greatly to the understanding of vibration behaviors of plates. The authors may find the work interesting since, among many other things, it deals with the relations between the frequency spectra of infinite and finite plates, which are applicable to beams as a special case. Thus, he described the process of obtaining the overtones of a finite plate from the frequency spectrum of an infinite plate and went on to point out that the adequacy of plate equations for the prediction of natural frequencies of finite plates depends on the ability of the plate equations to reproduce accurately the frequency spectrum of an infinite plate as compared with the result of the exact elasticity theory. Since the classical beam or plate equations yield only the lowest branch of the frequency spectrum of an infinite beam or plate, the frequency range that can be handled adequately will be limited when the equations are applied to a finite beam or plate. On the other hand, by including the effects of transverse shear and rotatory inertia, a second branch of the frequency spectrum can be generated by the Timoshenko beam theory, and, by further matching the cutoff frequency of this branch with the exact result of elasticity theory in the manner of Mindlin, the frequency range that can be handled adequately is greatly extended. For sandwiches and laminated composites, the transverse shear effect in general can be much more important and often must be considered even for the lowest branch of the frequency spectrum, as has been pointed out earlier. The effect of rotatory inertia plays the same role as in the case of a single-layered beam or plate in making it possible for the beam or plate equations to accommodate a second higher branch of the frequency spectrum. For sandwiches it was further shown by Yu (1960) that the cutoff frequency of this second branch can be much lower than for single-layered beams or plates, which makes the presence of the second branch that much more important for sandwiches. The same is expected to be true for laminated composites.

Finally, it is noted that the value of the shear factor $k = 5/3$ used by the authors below their equations (105) is unusually high.

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Authors' Closure[14]

We thank Professor Yu for his careful reading of our paper and shall reply to those comments related to its contents; Professor Yu's comments concerning high-frequency vibrations of plates are irrelevant.

First, we acknowledge a semantic problem: we should have written "...Timoshenko beam theory produces an erroneous *asymptotic* correction...". Second, because we take the depth of the beam to be $2H$, our definition of the shear coefficient k in Eq. $(15)_2$ is twice the conventional definition. We apologize for this confusion.

More fundamentally, we believe that Professor Yu's comments are based on a misunderstanding. The theories of Timoshenko and those of Yu for beams and plates are ad hoc—albeit useful—theories, based on *assuming* certain variations of displacements in the thickness direction. In general, these variations are not good point-wise approximations near edges, especially built-in edges, although they often lead to accurate gross quantities such as natural frequencies or buckling loads. In contrast, ours in an *exact* theory, based on a formal asymptotic analysis of the linear equations of elasticity. The limitations of such an analysis are well known: a statement that the remainder in an expansion is $O(\epsilon)$ says nothing about the implied constant that multiplies ϵ; it could be quite large. We acknowledged this limitation in the Conclusions section of our paper, as Professor Yu notes.

The difference between ad hoc and asymptotic theories is also reflected by the role of experiment. In the former, experiments are conducted to verify the soundness of the assumptions; in the latter—barring mathematical errors—the only reason for experiments would be (a) to confirm the underlying assumptions of elasticity theory, (b) to test the validity of idealized boundary conditions, or (c) to empirically estimate the constants associated with certain O-symbols.

On Energy Rate Theorems for Linear First-Order Nonholonomic Systems[15]

T. R. Kane[16] and D. A. Levinson.[17] The author points out a defect in a previously published Note (Kane and Levinson, 1988), but ignores the fact that this defect was remedied in a second Note by the same authors (Kane and Levinson, 1990), a Note whose publication in this journal preceded that of his paper by more than one year.

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[15]By John G. Papastavridis, and published in the June 1991 issue of the ASME Journal of Applied Mechanics, Vol. 58, pp. 536–544.

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