the prevention of trivial optimal solutions f*, c* for the scalar-input, scalar-output system (13). On the other hand, when u and/or y are vectors (the constant case of (2)), it is possible that trivial solutions for the columnas of the optimal matrix B* (rows of the optimal matrix B*) are prevented even when A is not cyclic. Thus, for the more general problem of maximizing the absolute value of det HH' (det RR'), as described in the "extension of Results" section of the paper, it is permissible (in certain cases) for two or more Jordan blocks (J, J) in (39) to share the same eigenvalue λ = λ (compare with footnote 10). For example, suppose A has the (very) noncyclic structure

$$A = \text{diag.} (λ_1, λ_2, \ldots, λ) = M, \text{for } n \times n \text{ identity matrix} \quad (4c)$$

$$λ = \text{real scalar}$$

in the constant case of (2). Then, it is readily established that

$$\det (HH') = (1 + \lambda^2 + \lambda^4 + \ldots + \lambda^{2n-1}) |det (FF')|$$

so that

$$\text{rank } H = n \Rightarrow \text{rank } F = n \Rightarrow r = n \text{ and } \det F \neq 0 \quad (5c)$$

and therefore |det HH'|, for this case, is maximized > 0 by maximizing |det F| > 0.

Additional References


