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EVALUATION OF PUMPING TESTS IN PARTIALLY PENETRATING WELLS

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Pumping tests in wells that partially penetrate aquifers have been evaluated by means of type curves and a simplified solution of nonsteady flow. The influence of skin effect on pumping tests in the partially penetrating wells and the application of the method to unconfined aquifers are discussed.

1. THE FLOW TO A PARTIALLY PENETRATING WELL IN AN INFINITELY DEEP AQUIFER BOUNDED ABOVE BY AN IMPERVIOUS LAYER

The flow to a partially penetrating well (called a "partial well" in the following) in an infinitely deep aquifer $\frac{l-d}{b} < 0.1$ bounded above by an impervious layer, or in an aquifer bounded above and below by an impervious layer, during a relatively short pumping time t, where

$$t < (2\,b - l - z)^2 \frac{S}{20\,b\,k} \tag{1}$$

may, according to Hantush (1961), be described by:

$$s(r,z,t) = \frac{Q}{4\pi\,k(l-d)}\,\frac{1}{2}\int_u^\infty \frac{e^{-y}}{y}\left[\operatorname{erf}\left(\frac{l-z}{r}\sqrt{y}\right) + \operatorname{erf}\left(\frac{l+z}{r}\sqrt{y}\right) - \operatorname{erf}\left(\frac{d+z}{r}\sqrt{y}\right) - \operatorname{erf}\left(\frac{d-z}{r}\sqrt{y}\right)\right] dy \tag{2}$$

The notations appear from Fig. 1, erf(X) is the error function, and

$$\int_u^\infty \frac{e^{-y}}{y}\,dy \equiv W(u) \text{ is the Theis well function,}$$

$$\int_u^\infty \frac{e^{-y}}{y}\left[\operatorname{erf}\left(\frac{l-z}{r}\sqrt{y}\right)\right] dy \equiv M(u,\beta), \tag{3}$$

$$\beta_1 \equiv \frac{l-z}{r},\ \beta_2 \equiv \frac{l+z}{r},\ \beta_3 \equiv \frac{d+z}{r},\ \beta_4 \equiv \frac{d-z}{r} \tag{4}$$

$$u \equiv \frac{r^2 S_s}{4\,k\,t},\ S_s \equiv \frac{S}{b}\ [m^{-1}]. \tag{5}$$

Equation (2) is valid on the assumption that the aquifer is homogeneous and isotropic in regard to permeability and specific storage coefficient S_s (defined as the volume of water released per unit volume of the aquifer per unit change of drawdown). Furthermore, it is assumed that the piezometric water level is

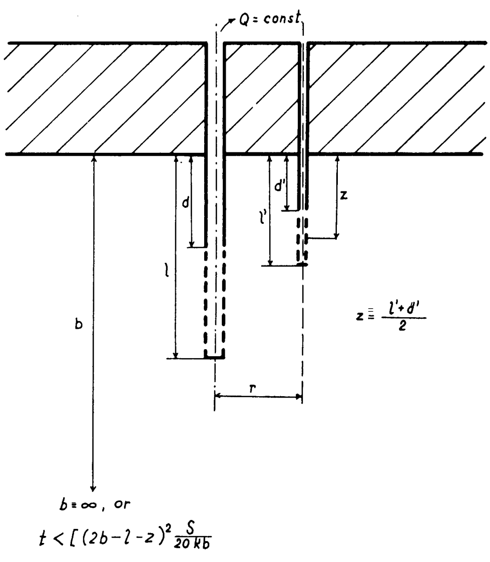

Fig. 1.
Flow to a partially penetrating well in an infinitely deep aquifer bounded above by an impervious layer, or in an artesian aquifer during a relatively short pumping time.

horizontal, and that pumping is at a constant rate. The storage in the well can be neglected. The aquifer may be infinite or finite; however, pumping time is relatively short, in order that the hydraulic impulse due to pumping should not reach the impervious bottom. The open lower part of the observation tube is relatively short, and the *z* coordinate represents the value in the middle of this open portion (Fig. 1).

By using (3) and (4), equation (2) is written as

$$s = \frac{Q}{4\pi k(l-d)} \frac{1}{2} [M(u,\beta_1) + M(u,\beta_2) - M(u,\beta_3) - M(u,\beta_4)] \qquad (6)$$

Since $\mathrm{erf}(-X) = -\mathrm{erf}(X)$, $M(u,-\beta) = -M(u,\beta)$ is valid. (7)

Values of the function M (u,β) are given in Hantush's tables within a sufficient range (Hantush 1961, 1964).

$$\text{When } u > \frac{5}{\beta^2},\ M(u,\beta) \approx W(u) \text{ is valid.} \qquad (8)$$

Equation (2) is the equation of flow towards the partial well in an infinitely deep aquifer (or in a bounded aquifer for a short pumping time, eq. (1) and $r < 2b$) confined above by an impervious top layer. The equation is a solution of the drawdown at the point determined by coordinates *r* and *z* (Fig. 2).

Assuming a uniform flow along the screen in a pumping well, it can be shown (Hantush 1961) that the drawdown may be calculated by substituting $r = r_0$ (the effective well radius) and $z = 1/2\ (l + d)$ or *z* equal the depth representing half of the well discharge measured, for instance, by the rotating meter, hot-wire anemometer, etc., in equation (6).

The application of this equation to the evaluation of the drawdown at an observation well with a long screen may, however, often lead to considerable

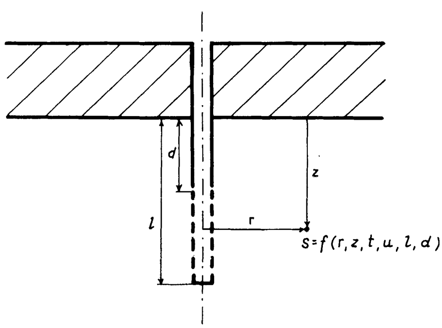

Fig. 2.
Lowering of the piezometric water level due to pumping in a partially penetrating well.

errors, particularly in those cases where the screen is not symmetrically situated in the hydrodynamic flow-net. The solution is approximately valid, if $\frac{l'}{l} < 2$. If $\frac{r}{l} > 1$ and $\frac{l'}{l} < 1$, the given solution is valid with sufficient accuracy for arbitrary values of z within the limits of $l' > z > 0$. In the case of a longer screen of the observation well, a solution as indicated in Hantush (1961) may apply. Long screens for observation wells are methodically inappropriate. Their application eliminates the possibility of calculating and setting up the space hydrodynamic flow-net, which is often the main problem in aquifers of great thicknesses.

1.1. Well Screen Testing Extending Downwards into the Aquifer from an Impervious Overlying Layer

In case the well screen touches upon an impervious overlying layer (d = 0 and the drawdown is to be determined at various depths ($z \neq 0$), equation (6) is simplified to

$$s = \frac{Q}{4\pi k l} \frac{1}{2} [M(u,\beta_1) + M(u,\beta_2)] \tag{9}$$

where

$$\beta_1 = \frac{l - z}{r}, \beta_2 = \frac{l + z}{r} \tag{10}$$

If z = 0, i. e. the drawdown is to be determined at the ceiling of the impervious overlying layer, equation (6) will be further simplified to

$$s = \frac{Q}{4\pi k l} M(u,\beta) \tag{11}$$

$$\text{where } \beta = \frac{l}{r} \tag{12}$$

This solution is often taken into consideration mainly when determining the drawdown, or the required pumped amount during dewatering. Then the drawdown will depend on the u value and ratio $\frac{l}{r}$. In this case, type curves of the pumping test may be set up using coordinates of log $M(u,\beta)$ versus log $\frac{1}{u}$. For a short duration of time: $u > 5/\beta^2$, the value of the $M(u,\beta)$ function is identical

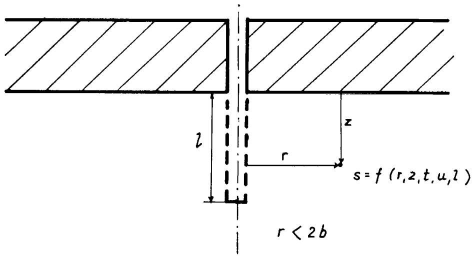

Fig. 3.
Lowering of the piezometric water level when the well screen extends downwards from an impermeable top layer.

with that of the W(u) function (equation (8) is valid), i. e. in case of great values of β, the pumping test may be evaluated according to Theis' method by using the well function W(u) (Appendix 1). The only difference is the substitution of the screen length in the calculation (equation (11)), the value of the filtration coefficient *k* and the specific storage coefficient S_s being the results.

1.2 Well Screen Detached from the Impervious Overlying Layer

In the cases where the well screen does not reach the impervious overlying layer $d \neq 0$, and the drawdown is to be determined at the top of the aquifer: $z = 0$, equation (6) may be rewritten into:

$$s = \frac{Q}{4\pi\, k\, (l-d)} [M(u,\beta_1) - M(u,\beta_2)], \qquad (13)$$

$$\text{where } \beta_1 = \frac{l}{r},\ \beta_2 = \frac{d}{r} \qquad (14)$$

From the practical point of view, it is interesting to determine the effect of screen depth on the drawdown of the piezometric level at the top of an infinitely deep aquifer. For screen lengths of 5 and 10 m, the ratio s/s_{max} between the drawdown s, resulting from a situation where the screen is detached from the overlying ($d > 0$) and the maximum drawdown s_{max}, in case the screen touches the upper stratum ($d = 0$), is calculated for various values of r and d for depth $z = 0$ (Fig. 4). This dependence can be used advantageously in optimizing the geometrical location of the well screen. A partial well with an immersed screen installed for dewatering produces a smaller drawdown than a dewatering well with a screen at the top of the aquifer. In water supply, a partial well with an immersed screen lowers the water level less than a well with its screen touching the aquifer ceiling at the same pumping rate (Fig. 4).

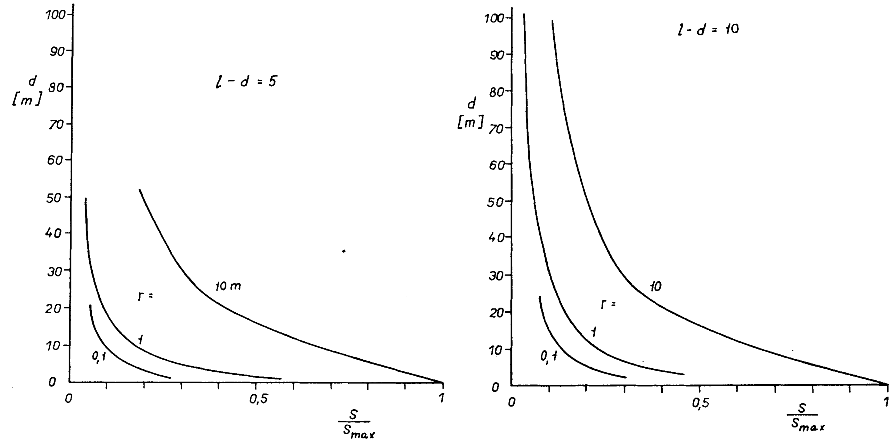

Fig. 4.
Proportional drawdown s/s_{max} for various values of r and $(l-d)$ related to depth of screen location.

1.3. Evaluation of Pumping Tests in a Partially Penetrating Well in an Artesian Infinitely Deep Aquifer

1.3.1. Evaluation of pumping tests by means of type curves.

The evaluation of pumping test includes the following steps:

- determine values of β_1, β_2, β_3, β_4 and calculate function values for various u:

$$\frac{1}{2}[M(u,\beta_1) + M(u,\beta_2) - M(u,\beta_3) - M(u,\beta_4)] = W(u,\beta_4,\beta_3,\beta_2,\beta_1) \tag{15}$$

from equation (6) or similarly from equations (9), (11), (13),

- plot these function values on log scale versus $\log \frac{1}{u}$, to obtain Theis' curve as modified by Hantush for any case of ground water flow towards the partial well,
- plot the values of drawdown versus the time of pumping tests on tracing paper on the same log scale (as in Theis' method of type curves),
- superimpose the curve on the pumping test curve, keeping the coordinates' axes parallel, to make them coincide. Select a match point and subtract its values s, t, $\frac{1}{u}$ and $W(u,\beta_1,\beta_2,\beta_3,\beta_4)$,
- substitute the values thus determined into equation (5) and (6), respectively (9), (11), (13), and calculate values of k and S_s.

If the pumping test curve deflects from the type curve towards higher drawdown values after a certain time: determine the time point of the initial deflection of its value $\frac{1}{u}$ and calculate the aquifer thickness approximately by means of equation

$$b \approx \frac{1}{2}\left(l + z + r\sqrt{5\,\frac{1}{u_{defl}}}\right) \tag{16}$$

The solution is equally valid for the pumping well and the observation well with a short screen.

1.3.2. Simplified solution.

Since

$M(u,\beta) \approx W(u)$ is valid, when $\frac{r^2S_s}{4\,k\,t} > \frac{5}{\beta^2}$

to calculate the initial portion of the graphical representation of pumping test (subh. 1.3.1), directly apply Theis' type curve

$$\frac{1}{u} = f\,[W(u)].$$

$$\text{When } \frac{r^2S_s}{4\,k\,t} \leqq 0.03 \tag{17}$$

replace function W(u) at an accuracy of 1 % by

$$W(u) \approx ln\,\frac{4\,k\,t}{S_s r^2} - 0.5772 \tag{18}$$

and continue the solution in accordance with Jacob's simplified method by linearizing the drawdown dependences on the time in a semilogarithmic graph. This solution is valid for a flow towards the partial well (for the case $d = 0$ and $z = 0$) if the following condition is satisfied:

$$0.03 > \frac{r^2S_s}{4\,k\,t} > \frac{5}{\beta^2} \tag{19}$$

From this condition follows that a solution only can be obtained for $\beta > 12.9$; $(\beta = \frac{l}{r})$.

The simplified solution is then valid for the following time interval of the pumping test:

$$\frac{r^2 \beta^2 S_s}{20k} > t > \frac{r^2 S_s}{0.12k} \qquad (20)$$

The applicability interval of the solution according to $\frac{l}{r}$ is illustrated by Fig. 5.

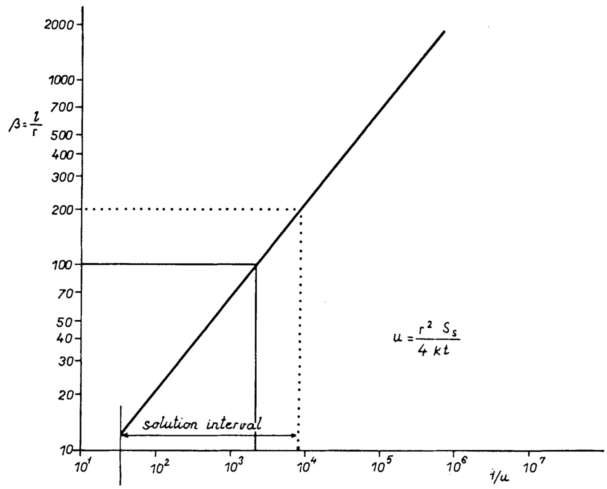

Fig. 5.
Diagrammatic representation of an interval applicability to a simplified solution.

In practice, the evaluation is carried out by plotting the respective values of drawdown *s* against log t of the pumping test (satisfying conditions (19) and (20)). The slope of the straight line thus obtained is expressed by equation

$$i = 0.1832 \frac{Q}{k\,l} = \frac{\Delta s}{\log t_2 - \log t_1}, \qquad (21)$$

from which by substituting the respective values magnitude *k* is calculated.

Extending the straight line portion into value s = 0, we determine time t_o, and from equation:

$$S_s \equiv \frac{2.246\, k\, t_o}{r^2} \qquad (22)$$

calculate S_s.

The indicated solution can also be applied generally to equation (6), if condition (17) is satisfied, and to all its conditions (8). For observation wells, the condition is usually not satisfied and, therefore, the simplified solution can be applied mainly to the evaluation of pumping wells.

2. FLOW TOWARDS A PARTIALLY PENETRATING WELL IN AN ARTESIAN AQUIFER DURING A RELATIVELY LONG PUMPING PERIOD

After a relatively long period of pumping in a partial well in an aquifer bounded by impervious layers above and below, the hydraulic impulse, created by the pumping, reaches both horizontal boundaries. The drawdown equation for this case was set up by Hantush (1961).

$$\text{For } t > \frac{b^2 S_s}{2\,k} \text{ and } r < 1.5\, b' \tag{23}$$

the solution is:

$$s = \frac{Q}{4\pi\, k\, b}\left\{ W(u) + \frac{4\, b}{\pi(l-d)} \sum_{n=1}^{\infty} \left(\frac{1}{n}\right) K_0\left(\frac{n\pi r}{b}\right) \cos\left(\frac{n\pi z}{b}\right) \times \left[\sin\left(\frac{n\pi l}{b}\right) - \sin\left(\frac{n\pi d}{b}\right)\right]\right\} \tag{24}$$

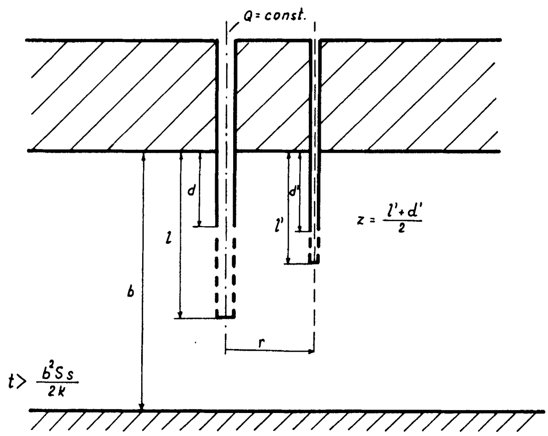

Fig. 6.
Flow towards the aquifer bounded above and below by impervious layers.

where K_0 is the zero-order modified Bessel function of the second kind. The equation is valid for a pumping or observation well with a relatively short screen. The solution for an observation well with a relatively long screen was given by Hantush (1961, 1964).

Equation (24) shows that the lowering of the water level at a constant pumping rate for a period exceeding t as defined by (23) depends on the function value W(u) to which the completing resistance not dependent on time is added (the terms in chain brackets in equation (24) is a constant magnitude). Equation (24) may be rewritten into:

$$s \equiv \frac{Q}{4\pi\, k\, b}\left\{ W(u) + \xi_0 \right\} \qquad (25)$$

where ξ_0 is a constant which hydraulically characterizes the partial well for a sufficiently long pumping time (23). Values of ξ_0 for cases $d \equiv 0$ and $z \equiv 0$ are shown in Fig. 7.

2.1. Evaluation of a Pumping Test

2.1.1. Evaluation of pumping tests by means of type curves.

The evaluation of pumping tests by this method consists of the following procedure:

- determine the thickness of the aquifer and calculate the value of ξ_0 by using equation (24) or Fig. 7,
- plot values $W(u) + \xi_0$ against $\frac{1}{u}$ on log scale for $u < \frac{r^2}{2b^2}$, (26)
- plot drawdown values versus time from the pumping test on the same log scale ($t > \frac{b^2 S_s}{2k}$ is valid), (27)
- superimpose the type curve on the pumping test curve, select a match point and subtract the respective values s, t, $\frac{1}{u}$ and $[W(u) + \xi_0]$,
- substitute into equation (25) and (5) and calculate the respective values k, S_s, S, T.

The solution indicated above represents a disadvantage in the case of a relatively shallow curve of pumping test-run (27) and a type curve for the condition (26). Provided no accurate data on the aquifer thickness *b* are available, the result is affected considerably and the solution becomes only approximate.

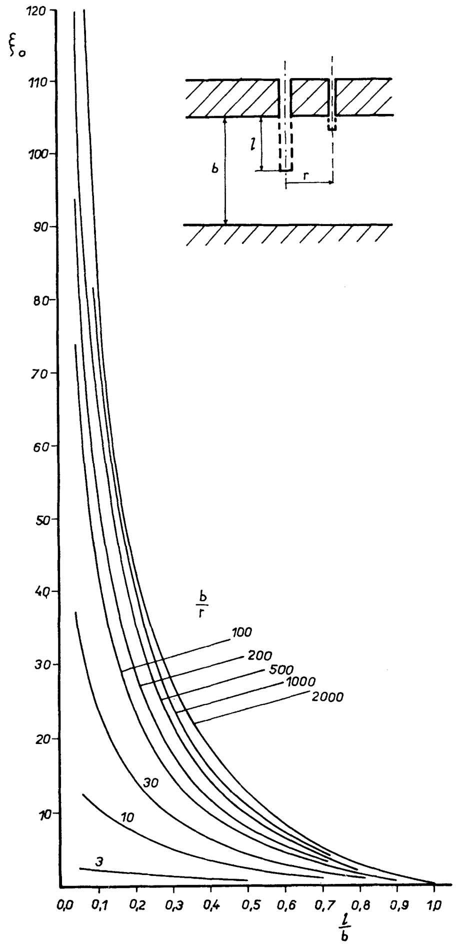

Fig. 7.
Values of function ξ_0 (equation 25).

2.1.2. Simplified solution.

Since the drawdown-run which, in time, satisfies condition (23), depends only on the function W(u), and because ξ_0 is a constant, the pumping test curve with coordinates *s* plotted against log t will be a straight line. Its slope will be similar to that of the full well under the same hydrogeological conditions and will depend only on the aquifer parameters and not on the partial well. The hydraulic parameters of the aquifer can be calculated according to Jacob's simplified method for partial wells with equation

$$T = k\,b = 0.1832\,\frac{Q}{i} \tag{28}$$

where $i = \dfrac{\Delta s}{\log t_2 - \log t_1}$.

The calculated value T is valid for the entire aquifer, as if calculated from a full well.

By extending the straight portion of the curve of the pumping test into value $s = 0$, time t_0 can be determined which can then be substituted into Jacob's method of simplified equation (25)

$$s \doteq \frac{2.303\,Q}{4\pi\,k\,b}\left[\log\frac{2.246\,k\,t_0}{S_s\,r^2} + \frac{\xi_0}{2.303}\right]. \tag{29}$$

For $s = 0$ must satisfy

$$\log\frac{2.246\,k\,t_0}{S_s\,r^2} + \frac{\xi_0}{2.303} \equiv 0,$$

which satisfies if

$$\frac{2.246\,k\,t_0}{S_s\,r^2} \times 10^{\left(\frac{\xi_0}{2.303}\right)} \equiv 1$$

Calculate then S_s with equation

$$S_s \doteq \frac{2.246\,k\,t_0}{r^2} \times 10^{\left(\frac{\xi_0}{2.303}\right)} \tag{30}$$

3. FLOW TOWARDS THE PARTIAL WELL IN AN ARTESIAN AQUIFER IF $r/b > 1.5$

For a relatively long distance from the pumping well, i. e. if $r > 1.5$ b, the drawdown equation will acquire the known form

$$s = \frac{Q}{4\pi\,k\,b}\,W(u) \tag{31}$$

where $u = \dfrac{r^2\,S}{4\,Tt}$.

Evaluation of a pumping test in this case is known as Theis' method of type curves. To satisfy a pumping test duration longer than

$$t > \frac{r^2\,S}{0.12\,k\,b}$$

apply Jacob's simplified solution to the evaluation of the pumping test.

4. GENERAL FLOW EQUATION OF GROUND WATER TOWARDS PARTIALLY PENETRATING WELLS

4.1. Method of Type Curves

According to Hantush (1961), the pumping test-run for the well itself and the observation tubes with a short screen can be generally expressed in the form (symbols shown in Fig. 1):

$$s \equiv \frac{Q}{4\pi\,k\,b}\left\{W(u) + f(u, \frac{r}{b}, \frac{l}{b}, \frac{d}{b}, \frac{z}{b})\right\} \tag{32}$$

where

$$f(u, \frac{r}{b}, \frac{l}{b}, \frac{d}{b}, \frac{z}{b}) \equiv \frac{2\,b}{\pi(l-d)}\sum_{n=1}^{\infty}\frac{1}{n}\left\{\sin\frac{n\pi l}{b} - \sin\frac{n\pi d}{b}\right\}\times$$

$$\cos\frac{n\pi z}{b}\,W(u, \frac{n\pi r}{b}). \tag{33}$$

If d = 0 and z = 0, equations (32) and (33) simplify into the form

$$s \equiv \frac{Q}{4\pi\,k\,b}\left\{W(u) + f(u, \frac{r}{b}, \frac{l}{b})\right\}, \tag{34}$$

where

$$f(u, \frac{r}{b}, \frac{l}{b}) \equiv \frac{2\,b}{\pi\,l}\sum_{n=1}^{\infty}\frac{1}{n}(\sin\frac{n\pi l}{b})\,W(u, \frac{n\pi r}{b}), \tag{35}$$

$$W(u, \frac{n\pi r}{b}) \equiv \int_u^{\infty}\exp - (y + \frac{\left(\frac{n\pi r}{b}\right)^2}{4y})\frac{dy}{y} \equiv \int_u^{\infty}\frac{dy}{y}\exp\left[-y - \frac{\left(\frac{n\pi r}{b}\right)^2}{4y}\right] \tag{36}$$

$$\text{and } u \equiv \frac{r^2\,S_s}{4\,k\,t}. \tag{37}$$

A series of type curves was constructed for this simplified case, where the screen reaches the impermeable top boundary and the drawdown is to be determined at its limit (Appendix 2). The use of type curves for the evaluation of pumping tests is identical to Theis' method of type curves. In the practical application of these methods of evaluation of pumping tests it is necessary to take the effects of boundary conditions, leakage conditions, non-homogeneous aquifer, etc. into consideration. With the known parameters of $\frac{l}{b}$ and $\frac{b}{r}$, equations (34) and (37) can be applied to determine the values k, T, S_s and S.

If $d \neq 0$, it is necessary in accordance with equations (32) and (33) to construct the respective type curves for the required parameters r/b, l/b, d/b, z/b.

4.2. Simplified Solution

The pumping test of a partial well can be generally divided into two substantial parts:

a) time interval, when the pumping test duration is shorter than

$$t < \frac{r^2 \beta^2 S_s}{20k}, \qquad \beta = \frac{l}{r}$$

but

$$t > \frac{r^2 S_s}{0.12\,k} \text{ (compare with section 1),}$$

b) time interval, when the pumping test duration is overlapping

$$t > \frac{b^2 S_s}{2k} \text{ (compare with section 2).}$$

In both cases the simplified solution can satisfy by linearizing the pumping test curves on a semilogarithmic scale *s* against log t (compare subheadings 1.3.2 and 2.1.2 and Fig. 8).

The slope of the linearized pumping test curve is expressed in case a) with equation

$$i_l \equiv 0.1832 \frac{Q}{k\,l} \equiv \left(\frac{\Delta s}{\log t_2 - \log t_1} \right)_l, \tag{38}$$

in case b) by equation

$$i_b \equiv 0.1832 \frac{Q}{k\,b} \equiv \left(\frac{\Delta s}{\log t_2 - \log t_1} \right)_b \tag{39}$$

and by extending this linearized curve to point $s = 0$, time t_0 is obtained while for case a) it is valid

$$S_s = \frac{2.246\,k\,t_{o \cdot l}}{r^2} \tag{40}$$

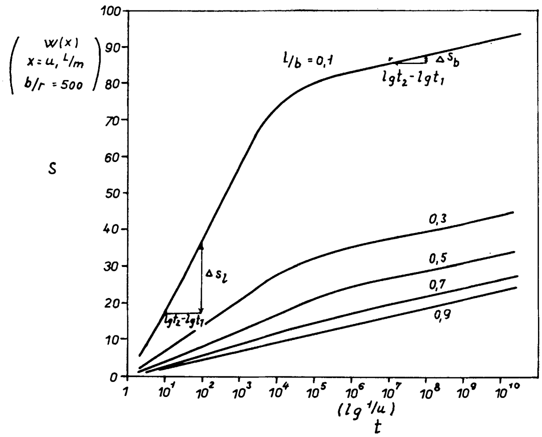

Fig. 8.
Pumping test curve of a partially penetrating well in an artesian aquifer for a coordinate system s against log t.

for case b)

$$S_s \equiv \frac{2.246\ k\ t_{o \cdot b}}{r^2} \times 10^{\left(\frac{\xi_o}{2.303}\right)} \tag{41}$$

by dividing equations i_l/i_b we obtain

$$\frac{i_l}{i_b} = \frac{\dfrac{\Delta s_l}{\log t_2 - \log t_1}}{\dfrac{\Delta s_b}{\log t_2 - \log t_1}} = \frac{b}{l} \tag{42}$$

and for similar time intervals $\log t_2 - \log t_1$ for example 1 log cycle

$$\frac{b}{l} = \frac{\Delta s_l}{\Delta s_b} \tag{43}$$

Similarly related are also equations (40) and (41) from which by dividing we get

$$\frac{t_{o,l}}{t_{o,b}} = 10^{\left(\frac{\xi_o}{2.303}\right)} \text{ and } \xi_o = 2.303 \log \frac{t_{o,l}}{t_{o,b}} \tag{44}$$

Calculate values k by substituting the respective data into equations (38) and (39) and the value of b from equation (42) or (43). Calculate S_s with equations (40) and (41) and calculate ξ_o with equation (44).

The indicated simplified solution is limited by condition (19) $l/r > 12.9$ in the first time interval of the pumping test. The condition satisfies mainly in the pumping wells (compare 1.3.2).

5. INFLUENCE OF THE SKIN EFFECT OF THE SCREEN OF A PARTIALLY PENETRATING WELL ON THE PUMPING TEST

When, in a pumping test, the partial well is combined with the skin effect, rewrite the drawdown equation into the form in which the coefficient of skin effect φ is added to the resistance term. For instance equation (6) for a short pumping time, rewrite into:

$$s = \frac{Q}{4\pi\, k(l-d)} \left\{ \frac{1}{2} [M(u,\beta_1) + M(u,\beta_2) - M(u,\beta_3) - M(u,\beta_4)] + \varphi \right\} \tag{45}$$

and equation (25) for a relatively long pumping time rewrite into:

$$s \equiv \frac{Q}{4\pi\, k\, b} \left\{ W(u) + \xi_o + \frac{b}{l \simeq d} \varphi \right\} \tag{46}$$

In the first case (45), the complementary resistance (coefficient of skin effect) φ acts on the length of the well screen expressed by the term $(l-d)$. In the second case, φ acts also only on the well screen length and should, therefore, be multiplied by $\frac{b}{l \simeq d}$ (compare constants preceding the brackets in both equations).

Rewrite also equation of water flow towards the partial well into the form

$$s \equiv \frac{Q}{4\pi\, k\, b} \left[W(u) + f(u, \frac{r}{b}, \frac{l}{b}, \frac{d}{b}, \frac{z}{b}) + \frac{b}{l-d} \varphi \right] \tag{47}$$

The skin effect caused by the constant value of coefficient φ appears, when evaluating in accordance with the method of type curves, in the line flattening of the pumping test through its whole run. The evaluation becomes difficult and it is necessary to add various values of φ to calculate the type curve, wherefrom a set of new type curves is obtained. From among these new type curves one may be found that best satisfies the pumping test.

Attention should be drawn to the fact that the skin effect appears only in the pumping well regardless of whether the well is completely or partially full. In evaluating the pumping test with the simplified solution, the influence of the skin effect does not appear in the calculation of k and T, because the slope of the straight portion of the pumping test curve does not change. However, the position of point t_0 for the calculation of S_s is affected.

In the case of short pumping time (25) will keep the form

$$i \equiv 0.1832 \frac{Q}{k\,l} \equiv \frac{\Delta s}{\log t_2 - \log t_1}; \tag{48}$$

equation (22) rewritten:

$$S_s \equiv \frac{2.246\,k\,t_0}{r^2} \times 10^{\left(\frac{\varphi}{2.303}\right)}; \tag{49}$$

for long pumping time, equation (28) remains in the form:

$$i \equiv 0.1832 \frac{Q}{k\,b} \equiv \frac{\Delta s}{\log t_2 - \log t_1} \tag{50}$$

and equation (30) will have the form:

$$S_s \equiv \frac{2.246\,k\,t_0}{r^2} \times 10^{\frac{\left\{\xi_0 + \frac{b}{(l-d)}\varphi\right\}}{2.303}} \tag{51}$$

6. STEADY-STATE FLOW TOWARDS THE PARTIALLY PENETRATING WELLS

Dupuit's equation applies for steady-state flow towards the partial well in an artesian aquifer if rewritten as:

$$s \equiv \frac{2.303\,Q}{2\pi\,b\,k}\left[\log \frac{r_2}{r_1} + 0.217\,\xi_0\right] \tag{52}$$

Adding the skin effect in the well screen, we get:

$$s \equiv \frac{2.303\,Q}{2\pi\,b\,k}\left[\log \frac{r_2}{r_1} + 0.217\,\xi_0 + 0.217\,\frac{b}{l-d}\varphi\right] \tag{53}$$

The values of ξ_0 are calculated from the constant resistance term of equation (26) or read from the graph in Fig. 7.

In an infinitely deep aquifer ($l/b < 0.1$), for $d = 0$, $z = 0$ a steady-state flow occurs after a certain time (equation (11) and Appendix 1), which for various values of $\frac{l}{r}$ depends on the function value $M(u,\beta)$. These function values for $u \equiv 0$ are indicated in Hantush's work (1961). The equation of the steady-state flow in an infinite aquifer can for $d \equiv 0$ be written as:

$$s \equiv \frac{Q}{4\pi\, k\, l} M(\beta_{u=0}) \tag{54}$$

where $\beta = \frac{l}{r}$.

It might also be mentioned that

$$\lim_{u \to 0} (M(u,\beta)) = M(\beta_{u=0}) = 2 \sinh^{-1}\beta \equiv 2\, ln\, (\beta + \sqrt{\beta^2 + 1})$$

from which follows:

$$s \equiv \frac{Q}{2\pi\, k\, l}\, ln \left(\frac{l}{r} + \sqrt{1 + \left(\frac{l}{r}\right)^2}\right)$$

an expression often found in literature.

The dependence of function values $M(\beta)$ from β for $u \equiv 0$, $d \equiv 0$ and $z \equiv 0$ is graphically illustrated in Fig. 9.

For a skin effect, equation (54) will read:

$$s \equiv \frac{Q}{4\pi\, k\, l} \left\{ M(\beta_{u=0}) + \varphi \right\} \tag{55}$$

A similar dependency can be derived from equation (6) for a steady-state flow of $d \neq 0$ and $z \neq 0$.

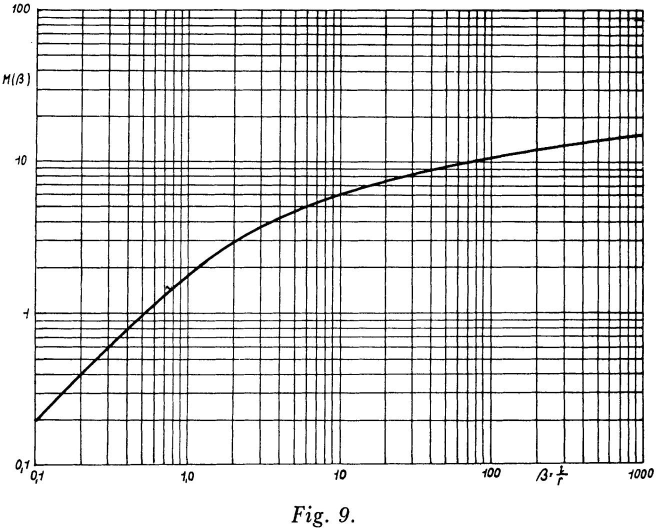

Fig. 9.
Dependence of function $M(\beta)$ on β (for $u = 0$).

From the above it follows that great aquifer thickness, usually $b > 10\,l$, affects the pumping test similarly as the boundary condition $H \equiv$ const., which may lead to errors when evaluating the pumping test.

7. APPLICATION OF EQUATIONS FOR CONFINED AQUIFERS TO UNCONFINED AQUIFERS

Under certain conditions it is possible to utilize the solution of the drawdown of the water level during pumping tests in a confined aquifer as an approximation also in the case of an unconfined aquifer. In an unconfined aquifer, Jacob (1944), for fully penetrating wells, adjusted real drawdown of s into an equivalent drawdown of s_{eq} according to the relation

$$s_{eq} \equiv s - \left(\frac{s^2}{2b}\right). \tag{56}$$

For flow equation towards the partial well in an infinitely deep aquifer, or in a finite aquifer for a relatively short pumping period, rewrite equation (56) as

$$s_{eq} \equiv s - \left(\frac{s^2}{2l}\right). \tag{57}$$

Jacob's adjustment of s to s_{eq} initiates in Dupuit's equation comparison for flow in an unconfined and confined aquifer. Complementary screen resistances due to the partial penetration of the well and the skin effect in a confined and unconfined aquifer are similar (Verigin 1962). They are to be calculated for the real length of the well screen (in an immersed screen: total screen length; in a non-immersed screen: total inflow length). This means that to calculate an equivalent drawdown, it is necessary to reduce the drawdown due to the partial well and the skin effect. Then rewrite equation (56) for instance for flow towards the partial well in an unconfined aquifer and underlying impervious layer for a relatively long pumping duration (section 2, equation (25)) as

$$s_{eq} \equiv \left[s - \frac{Q}{4\pi k b}\left(\xi_0 + \frac{b}{l-d}\,\varphi\right)\right]\left[1 - \frac{s - \frac{Q}{4\pi k b}\left(\xi_0 + \frac{b}{l-d}\,\varphi\right)}{2b}\right] \tag{58}$$

Calculate s_{eq} in flow towards the partial well in an infinite unconfined aquifer for $d \equiv 0$ and $z \equiv 0$ (equation (11), subheading 1.1) from

$$s_{eq} \equiv \left\{ s - \frac{Q}{4\pi\, k\, l} \left[M(u,\beta) - W(u) \right] \right\} \left\{ 1 - \frac{s - \frac{Q}{4\pi\, k\, l}\left[M(u,\beta) - W(u) \right]}{2l} \right\} \qquad (59)$$

and similarly for the remaining cases.

All equations for calculating S and S_s when the well screen approaches an unconfined aquifer are to be adjusted, so as to make the resulting values represent the coefficient of the aquifer yield at its water table. In order to maintain the dimension, rewrite equation (5) as

$$u \equiv \frac{r^2\, S}{4k\, b\, t}, \text{ or } u \equiv \frac{r^2\, S}{4k\, l\, t}, \qquad (60)$$

according to the equation used. Calculated values of S represent the coefficient of water yield.

For an aquifer lying at a considerable depth below an unconfined aquifer in equations for relatively short durations,

$$t < (2b - l' - z')^2 \frac{S}{20\, kb}, \qquad (61)$$

where l' means the depth of the upper screen portion below the unconfined aquifer and z' means the depth point where the drawdown is calculated from the unconfined aquifer, values S and S_s remain expressions of the properties of aquifer elasticity. For longer pumping durations than envisaged in (23), calculated values of S represent the total coefficient of water yield. Between the two time intervals, the values of S, representing the elasticity values, gradually pass over to the coefficient of the water yield.

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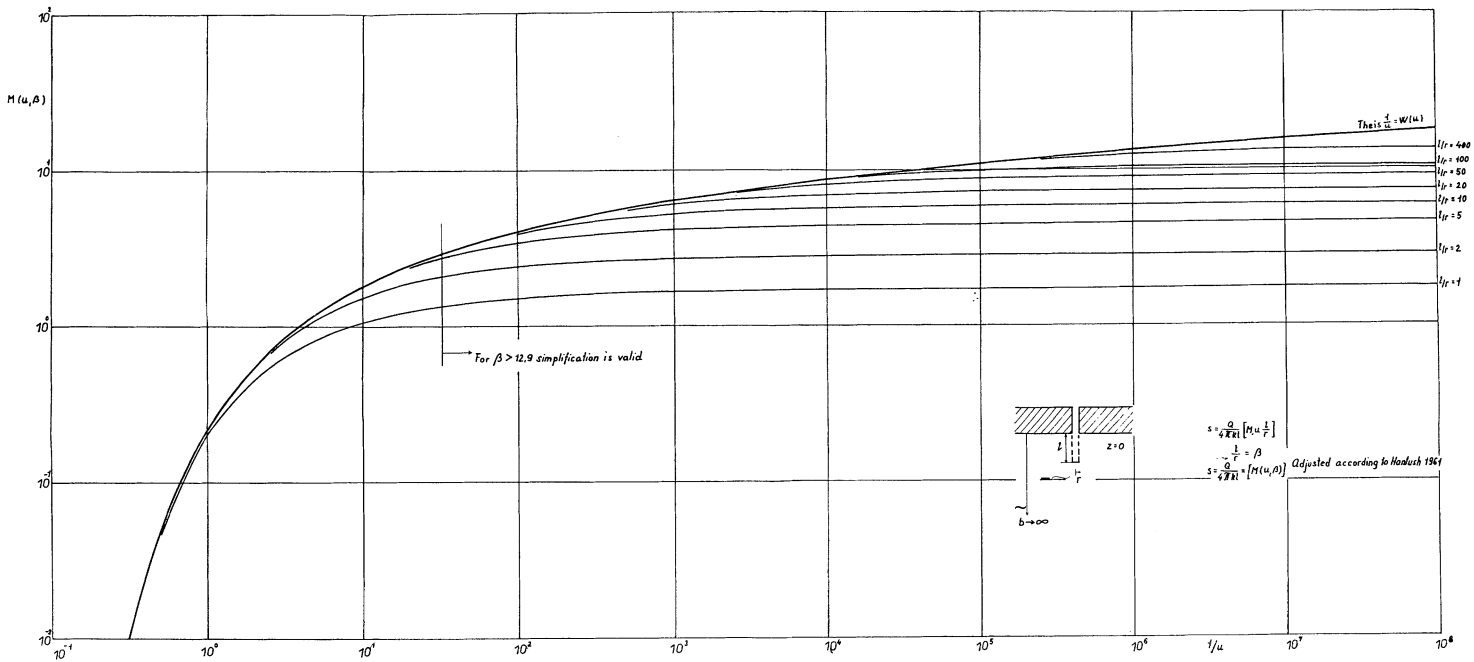
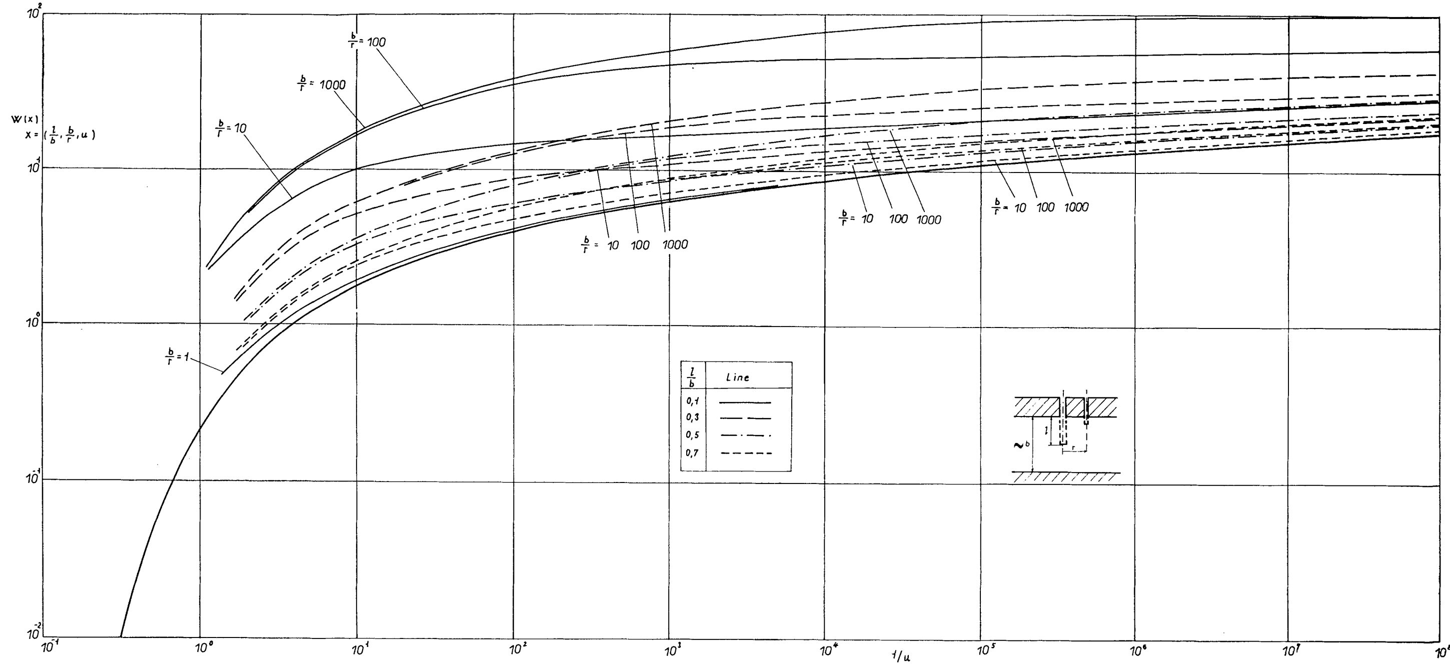


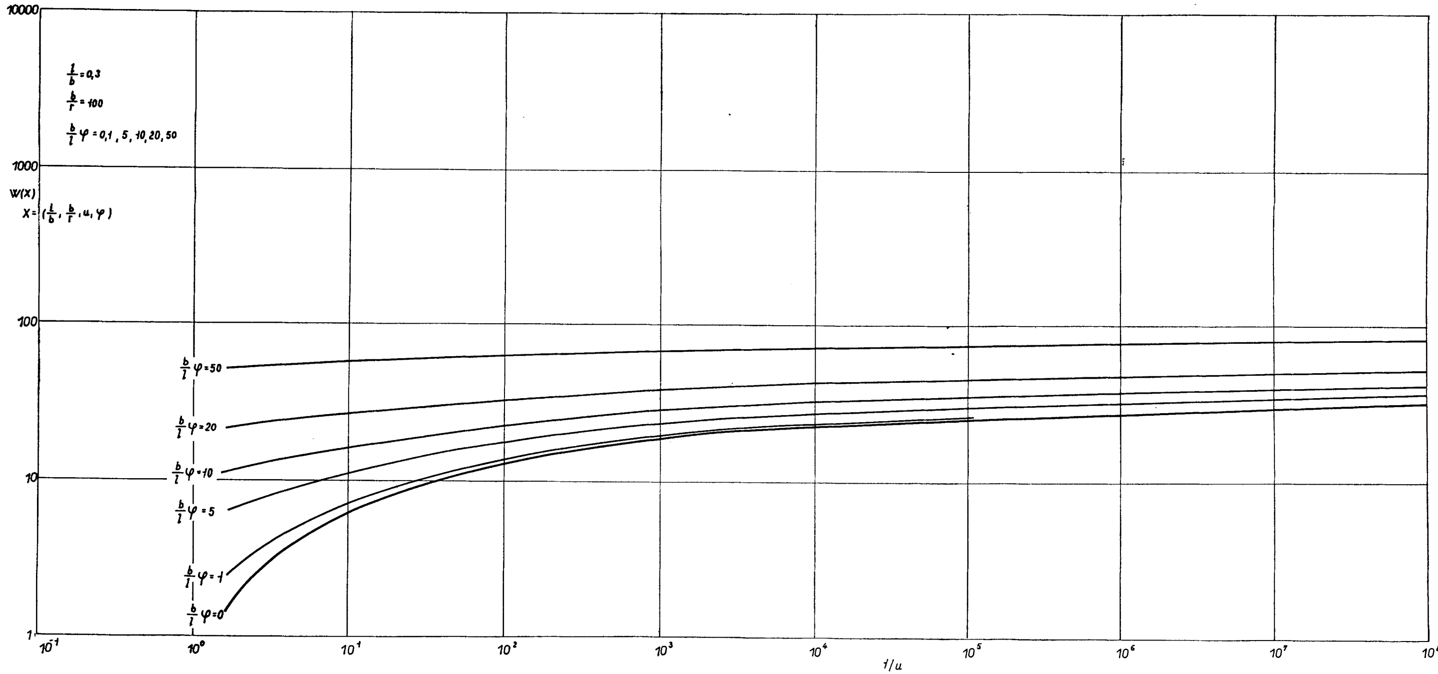
Appendix 1.

Type curves for ground water flow towards the partially penetrating well.



Appendix 2.

Type curves for ground water flow towards the partially penetrating well.



Appendix 3.

Influence of skin effect on type curves.

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