Nordic Hydrology 6, 1975, 242–262
Published by Munksgaard, Copenhagen, Denmark

STOCHASTIC TIME DISTRIBUTION OF STORM RAINFALL

W. G. KNISEL, JR. and W. M. SNYDER

Southeast Watershed Laboratory, Athens, Georgia, U.S.A.

Increments of rainfall during a storm event were treated as stochastic variates. Statistical distributions of accumulated rain were developed at 2-hour intervals. Three-parameter lognormal distributions were then fitted simultaneously to the twelve 2-hour histograms by making two of the three parameters continuous functions of elapsed time from beginning of rainfall. After evaluation of the parametric functions by nonlinear least squares, stochastic patterns of storm rainfall can be simulated.

Watershed research by the U. S. Agricultural Research Service is directed toward development of rainfall-runoff relationships and mathematical hydrologic models for predicting rates and volumes of streamflow from watersheds. Runoff predictions are generally based on some experienced climatic record. However, precipitation events will not repeat exactly in amount, temporal distribution or spatial distribution. Rainfall depth-area-duration curves have been developed for commonly-used recurrence intervals. However, few models deal with temporal distribution of rainfall during a storm period. The purpose of this paper is to present a method of fitting such a model to historical records of time-distributed rainfall. Synthetic storm rainfall distributions are also generated.

Rainfall during storm events cannot be described mathematically in continuous form. Linsley, Kohler & Paulhus (1949) presented "typical" mass rainfall curves for 1-hour thunderstorms and various rainfall amounts of 1 to 5 inches. Water resources planners often use project or design storms established for a particular purpose in a given climatic region. The U. S. Soil Conservation Service uses a rainfall volume of varying recurrence interval depending upon the class of hydraulic structure involved. The design storm has a 6-hour duration, and the time distribution of storm rainfall is generalized as an S-shaped curve. The rainfall amount in time is expressed as a percentage of the design storm total rainfall. For durations longer than 6 hours, a generalized adjustment factor is used (U. S. Dept. Agr., 1964). Other agencies use the maximum historical event of recorded rainfall amounts, with observed temporal and spatial distribution in a particular region.

Mills (1965) analyzed rainfall data for selected storms in Southern Florida and separated storms into two groups: one with durations less than 12 hours, and the second with durations greater than 12 hours. He presented a dimensionless plot of percent accumulated rainfall to total storm rainfall versus percent accumulated time to total storm duration. The plot showed larger percentages of the total storm rainfall in shorter percentages of the total duration for storms with less than 12-hour durations.

In the development of an operational scheme for the Central and Southern Florida Flood Control District, Sinha & Khanal (1971) used conditional probability and rainfall persistence to estimate the hour when rainfall began on a "rain" day. A linear regression model was developed for each of 10 rainfall classes to estimate 24 hourly rainfall amounts during the rainy season months of June through September.

Heaton (1968) investigated point and areal hourly rainfall in Southwest Texas, and used conditional probability to estimate the class of hourly rainfall that might occur after an hour of no rainfall or after an hour with rainfall. The investigation led to the probabilistic estimate of hourly sequential rainfall of different class magnitudes.

Snyder & Wallace (1974) developed the three-parameter lognormal distribution as a functional variate transform of an embedded normal distribution with unit variance. The mean of the embedded normal distribution and two parameters in the transform function were evaluated by nonlinear least squares. In an extension of this work, Snyder (1975a,b) considered monthly rainfall amounts distributed log-normally and expressed distribution parameters as seasonally continuous functions. He fitted the 12 monthly distributions simultaneously to provide monthly continuity.

This paper presents a method for constructing distributions of cumulative storm rainfall amounts at successive time intervals after the storm began. The distributions are changed by a mathematically continuous function of time throughout storm duration.

METHODOLOGY

Rainfall occurrence during storm events is highly variable from storm to storm. A sample of storm events at a location provides a distribution of cumulative rainfall amounts for any specified time interval after the storms began. Distributions successive in time must have some degree of continuity since storms are more or less continuous within their durations. The hypothesis of continuity between distributions within the storms was tested at one location in each of the states Georgia, North Carolina, and Florida.

In order to test the methodology, length of storm was arbitrarily defined as a 24-hour period of time. The time interval within storms was also arbitrarily selected as 2 hours, thus providing 12 distributions for the 24-hour duration. Any desired duration and time interval could be used.

Storms considered hydrologically significant were selected for study. All recorded events with total storm rainfall equal to or greater than 1.0 inch in a 24-hour period were selected. Since differences in storm rainfall are seasonal, storm data were divided into two arbitrary seasons as follows:

winter – October 1 through April 30
summer – May 1 through September 30.

The seasons do not completely separate storm types, but provide some differentiation to determine if time-distribution characteristics differ by type.

Since different storms have different durations, two alternative schemes were considered. The ending of storms with the 2-hour interval of actual duration would result in an unequal number of events for each of the twelve 2-hour intervals. Thus, a 12-hour duration storm would result in data for only six intervals. This poses no particular problem except that few historical events would be available for fitting in the latter intervals. An alternative procedure was used whereby the total 24-hour period was filled with the storm rainfall total beyond the 2-hour interval at the end of the storm. Thus a 12-hour storm with 3.00 inches of total rainfall would be recorded as 3.00 inches for the remaining six 2-hour intervals. The second alternative provides an equal number of events for each interval and eliminates storm duration from the stochastic model.

A summary of the storm data is given in Table 1 for the three locations used in the study. The data were organized into histograms for each 2-hour interval with an assumed class width of 0.25 inch for all locations and seasons.

The three-parameter lognormal distribution function was used in the present study for fitting distributions of cumulative storm rainfall amounts at 2-hour time intervals within the storms. Parameters were evaluated using the method of nonlinear least squares (Snyder 1972, 1975a).

Table 1.
Summary of selected storms.

Location	Period of record	Season	No. storms	Maximum storm rainfall
R. G. No. 3,		Summer	50	5.00
Ahoskie Creek, N. C.	7/54–12/72	Winter	35	4.50
R. G. No. 3,		Summer	96	4.47
Taylor Creek, Fa.	10/61–9/71	Winter	45	5.18
		Summer	102	4.34
USWB, Atlanta, Ga.	6/50–12/73	Winter	189	4.82

The lognormal probability density function is expressed as

$$p(v) = \frac{1}{\sqrt{2\pi}\, k\,(v-a)} \exp\left\{ -\tfrac{1}{2} \left[\frac{\ln(v-a)}{k} - m \right]^2 \right\} \quad (1)$$

where the variate transform function is

$$\ln(v-a) = kx. \quad (2)$$

In equations (1) and (2), p(v) is the probability density of variate v; x is the variate of the embedded normal distribution of mean, m, and unit variance, and a and k are mathematical parameters. Parameter k controls the shape of the distribution and parameter a shifts the distribution from zero.

The usual procedure in fitting historical distributions is to evaluate the three parameters a, k, and m, for each distribution independently. For the twelve 2-hour distributions, this would require evaluation of 36 parameters without continuity between time intervals. In order to provide time-continuity within storms, parameters a and k were expressed as functions of time interval and the twelve distributions were fitted simultaneously (Snyder 1975b).

Cumulative rainfall reaches the storm total at some point in time equal to the storm duration. The minimum storm selection criteria, 1.0 inch in this study, represents the maximum shift of the histogram from zero, and therefore is a maximum value for the function of parameter a. The boundary parameter a was made to vary with duration as in equation (3).

$$a_i = LCL - b_1 \left\{ \exp\left[-b_2 (i-1)^2\right] \right\} \quad (3)$$

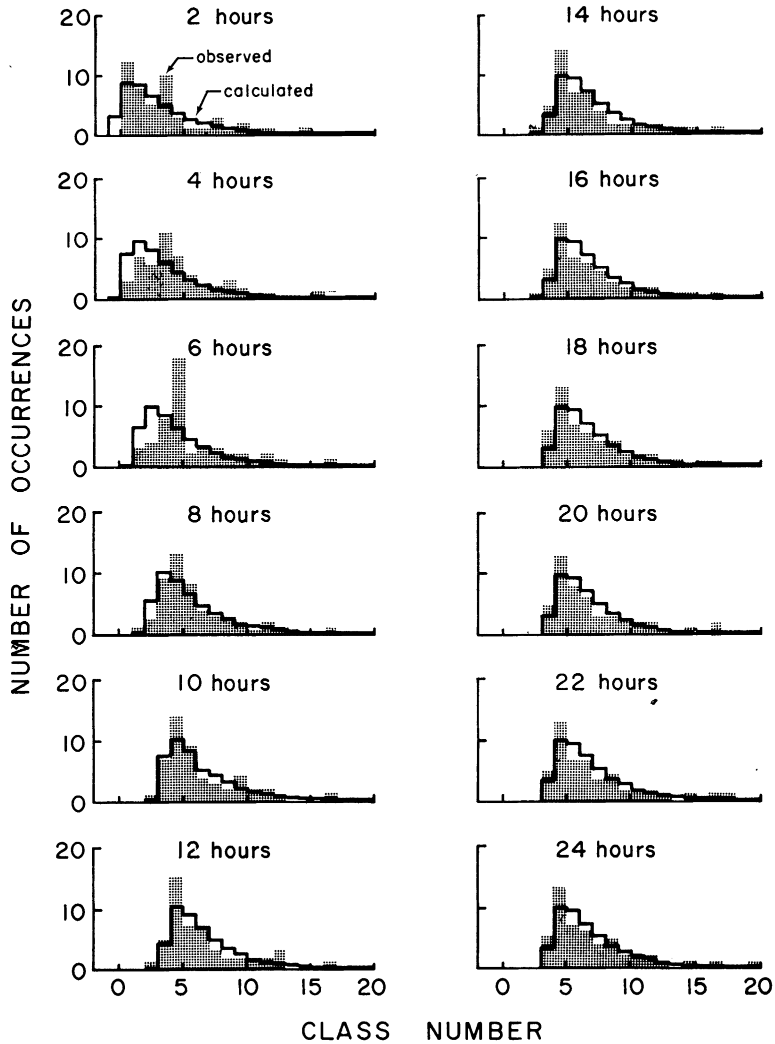

Fig. 1.

Observed and predicted histograms for 24-hour rainfall at 2-hour intervals, 0.25-inch class width, summer storms, Ahoskie Creek, North Carolina, rain gage No. 3.

i is the i^{th} 2-hour interval; LCL is the minimum storm selection criterion, and b_1 and b_2 are mathematical coefficients. Parameter a has, therefore, a minimum value of LCL-b_1 for the first 2-hour distribution and approaches LCL exponentially with i.

The parameter k was made to vary inversely with i, since most change of shape of the distributions occurred early in the storms. The function used is given in equation (4).

$$k_i = \frac{c_1}{i} + c_2 \qquad (4)$$

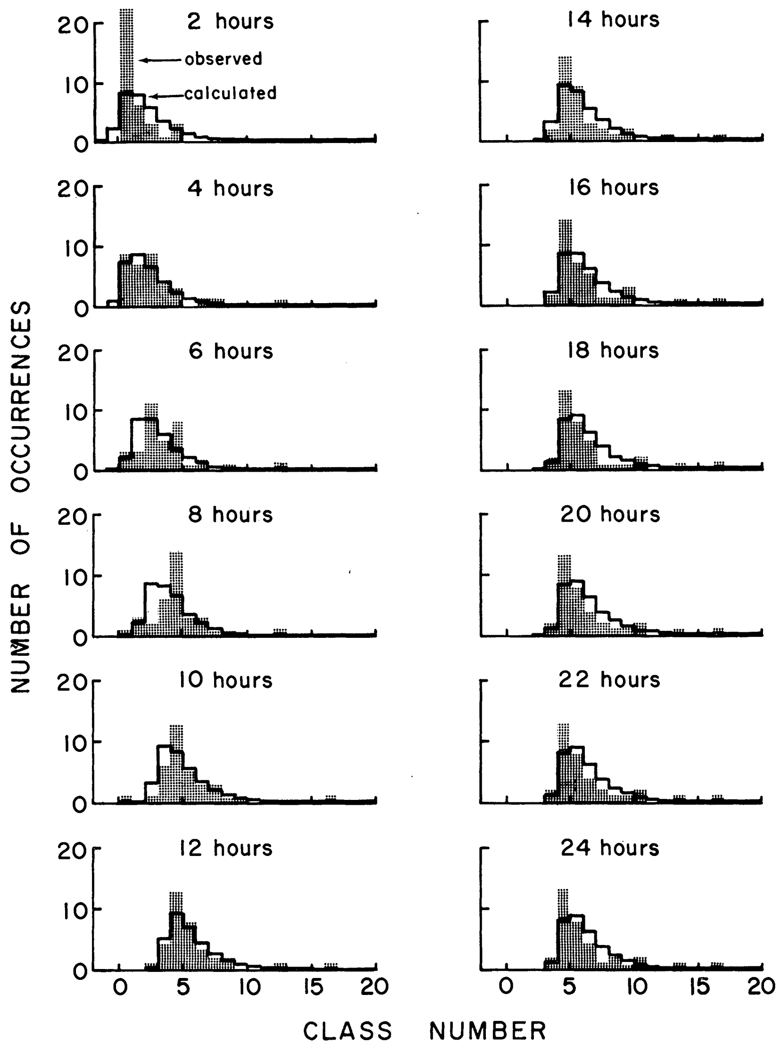

Fig. 2.
Observed and predicted histograms for 24-hour rainfall at 2-hour intervals, 0.25-inch class width, winter storms, Ahoskie Creek, North Carolina, rain gage No. 3.

i is the i^{th} 2-hour interval as before, and c_1 and c_2 are mathematical coefficients.

In the simultaneous fitting of the 12 distributions, parameter m was specified as having the same value for all time intervals. Thus, five parameters are evaluated by fitting; m and the four coefficients of equations (3) and (4). This

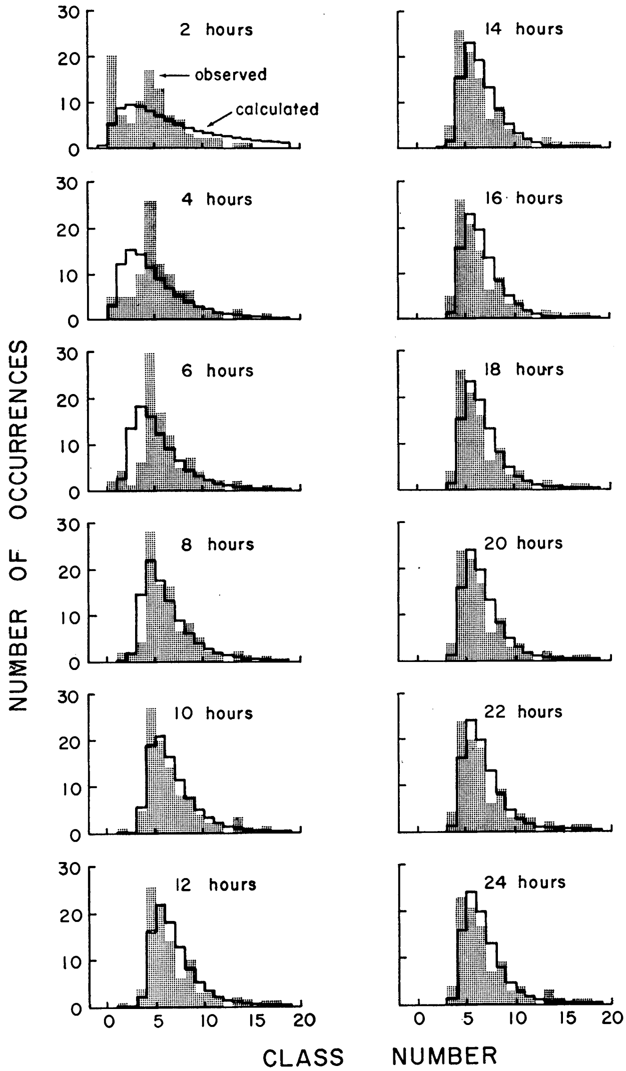

Fig. 3.

Observed and predicted histograms for 24-hour rainfall at 2-hour intervals, 0.25-inch class width, summer storms, Taylor Creek, Florida, rain gage No. 3.

results in an approximate 7-to-1 gain in efficiency when compared with 36 parameters required for fitting the distributions independently.

Storm rainfall data were organized into histograms by time interval as shown in Figures 1 through 6. A class width of 0.25 inch was assumed. It should be

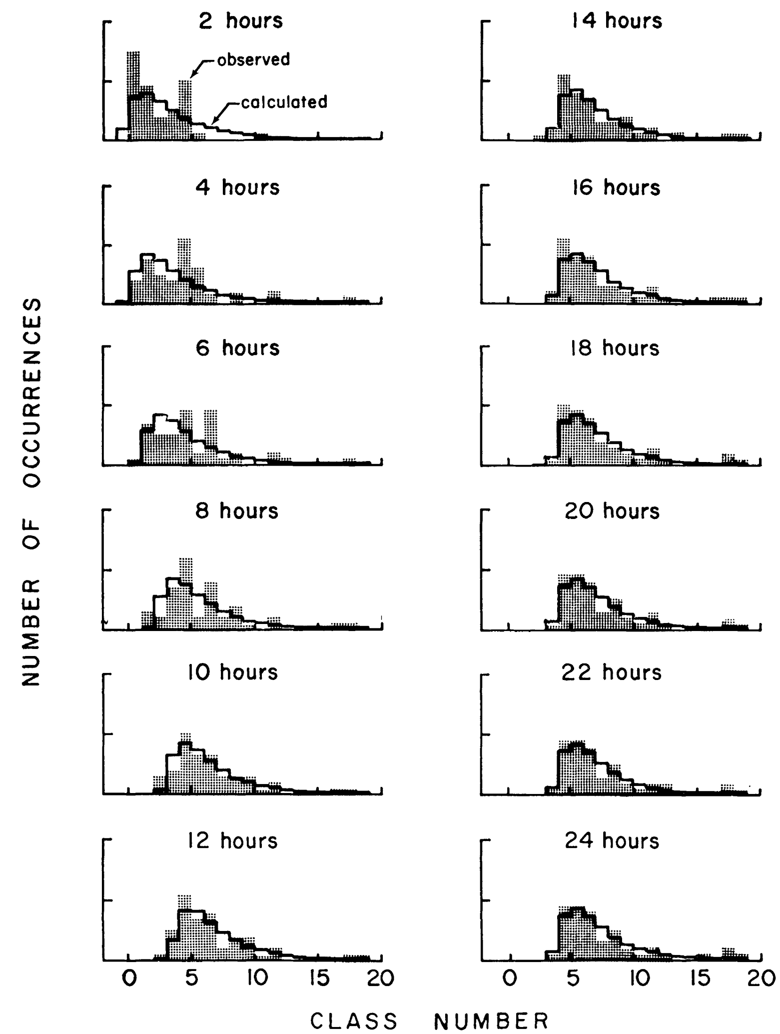

Fig. 4.

Observed and predicted histograms for 24-hour rainfall at 2-hour intervals, 0.25-inch class width, winter storms, Taylor Creek, Florida, rain gage No. 3.

noted that the base length is the same for all histograms for a season at a location. That is, as in Figure 1, the base length for 24 hours is 20 classes. The same base length was used for the first 11 time intervals also to provide uniformity in the simultaneous fitting. This was achieved by adding empty classes beyond

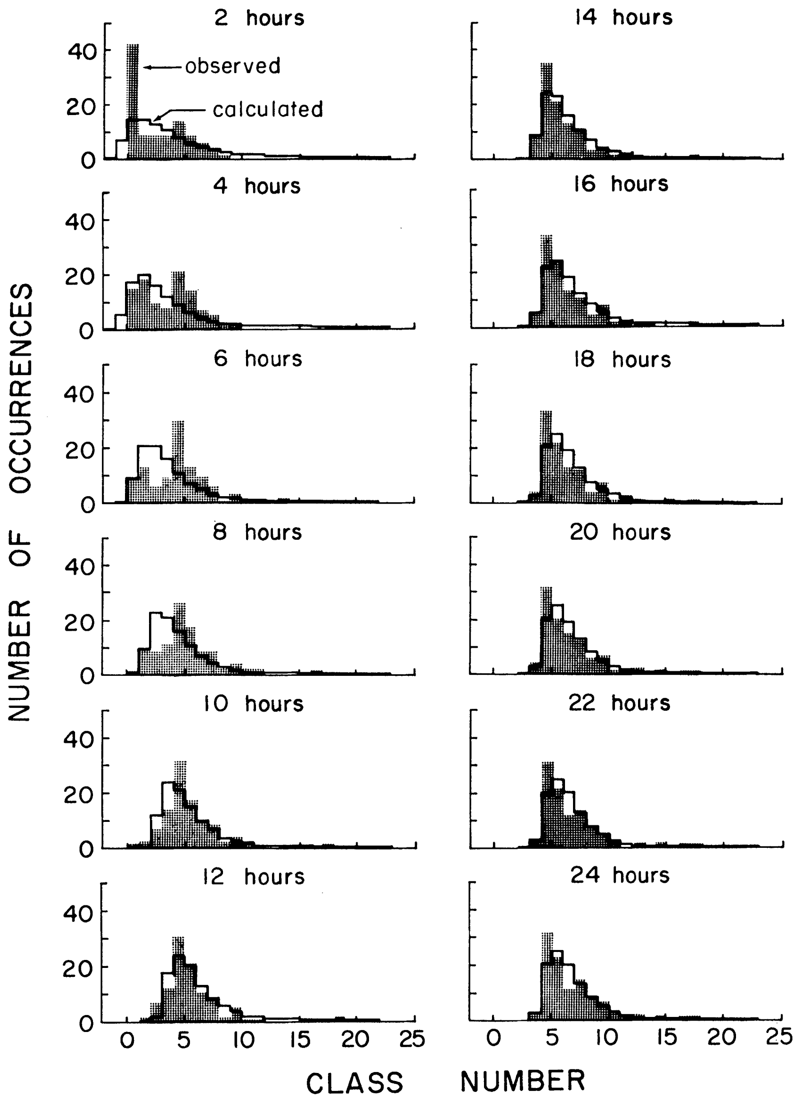

Fig. 5.

Observed and predicted histograms for 24-hour rainfall at 2-hour intervals, 0.25-inch class width, summer storms, USWB, Atlanta, Georgia.

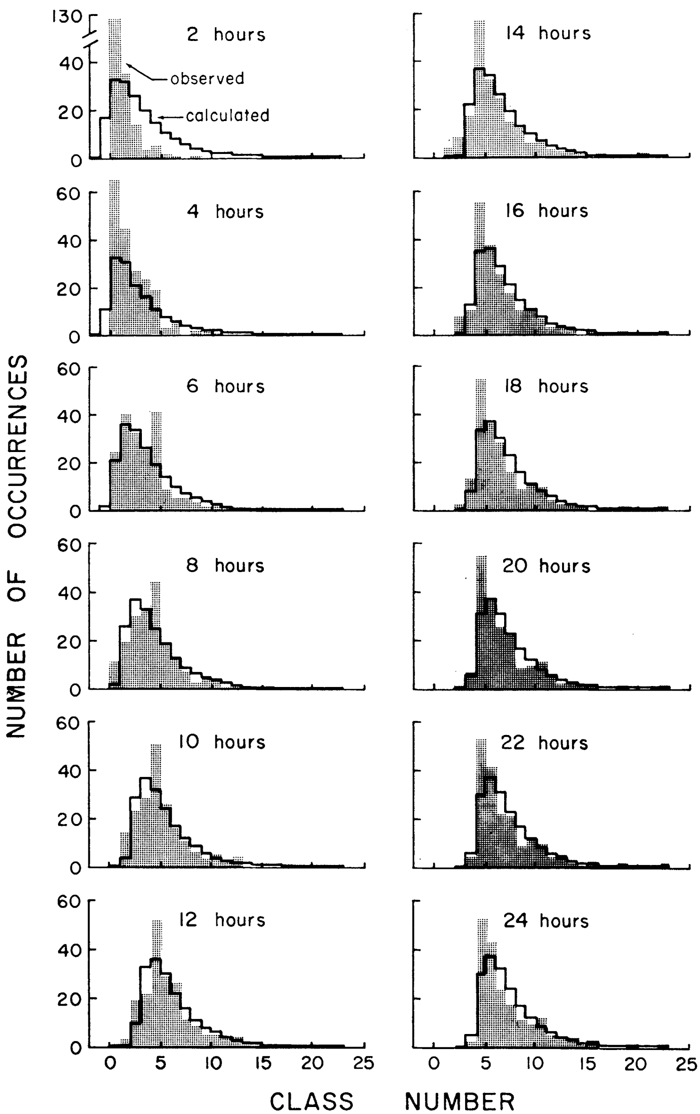

Fig. 6.

Observed and predicted histograms for 24-hour rainfall at 2-hour intervals, 0.25-inch class width, winter storms, USWB, Atlanta, Georgia.

Table 2.
Optimized storm-rainfall model coefficients and correlation coefficients.

	Eqn (3)		Eqn (4)		m	Corr. Coef.
	b_1	b_2	c_1	c_2		
Ahoskie Creek						
Summer	3.997	0.1268	0.0792	0.7712	1.519	0.873
Winter	4.200	0.0727	0.0299	0.5981	1.749	0.841
Taylor Creek						
Summer	3.740	0.1351	0.3669	0.5094	2.180	0.846
Winter	4.092	0.1003	0.0354	0.7084	1.811	0.868
Atlanta WB						
Summer	4.465	0.0630	0.2012	0.5686	1.955	0.864
Winter	4.515	0.0526	0.0339	0.6772	1.907	0.838

historical data for each of the first 11 intervals. The concept of empty classes applies (Snyder 1972), and they are part of the total information set for fitting. The sum of squares was minimized for the total 240 classes (20 classes by 12 time intervals). The simultaneous fitting did not result in a best-fit for any particular histogram, but rather a best-fit of all 12 histograms which provided an averaging and smoothing of any irregularities. Such an overall best-fit tends to further reduce the problem of individual outliers common in variates of hydrologic data samples.

Grant (1973) weighted the errors by classes between observed and predicted histograms to prevent a bias when fitting a two-parameter gamma distribution by least squares fitting techniques. The weight was calculated as $p_c(v)^{\gamma}$. $p_c(v)$ is the average value of the probability density function for the class. γ was found to be 0.75. A γ of 0.75 in this study resulted in a heavy tail for the predicted histograms. A value of 0.5 was selected and optimization of the parameter and coefficients was achieved efficiently. Values of the function coefficients, parameter m and correlation coefficients are shown in Table 2. Values of parameters a and k are shown for each time interval in Table 3. The observed histograms in Figures 1 through 6 clearly show the maximum number of occurrences in the first class interval for time interval number 1 (2 hours) at each location and season. This shape results from a combination of storm characteristics and the

Table 3.
Parameters a and k from optimized coefficients for storm-rainfall distribution by 2-hour interval.

	2-Hour time interval											
	1	2	3	4	5	6	7	8	9	10	11	12
Ahoskie Creek												
Summer												
a eqn (3)	1.000	−0.523	0.592	1.723	2.474	2.832	2.958	2.992	2.999	3.000	3.000	3.000
k eqn (4)	0.850	0.811	0.798	0.791	0.787	0.784	0.783	0.781	0.780	0.779	0.778	0.778
Winter												
a eqn (3)	−1.200	−0.905	−0.140	−0.817	1.687	2.317	2.693	2.881	2.960	2.988	2.997	2.999
k eqn (4)	0.628	0.613	0.608	0.606	0.604	0.603	0.602	0.602	0.601	0.601	0.601	0.601
Taylor Creek												
Summer												
a eqn (3)	−0.740	−0.267	0.822	1.892	2.570	2.872	2.971	2.995	2.999	3.000	3.000	3.000
k eqn (4)	0.876	0.693	0.632	0.601	0.583	0.571	0.562	0.555	0.550	0.546	0.543	0.540
Winter												
a eqn (3)	−1.092	−0.702	0.260	1.340	2.177	2.666	2.889	2.970	2.993	2.999	3.000	3.000
k eqn (4)	0.744	0.726	0.720	0.717	0.716	0.715	0.714	0.713	0.712	0.712	0.712	0.711
Atlanta												
Summer												
a eqn (3)	−1.465	−1.192	−0.470	0.467	1.370	2.076	2.538	2.796	2.921	2.973	2.992	2.998
k eqn (4)	0.770	0.669	0.636	0.619	0.609	0.602	0.597	0.594	0.591	0.589	0.587	0.585
Winter												
a eqn (3)	−1.515	−1.284	−0.659	0.187	1.053	1.787	2.319	2.656	2.844	2.936	2.976	2.992
k eqn (4)	0.711	0.694	0.689	0.686	0.684	0.683	0.682	0.681	0.681	0.681	0.680	0.680

Table 4.
Ahoskie Creek R. G. 3, summer storms (5/1–9/30), 24-hour dur., 2-hour int., 50 24-hour synthetic storms at 2-hour increments.

Accumulated rainfall, inches

No.	1	2	3	4	5	6	7	8	9	10	11	12
1	0.74	0.88	1.16	1.16	1.16	3.95	3.95	3.95	3.95	3.95	3.95	3.95
2	0.53	0.53	1.46	1.90	1.90	1.90	1.90	2.98	2.98	2.98	2.98	2.98
3	0.26	0.71	1.52	1.52	1.72	2.43	2.43	2.43	2.43	2.43	2.77	2.77
4	0.23	1.09	1.42	1.42	2.25	2.25	3.74	3.74	3.74	3.74	3.74	3.74
5	0.17	0.35	1.80	1.80	2.37	2.37	2.37	2.37	2.37	2.65	2.65	2.65
6	1.06	1.06	2.52	3.90	3.90	3.90	3.90	3.90	3.90	3.90	4.02	4.02
7	0.47	0.70	0.70	6.24	6.24	6.24	6.24	6.24	6.24	6.24	6.24	6.24
8	1.70	1.70	1.70	4.43	4.43	4.43	4.43	4.43	4.43	4.43	4.43	4.43
9	0.82	1.61	1.61	1.61	1.61	1.61	4.30	4.30	4.30	4.30	4.30	4.30
10	2.19	2.19	2.19	2.19	2.19	2.19	2.19	5.04	5.04	5.04	5.04	5.04
11	1.78	2.65	2.65	2.65	2.65	2.65	2.65	2.65	2.65	2.65	2.65	2.65
12	0.74	0.74	2.18	2.18	2.18	2.18	2.18	2.18	2.18	2.18	2.18	2.18
13	1.23	1.23	1.23	1.54	1.88	3.96	3.96	3.96	3.96	3.96	3.96	4.80
14	0.55	0.55	0.57	1.85	1.85	1.85	1.95	2.03	2.04	2.73	2.73	2.73
15	0.75	1.40	1.40	1.40	2.22	2.22	2.22	2.22	2.46	2.46	2.46	2.46
16	1.42	2.59	2.59	2.59	2.59	2.59	2.59	2.60	2.60	3.37	3.37	3.37
17	0.45	1.03	1.03	1.03	1.26	1.98	3.44	3.44	3.44	3.44	3.44	3.44
18	0.39	1.24	2.48	2.48	2.48	3.05	3.05	3.05	3.05	3.05	3.05	3.05
19	0.56	1.97	1.97	1.97	1.97	1.97	1.97	2.14	2.14	2.72	2.72	2.72
20	0.11	1.64	1.64	1.64	1.64	1.64	1.64	1.65	1.65	2.66	2.66	2.66
21	0.73	0.73	0.85	1.84	4.13	4.13	4.13	4.13	4.13	4.13	4.13	4.13
22	0.61	0.61	0.61	0.92	3.01	3.01	3.01	3.01	3.01	3.01	3.01	3.01
23	0.20	0.83	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87
24	0.17	3.43	3.43	3.43	3.43	3.43	3.43	3.43	3.43	3.43	3.43	3.43
25	0.58	1.34	1.53	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69	2.69

data available for the study. Clock-hour rainfall data were used in the study for both Taylor Creek and Atlanta. The starting time of the storm rainfall is defined only within a particular clock-hour. This may have resulted in slightly erroneous data for the first 2-hour time interval, especially for low intensity storms.

Table 4. (cont.)

No.	1	2	3	4	5	6	7	8	9	10	11	12
26	1.56	2.96	2.96	2.96	3.25	3.25	3.25	3.25	3.25	3.25	3.25	3.25
27	1.86	1.86	3.04	3.04	3.04	3.56	3.56	3.56	3.56	3.56	3.56	3.56
28	0.10	0.22	1.67	1.67	1.67	4.16	4.16	4.16	4.16	4.16	4.16	4.16
29	0.81	0.81	0.81	1.57	1.62	1.62	1.62	1.65	5.25	5.25	5.25	5.25
30	1.23	1.23	1.23	3.23	3.23	3.94	3.94	3.94	3.94	3.94	3.94	3.94
31	0.83	0.83	0.83	1.81	1.81	1.81	1.96	1.96	1.96	1.96	1.96	2.20
32	2.47	2.47	2.47	2.82	2.82	2.82	2.82	2.82	2.82	2.82	2.82	2.82
33	2.21	2.21	2.21	2.21	2.21	2.21	3.80	3.80	3.80	3.80	3.80	3.80
34	0.01	1.52	1.52	1.52	2.49	2.49	2.49	2.49	2.49	2.49	2.49	2.49
35	0.09	0.09	0.75	1.08	2.14	2.14	2.14	2.40	4.34	4.34	4.34	4.34
36	1.73	1.73	1.73	3.23	3.23	3.57	3.57	3.57	3.57	3.57	3.57	3.57
37	2.56	2.56	2.56	2.56	2.56	2.56	2.56	2.56	4.12	4.12	4.12	4.12
38	0.05	2.09	2.09	2.09	2.57	2.57	2.57	2.57	2.57	2.57	2.57	2.57
39	0.01	0.50	1.53	1.53	2.39	2.39	2.39	2.39	2.39	2.42	2.42	3.81
40	1.46	1.46	1.46	1.46	1.46	3.76	3.76	3.76	3.76	5.31	5.31	5.31
41	2.48	2.48	2.48	2.48	2.48	2.58	2.58	2.58	2.58	2.58	2.58	2.58
42	0.50	1.06	1.58	1.58	1.58	3.11	3.11	3.11	3.11	3.11	3.11	3.11
43	1.37	1.37	1.37	1.42	2.36	2.36	2.90	2.90	2.90	2.90	2.90	2.90
44	0.58	0.58	1.73	1.73	1.78	2.86	2.86	3.99	3.99	3.99	3.99	3.99
45	0.97	0.97	3.32	3.32	3.32	3.32	3.32	3.32	3.32	3.32	3.32	3.32
46	0.03	0.26	3.21	3.21	3.21	3.21	3.21	3.21	3.21	3.21	3.21	3.21
47	4.79	4.79	4.79	4.79	4.79	4.79	4.79	4.79	4.79	4.79	4.79	4.79
48	0.83	0.83	0.92	1.77	1.77	1.77	1.77	1.77	2.31	2.31	2.31	2.31
49	0.66	0.66	0.97	2.11	2.11	2.11	2.11	2.11	2.11	2.11	2.88	2.88
50	0.98	1.49	1.49	1.49	1.49	2.44	2.44	3.26	3.26	3.26	3.26	3.26

GENERATION OF SYNTHETIC STORM RAINFALL

The method of evaluating statistical frequency functions as presented in this paper is useable in generating synthetic values of storm rainfall. Pseudo-random normal numbers of zero mean and unit variance can be generated. The random numbers are transformed to variates of the embedded normal by adding the

Table 5.
Atlanta WB, winter storms, 24-hour duration, 2-hour int., 50 24-hour synthetic storms at 2-hour increments.

Accumulated rainfall, inches

No.	1	2	3	4	5	6	7	8	9	10	11	12
1	0.66	0.76	0.93	0.93	0.93	3.48	3.48	3.48	3.48	3.48	3.48	3.48
2	0.48	0.48	1.20	1.56	1.56	1.56	1.56	2.86	2.86	2.86	2.86	2.86
3	0.22	0.60	1.26	1.26	1.44	2.19	2.19	2.19	2.19	2.23	2.76	2.76
4	0.19	0.95	1.16	1.16	1.92	1.92	3.42	3.42	3.42	3.42	3.42	3.42
5	0.13	0.25	1.50	1.50	2.03	2.03	2.03	2.03	2.03	2.65	2.65	2.65
6	0.94	0.94	2.11	3.24	3.24	3.24	3.24	3.24	3.24	3.24	3.83	3.83
7	0.42	0.59	0.59	5.03	5.03	5.03	5.03	5.03	5.03	5.03	5.03	5.03
8	1.46	1.46	1.46	3.65	3.65	3.65	3.65	4.01	4.01	4.01	4.01	4.01
9	0.55	1.40	1.40	1.40	1.40	1.40	3.87	3.87	3.87	3.87	3.87	3.87
10	1.83	1.83	1.83	1.83	1.83	1.83	1.86	4.55	4.55	4.55	4.55	4.55
11	1.52	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25
12	0.66	0.66	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83
13	1.08	1.08	1.08	1.23	1.60	3.49	3.49	3.49	3.49	3.49	3.49	4.46
14	0.49	0.49	0.49	1.52	1.52	1.52	1.87	2.02	2.07	2.72	2.72	2.72
15	0.67	1.22	1.22	1.22	1.90	1.90	1.90	1.90	2.46	2.46	2.46	2.46
16	1.23	2.21	2.21	2.21	2.21	2.21	2.21	2.53	2.53	3.27	3.27	3.27
17	0.40	0.90	0.90	0.90	1.00	1.79	3.17	3.17	3.17	3.17	3.17	3.17
18	0.35	1.08	2.08	2.08	2.08	2.73	2.73	2.73	2.73	2.73	2.73	2.73
19	0.50	1.70	1.70	1.70	1.70	1.70	1.70	2.12	2.12	2.71	2.71	2.71
20	0.07	1.42	1.42	1.42	1.42	1.42	1.42	1.66	1.66	2.66	2.66	2.66
21	0.66	0.66	0.66	1.51	3.50	3.50	3.50	3.50	3.50	3.50	3.50	3.50
22	0.55	0.55	0.55	0.63	2.59	2.59	2.59	2.59	2.59	2.59	2.59	2.59
23	0.16	0.71	2.40	2.40	2.40	2.40	2.40	2.40	2.40	2.40	2.67	2.67
24	0.13	2.86	2.86	2.86	2.86	2.86	2.86	2.86	2.86	2.86	2.86	2.86
25	0.52	1.17	1.26	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.39	2.39

parametric mean, m, of equation (1). The variates are transformed by the transform function of equation (2) and multiplied by the class width to produce synthetic values.

In this generation of synthetic storm rainfall values, successive 2-hour cumulative amounts are not independent as in most other generation schemes. The

Table 5. (cont.)

No.	1	2	3	4	5	6	7	8	9	10	11	12
26	1.34	2.49	2.49	2.49	2.79	2.79	2.79	2.79	2.79	2.79	2.79	2.79
27	1.58	1.58	2.53	2.53	2.53	3.16	3.16	3.16	3.16	3.16	3.16	3.16
28	0.06	0.11	1.39	1.39	1.39	3.65	3.65	3.65	3.65	3.65	3.65	3.65
29	0.72	0.72	0.72	1.26	1.36	1.36	1.36	1.66	4.77	4.77	4.77	4.77
30	1.08	1.08	2.51	2.51	3.30	3.30	3.30	3.30	3.30	3.30	3.52	3.52
31	0.43	1.14	1.14	1.14	1.57	1.57	1.78	1.78	1.78	2.25	2.94	2.94
32	1.50	2.05	2.05	2.05	2.05	2.44	2.44	2.44	2.44	2.44	2.77	2.77
33	0.43	0.43	0.92	1.54	3.17	3.17	3.17	3.17	3.17	3.17	3.17	3.17
34	0.68	1.61	1.61	1.61	1.64	1.64	1.64	1.87	1.88	1.88	1.88	1.88
35	0.40	1.29	1.29	1.29	1.97	3.80	3.80	3.80	3.80	3.80	3.80	3.80
36	2.51	2.51	2.51	2.51	2.51	2.51	25.1	2.51	2.97	2.97	2.97	2.97
37	0.64	0.64	0.64	3.26	3.26	3.26	3.26	3.26	3.26	3.26	3.26	3.26
38	1.73	1.73	1.73	1.73	1.73	1.73	1.73	1.73	1.73	1.73	2.14	2.14
39	1.55	1.55	1.55	1.55	1.55	2.17	2.17	3.59	3.59	3.59	3.59	3.59
40	0.05	2.67	2.67	2.67	2.67	4.60	4.60	4.60	4.60	4.60	4.60	4.60
41	0.05	1.62	1.62	1.62	1.62	1.62	2.34	2.34	2.34	2.34	2.34	2.34
42	0.02	2.10	2.10	2.10	2.10	2.10	2.10	2.70	2.70	2.70	2.70	2.70
43	1.52	1.52	2.01	2.01	2.01	2.01	2.01	2.43	2.43	2.43	2.43	2.43
44	0.94	1.88	1.88	3.14	3.14	3.14	3.14	3.14	3.14	3.14	3.59	3.59
45	1.25	1.54	1.54	1.62	1.77	1.77	1.77	1.77	1.77	3.49	3.49	3.49
46	0.58	1.46	1.46	1.46	1.46	2.31	2.31	2.31	4.32	4.32	4.32	4.32
47	0.93	0.93	0.93	1.56	2.47	2.47	2.47	2.47	2.47	2.47	2.47	2.47
48	0.19	0.19	0.79	0.95	1.89	1.89	1.89	1.89	1.89	1.89	1.89	2.42
49	0.00	0.27	0.27	1.37	1.37	1.37	2.71	2.71	2.71	2.71	2.71	2.71
50	0.43	1.50	1.50	2.52	2.52	2.52	2.52	2.52	4.01	4.01	4.01	4.01

cumulative values of storm rainfall cannot decrease for successive 2-hour intervals, i. e., negative increments of rainfall. Fiering & Jackson (1971) suggest regeneration until equal or greater values are obtained. However, this would result in most generated storms having continuously increasing amounts of rainfall for a total duration of 24 hours. As indicated in the previous section, differ-

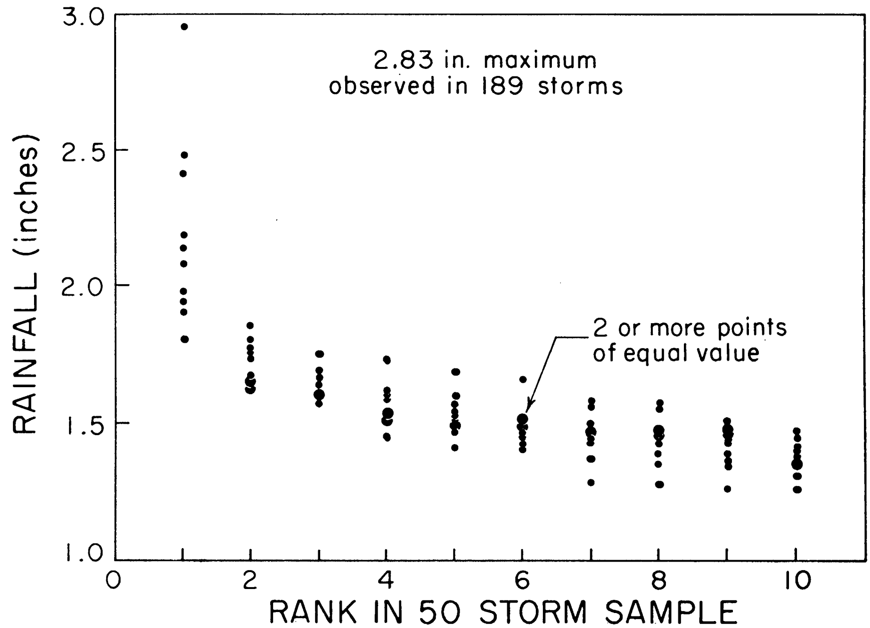
2.83 in. maximum
observed in 189 storms
2 or more points
of equal value
RAINFALL (inches)
3.0
2.5
2.0
1.5
1.0
0
2
4
6
8
10
RANK IN 50 STORM SAMPLE

Fig. 7.

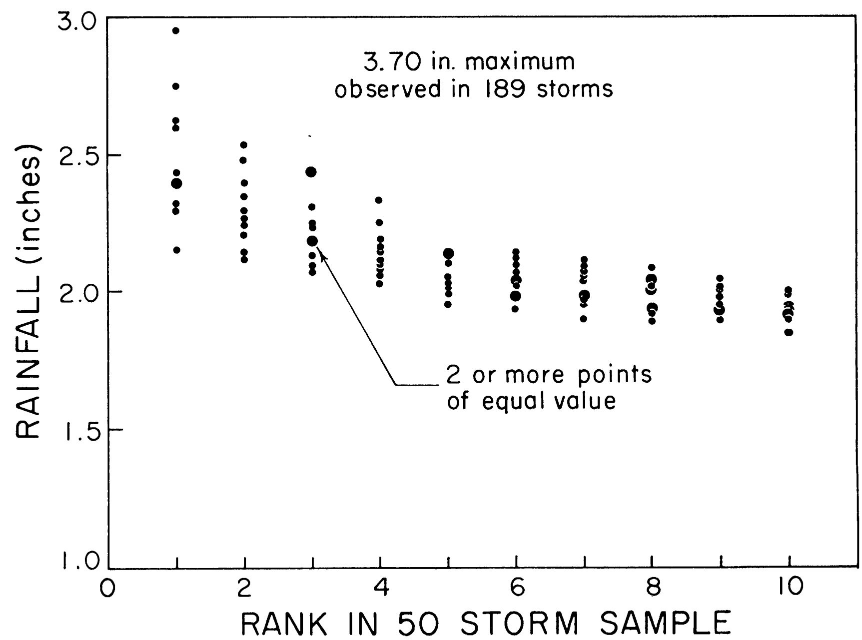
3.70 in. maximum
observed in 189 storms
2 or more points
of equal value
RAINFALL (inches)
3.0
2.5
2.0
1.5
1.0
0
2
4
6
8
10
RANK IN 50 STORM SAMPLE

Fig. 8.

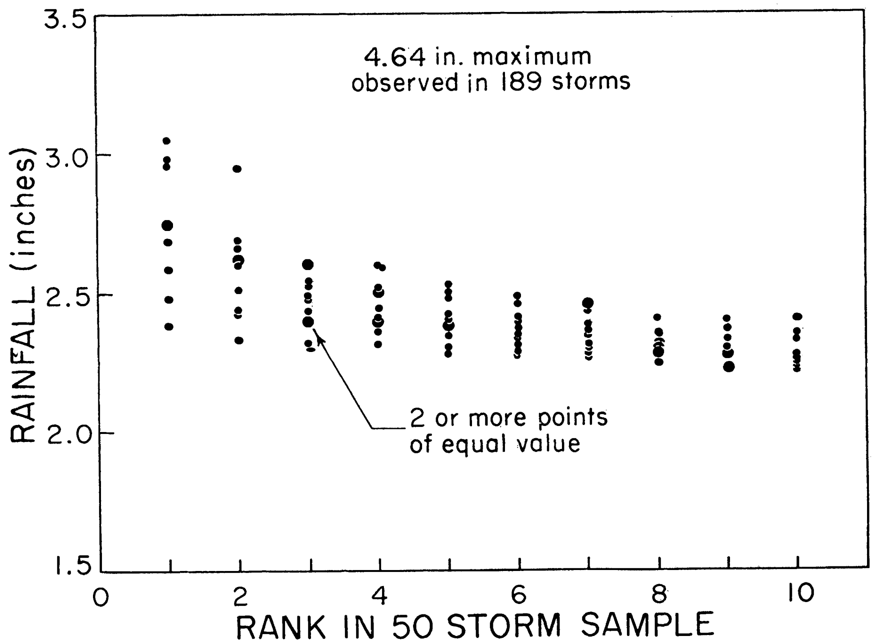

Fig. 9.
Maximum generated 18-hour rainfall, highest 10 from ten 50-storm samples, Atlanta, Georgia, winter storms.

ent storms have different durations and there are lapsed times during storms without rainfall occurrence. The procedure used in this study consisted of testing generated values against the previous value, and, if less than the previous value, it is set equal to that value. This method results in some lapsed intervals, without rainfall, as actually occur in historical data. This is especially true for the Taylor Creek region.

Fig. 7.
Maximum generated 6-hour rainfall, highest 10 from ten 50-storm samples, Atlanta, Georgia, winter storms.

Fig. 8.
Maximum generated 12-hour rainfall, highest 10 from ten 50-storm samples, Atlanta, Georgia, winter storms.

Two hundred 24-hour cumulative rainfall values were generated for Ahoskie Creek, Taylor Creek, and Atlanta for each season. Samples of 50 generated storms are shown in Tables 4 and 5 for Ahoskie Creek summer season and Atlanta winter season, respectively. Extreme values can be noted: 4.79 inches during the first 2-hour interval at Ahoskie Creek, 5.18 inches for Atlanta summer, 6.87 inches for Taylor Creek summer, and 4.02 inches for Taylor Creek winter. The largest generated 24-hour amount was 6.87 inches for Taylor Creek summer.

An additional ten sets of 50 storms were generated using the optimized parameter and coefficients for Atlanta Weather Bureau winter storms. Four 2-hour time intervals were selected for presentation: 3rd, 6th, 9th, and 12th. The ten largest generated values in each set of 50 storms were plotted for the four selected intervals. Plotted values are shown in Figures 7, 8, 9, and 10 for the 3rd, 6th, 9th, and 12th 2-hour intervals, respectively. Maximum values observed for each interval during the 23-year record are shown on each figure.

The largest range of values occurs in rank 1 of each 50-storm set. In general, ranges are reduced for ranks 2 through 10. As expected, the simulated highest of 50 storms are significantly smaller than observed highest of 189 storms.

SUMMARY

The three-parameter lognormal distribution was used to approximate histograms of 2-hour storm rainfall. Nonlinear least squares were used to fit the distributions of twelve 2-hour intervals simultaneously. The simultaneous fitting provides continuity between successive time intervals. The method was applied to historical rainfall data for summer and winter storms at Ahoskie Creek, North Carolina, Taylor Creek, Florida, and Atlanta, Georgia.

Optimized transform parameters and mean of the embedded normal were used with a random number generating scheme to generate synthetic values of time-distributed storm rainfall. Generated synthetic values are realistic for the climatic regions represented.

Further investigations are needed to evaluate other models. The methodology is appropriate for 2-hour intervals of storm rainfall. Shorter intervals should be investigated along with other criteria of storm selection, season, and climatic region.

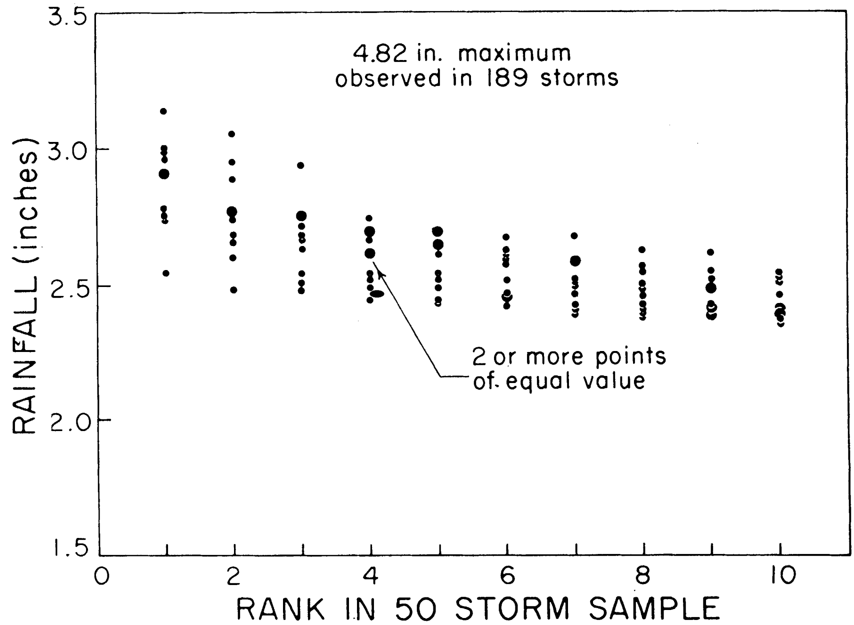

Fig. 10.
Maximum generated 24-hour rainfall, highest 10 from ten 50-storm samples, Atlanta, Georgia, winter storms.

REFERENCES

Fiering, M. B. & Jackson, B. B. (1971) *Synthetic Streamflows.* Water Resources Monograph 1. Amer. Geophysical Union, Washington, D. C.

Grant, J. L. (1973) Statistical Frequency Analysis by Optimization of Density Functions to Histograms. Unpublished Doctoral Dissertation, Georgia Institute of Technology, Atlanta, Georgia. October 1973.

Heaton, L. R. (1968) An Investigation of the Variability of Rainfall Statistics Over a Small Area with Varying Rain Gage Density. Unpublished Master's Thesis, Texas A & M University, College Station, Texas. May 1968.

Linsley, R. K., Jr., Kohler, M. A. & Paulhus, J. L. H. (1949) *Applied Hydrology.* McGraw-Hill, New York. 689 pp.

Mills, W. C. (1965) Storm rainfall in central and southern Florida. *Proc. Soil and Crop Sci. Society of Florida,* Vol. 25, pp. 311–319.

Sinha, L. K., & Khanal, N. N. (1971) Estimation of Rainfall for the Kissimmee River Basin. Paper No. 71–738, Amer. Soc. of Agr. Engr., Winter Meeting, Chicago, Ill. Dec. 7–10, 1971.

Snyder, W. M. (1972) Fitting of distribution functions by nonlinear least squares. *Water Resources Research,* Vol. 8, No. 6. December 1972.

Snyder, W. M. (1975a) Time Series Data Analysis and Synthesis for Research Watersheds. U. S. Dept. of Agr., Agr. Res. Ser. ARS-S-76.

Snyder, W. M. (1975b) Continuous Seasonal Probability of Extreme Rainfall Events. Bulletin of the IAHS. Vol. 20, No. 2.

Snyder, W. M. & Wallace, J. R. (1974) Estimating the parameters of the log normal distribution. *Nordic Hydrology 5,* pp. 129–145.

U. S. Dept. of Agr., Soil Conservation Service (1964) Hydrology. National Engineering Handbook, NEH4.

Dept. of Comm., U. S. Weather Bureau (1973) Climatological Data, Hourly Precipitation Data, Georgia Section, September 1947 – August 1973.

Received August 1975.

Address:

Dr. W. G. Knisel,
U. S. Department of Agriculture,
Agricultural Research Service.

Mr. W. M. Snyder,
Hydraulic Engineers, USDA-ARS,
Southeast Watershed Laboratory,
Athens, Georgia 30601.