Letters to the Editor

Charge Distribution in Be\textsuperscript{9}

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Scattering of electrons by nuclei has been a very useful tool for determining the charge distributions in various nuclei. The general procedure has been to choose a model distribution with some empirical parameters, evaluate the scattering cross-sections, and compare them with the experimental results. It is possible thus to eliminate certain types of charge distributions, such as uniform charge distribution, etc., but in general, one is still left with a family of models which all give similar results. For the purpose of studying the systematics of variation of parameters of nuclear charge distributions it would be desirable to use a single model for a large range of nuclei. For heavy nuclei the Fermi distribution\textsuperscript{1} (or a very similar "folded distribution"\textsuperscript{2}) has been successfully used to analyse the electron scattering data. Elton, Hiley and Price\textsuperscript{3} have analysed the C\textsuperscript{12} data also in terms of the Fermi distribution. Partly in order to extend this analysis to other light nuclei, we have calculated the electron scattering of Be\textsuperscript{9} on the basis of the Fermi model. This nucleus is also of particular interest, since it is in the middle of the \textit{p}-shell, hence one may expect a certain amount of quadrupole deformation of the nucleus to manifest itself. Recently Meyer-Berkhout, Ford and Green\textsuperscript{4} have carried out a fairly com-
plete analysis of the $p$-shell nuclei; however, their results for Be$^9$ parameters are somewhat different from ours, and since we have consistently employed the Fermi distribution even for evaluation of the quadrupole scattering effects, we present here our results for Be$^9$.

The Fermi distribution for a spherical nucleus is given by

$$\rho_0(r) = \rho_0 \left[ 1 + \exp \left( \frac{r - \zeta}{\zeta} \right) \right]^{-1}.$$ 

If we take into account the quadrupole deformation of the nucleus by replacing $r$ by $r(1 + \epsilon Y_2^0(\cos \theta))$, we may write

$$\rho(r) = \rho_0(r) + \frac{d\rho_0(r)}{dr} \epsilon r Y_2^0(\cos \theta)$$

$$= \rho_0(r) + \rho_4(r) Y_2^0(\cos \theta).$$

The scattering cross-section in the Born approximation is given by the well-known results:

$$\sigma = \left( \frac{2 \epsilon^2}{2E} \right) \cos^2(\theta/2) \left[ |F_0|^2 + |F_2|^2 \right]$$

where

$$F_0 = 4\pi \int \rho_0(r) j_0(qr) r^2 dr$$

$$F_2 = -\sqrt{20\pi} \int \rho_4(r) j_2(qr) r^2 dr.$$ 

The quadrupole moment $Q$ is given by

$$Q = \sqrt{\frac{16\pi}{5}} \int \rho_4(r) r^4 dr.$$ 

The Born approximation may be expected to give a valid result for such light nuclei except in the immediate neighbourhood of the diffraction minimum.

First we take $\epsilon = 0$, and determine the

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Fig. 1. Elastic scattering of 300 Mev electrons by Be$^9$

The continuous curve shows the monopole scattering, the dotted curve the quadrupole contribution and the dots are the experimental points.

Fig. 2. Calculated curve for 420 Mev electron scattering by Be$^9$

The continuous curve represents the monopole scattering and the dotted curve shows the quadrupole scattering.
best values of the parameters \( c, z \) by fitting the experimental data at small angles for scattering of 300 Mev electrons as given in reference 4). Since we later find that the quadrupole scattering at these angles is very small, the analysis at small angles in terms of monopole scattering alone is valid. We find the best choice of the parameters to be \( c=1.4f, z=0.6f \). The experimental results and the contribution of the monopole scattering to the cross-section given by \(|F_0|^2\) alone is shown in Fig. 1. It is clear that for larger angles (\( \theta > 75^\circ \)), the monopole scattering alone is inadequate and no variation of parameters can improve the situation.

The value of \( \epsilon \) is now fixed by evaluating the quadrupole moment of Be\(^9\) with the above values of \( c, z \) and comparing with the experimental value, which we take to be \( Q=2\times10^{-30} \text{ cm}^3 \). This gives us \( \epsilon=0.256 \). Now all the parameters are determined and the quadrupole scattering cross-section as given by \(|F_1|^2\) is evaluated and the plot is shown in Fig. 1.

Thus the Fermi charge distribution with the inclusion of quadrupole distortion appears to fit the experimental results at 300 Mev fairly satisfactorily. Perhaps an exact evaluation of the monopole cross-section would improve the agreement near the diffraction minimum. At this energy the diffraction minimum is not clearly seen. It would, however, be expected to be seen clearly at the electron energy of 420 Mev. In view of the possibility that these experiments may be performed, we also give in Fig. 2 the results of our model for this energy. Here again the calculations are made in the Born approximation.

The values \( c, z \) obtained here are different from those of reference 4), who obtain \( c=0.9f, z=0.79f \), values that appear to be somewhat extreme. In view of the values \( c=2.60f, 2.35f \) and \( z=0.43f, 0.42f \) for O\(^{16}\) and C\(^{12}\) respectively, our value of \( z \) appears rather high, indicating that Be\(^9\) nucleus is almost all surface. The r.m.s. radius obtained with our values of the charge distribution parameters is 2.48\( f \), compared to 2.99\( f \) reported by Meyer-Berkhout et al. for their values of the parameters, and 2.2\( f \) for the Harmonic oscillator model.

The deformation parameter \( \epsilon \) as determined here for Be\(^9\) has the value \( \epsilon=0.256 \), which indicates quite a sizable quadrupole deformation.

In conclusion, we may say that the Fermi charge distribution model including the effects of possible quadrupole deformation satisfactorily accounts for the observed scattering cross-section of Be\(^9\) for 300 Mev electrons.

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2) R. Helm, Phys. Rev. 104 (1956), 1466.