Simulation on combined rapid gravity filtration and backwash models
S. J. Han, C. S. B. Fitzpatrick and A. Wetherill

ABSTRACT
Combined rapid gravity filtration and backwash models have been applied to simulate filtration and backwash cycles. The simulated results from the backwash model suggest that an optimum air flow rate exists to maximise particle removal efficiency in the backwash operation for a certain backwash system. The simulation of combined rapid gravity filtration and backwash models suggests that the filter shouldn’t be completely cleaned up in the backwash and a certain amount of particles retained on filter grains after backwash can be beneficial for subsequent filtration runs. This is consistent with the experimental results in the literature.

Key words | backwash, rapid gravity filtration, simulation

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>particle concentration (mg l$^{-1}$)</td>
</tr>
<tr>
<td>$c_e$</td>
<td>effluent particle concentration (mg l$^{-1}$)</td>
</tr>
<tr>
<td>$c_{in}$</td>
<td>influent particle concentration (mg l$^{-1}$)</td>
</tr>
<tr>
<td>$d_c$</td>
<td>clean grain diameter (m)</td>
</tr>
<tr>
<td>$d_p$</td>
<td>particle diameter (m)</td>
</tr>
<tr>
<td>$D_z$</td>
<td>Axial dispersion coefficient (m$^2$s$^{-1}$)</td>
</tr>
<tr>
<td>$h$</td>
<td>head loss (m)</td>
</tr>
<tr>
<td>$J$</td>
<td>hydraulic gradient (-)</td>
</tr>
<tr>
<td>$J_0$</td>
<td>clean bed hydraulic gradient (-)</td>
</tr>
<tr>
<td>$L$</td>
<td>filter bed depth (m)</td>
</tr>
<tr>
<td>$\Delta P$</td>
<td>pressure drop (m)</td>
</tr>
<tr>
<td>$t$</td>
<td>time (s)</td>
</tr>
<tr>
<td>$t_c$</td>
<td>filtration time when the specific deposit reaches the value $\sigma_c$</td>
</tr>
<tr>
<td>$u$</td>
<td>superficial velocity (m s$^{-1}$)</td>
</tr>
<tr>
<td>$z$</td>
<td>position in the filter bed (m)</td>
</tr>
<tr>
<td>$\gamma_0$</td>
<td>constant (-)</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>porosity in filter (-)</td>
</tr>
<tr>
<td>$\varepsilon_0$</td>
<td>porosity in clean filter bed (-)</td>
</tr>
<tr>
<td>$\eta_0$</td>
<td>particle/filter grain transport efficiency in the clean filter bed (-)</td>
</tr>
<tr>
<td>$\eta_p$</td>
<td>particle/particle transport efficiency in the clean filter bed (-)</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>detachment coefficient in the backwash (s$^{-1}$)</td>
</tr>
<tr>
<td>$\mu$</td>
<td>water viscosity (Pa s)</td>
</tr>
<tr>
<td>$\rho_p$</td>
<td>particle density (kg m$^{-3}$)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>specific deposit (mg l$^{-1}$)</td>
</tr>
<tr>
<td>$\sigma_0$</td>
<td>specific deposit along the filter in the end of filtration run (mg l$^{-1}$)</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>transitional specific deposit (mg l$^{-1}$)</td>
</tr>
<tr>
<td>$\tau$</td>
<td>detachment coefficient in the filtration (s$^{-1}$)</td>
</tr>
<tr>
<td>$\pi_0$</td>
<td>constant (s$^{-1}$)</td>
</tr>
</tbody>
</table>

Greek symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>particle/filter grain attachment efficiency (-)</td>
</tr>
<tr>
<td>$\alpha_p$</td>
<td>particle/particle attachment efficiency (-)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>factor of particles deposited on the filter grain which act as additional collectors (-)</td>
</tr>
</tbody>
</table>

INTRODUCTION
Rapid gravity sand filtration is a process widely used in water treatment for removal of suspended particulate materials. The water to be treated flows through a packed...
Rapid gravity filtration and backwash models

A rapid gravity filtration model

A rapid gravity filtration model has been developed by the authors (Han et al. 2008) to describe an entire filtration run consisting of ripening, working and breakthrough stages. The model is summarized as below.

The mass balance within the filter is simplified as

$$ u \frac{dc}{dz} + \frac{dc}{dt} = 0 $$

(1)

For filtration with a clean filter, initial and boundary conditions can be defined as

$$ c = 0, \sigma = 0 \quad \text{for} \quad z \geq 0, t = 0 $$

(2)

$$ c = c_m \quad \text{for} \quad z = 0, t > 0 $$

(3)

The two-stage hypothesis is applied to describe the whole rapid gravity filtration process. The deposition rate at the first stage of the filtration is described as

$$ \frac{d \sigma}{dt} = \frac{3}{2} \frac{1 - \varepsilon_0}{d_c} \alpha \eta_0 \left[ 1 + 3 \right] \eta_0 \left( \frac{d_c}{d_c} \right)^2 \left( \frac{d_c}{d_c} \right)^2 \int_{0}^{t} u c dt \quad \sigma \leq \sigma_c $$

(4)

The deposition rate at the second stage of the filtration can be calculated by

$$ \frac{d \sigma}{dt} = \frac{3}{2} (1 - \varepsilon) \left( \alpha \eta_0 + \alpha \eta_0 \left( \frac{d_c}{d_c} \right)^2 \right) \left( N \right) uc - \sigma \eta_0 \left( \sigma - \sigma_c \right) \quad \sigma > \sigma_c $$

(5)

where

$$ N = \alpha \eta_0 \left( \frac{3}{2 \times 1000 \times \rho_f} \right) \left( \frac{d_c}{d_c} \right)^2 \int_{0}^{t} uc dt $$

$$ + \int_{0}^{t} \cdot \frac{1}{1000 \times \rho_f (1 - \varepsilon)} \left( \frac{d_c}{d_c} \right)^3 \frac{d \sigma}{dt} dt $$

N is an associated number of deposited particles which serve as additional collectors for a single filter grain. The first term and the second term on the right hand side of Equation (6) represent the number of deposited particles at

bed of granular filter media and particulate materials are collected by the granular filter media by various physico-chemical mechanisms. To avoid turbidity breakthrough periodic backwashing is required to dislodge the deposits from the filter. The cleaned up filter is then used for the next filtration run. The filtration and backwashing processes are inextricably linked for the successful operation of a rapid gravity filter.

Mathematical models with predictive capabilities are important tools for the design and operation of rapid gravity filters. There have been a few decades for attempting to mathematically describe rapid gravity filtration (Adin & Rehbn 1978; O’Melia & Ali 1978; Tien et al. 1979; Vigneswaran & Tulachan 1988; Darby et al. 1992; Tobiason & Vigneswaran 1994; Stevenson 1997). Generally speaking, there are two approaches applied for backwash modelling. A number of theoretical backwash studies have been carried out to predict the optimum backwashing condition for a certain backwash system. Some other researchers attempted to describe detachment rate during the backwash for the optimum backwashing condition exists in terms of efficiency and effectiveness.

As far as the authors are aware, there are no papers published for the simulation on a sequence of filtration and backwash cycles. Hall & Fitzpatrick (1998) developed a backwash model and applied the filtration model developed by Ives (1975) to calculate specific deposits along the filter in the end of filtration run as initial condition for the simulation of backwash model. However, they didn’t simulate the effect of backwash efficiency on a sequence of filtration runs. In this study, rapid gravity filtration and backwash models will be combined to simulate a sequence of filtration and backwash cycles. The purpose is to assess the interaction between filtration and backwash operations for a sequence of filtration/backwash cycles.
the first and second stages respectively. \( t_c \) corresponds to the transient concentration \( \sigma_c \). The detachment coefficient \( \sigma \) is calculated by

\[
\sigma = \sigma_0 u^{1.75} \quad (7)
\]

\( \sigma_0 \) is a constant for a specific filtration system.

On the basis of the model developed by Mays & Hunt (2005), the head loss through the filter can be calculated by

\[
h = J_0 \int_0^L \left( 1 + \frac{\gamma_0}{\rho_p} u^{-0.55} \sigma \right)^2 \, dz \quad (8)
\]

Suspended particle concentration and specific deposit profiles within the filter in time and space can be simulated by solving Equations (1)–(7). The head loss through the filter is calculated by Equation (8). The hydraulic gradient in clean filter bed \( J_0 \) can be experimentally obtained or calculated by the Carman-Kozeny equation

\[
\frac{\Delta \pi}{L} = \frac{150 \mu u (1 - \varepsilon_0)^2}{\varepsilon_0^2 \varepsilon_0^3} \quad (9)
\]

**A combined air-scour and water backwash model**

The traditional operation of backwash is by the use of only water with full fluidization. It has been well accepted that backwash by water fluidization alone is a weak washing method. The reason for that weakness, as discussed by Amirtharajah (1978), is attributed to a lack of any abrasion between the grains in a fluidized bed. Backwash is therefore usually assisted by an auxiliary scouring system such as surface wash or air scour. For combined water and air scour backwash, Amirtharajah (1984) observed a “collapse-pulsing” condition where there is the optimum backwash is in terms of efficiency and effectiveness. At “collapse-pulsing” backwashing operation, water velocity is in the range of 20–50% of the minimum fluidisation velocity. The simultaneous air scour and water backwash has been accepted as one of the best operation modes. In this study, the discussion will only focus on this backwash operation mode.

By neglecting radial flow and dispersion during the backwash, one can get the following mass balance

\[
u \frac{\partial c}{\partial z} - D_z \frac{\partial^2 c}{\partial z^2} + \frac{\partial \sigma}{\partial t} = 0 \quad (10)
\]

where \( D_z \) represents axial dispersion coefficient.

Huang & Basagoiti (1989) proposed that particle detachment rate at any time was assumed to be proportional to the specific deposit within the filter bed at that time

\[
\frac{d\sigma}{dt} = -\kappa \sigma \quad (11)
\]

With an assumption that detachment constant \( \kappa \) is independent of specific deposit, integrating Equation (11) gives

\[
\sigma = \sigma_0 \exp(-\kappa t) \quad (12)
\]

where \( \sigma_0 \) is specific deposit along the filter column at the start of backwash.

Differentiating Equation (12) gives

\[
\frac{d\sigma}{dt} = -\kappa \sigma_0 \exp(-\kappa t) \quad (13)
\]

Substituting Equation (13) into Equation (10), one can get

\[
u \frac{\partial c}{\partial z} - D_z \frac{\partial^2 c}{\partial z^2} = \kappa \sigma_0 \exp(-\kappa t) \quad (14)
\]

Boundary conditions are given by (Kalyuzhnyi et al. 2006)

\[
D_z \frac{\partial c}{\partial z} = u[c - c_{im}] \quad \text{for} \quad z = 0 \quad (15)
\]

\[
D_z \frac{\partial c}{\partial z} = 0 \quad \text{for} \quad z = L \quad (16)
\]

For the given values of specific deposit profile \( \sigma_0 \), axial dispersion coefficient \( D_z \) and detachment coefficient \( \kappa \), Equation (14) can be numerically solved to simulate effluent concentration profile with backwash time.

**RESULTS AND DISCUSSION**

**Model simulation**

The simulation of the combined filtration and backwash model is illustrated in Figure 1. In the simulation of the first filtration run, the filter bed is assumed clean. The outputs from the filtration model simulation include outlet...
concentrations, head loss, specific deposits. The simulation on filtration model has been discussed in detail in our previous paper (Han et al. 2008). The specific deposits along the filter in the end of filtration run will be used as the initial condition for the simulation on backwash model. The backwash model simulation will provide the data about effluent concentrations and specific deposits retained within filter media with backwash time. After the first filtration run, the specific deposits within filter media at the end of each backwash simulation will be used as the initial condition for the simulation on filtration model for the next filtration run.

Simulation of backwash model

For the backwash model, the specific deposits along the filter prior to backwash operation are calculated by filtration model with model parameters shown in Table 1. The undetermined parameters in backwash model are detachment coefficient $\kappa$ and axial dispersion coefficient $D_z$. The values of these two parameters are related to backwash velocity and physical and chemical properties of filter media and particles. The simulation will be carried out to analyze the effects of two parameters on backwash. In the simulation, backwashing water velocity was 8 m hr$^{-1}$ for 4 minutes.

Figure 2 shows effluent concentrations with backwash time at different values of detachment coefficients. The simulated results demonstrate the typical effluent concentration profiles in the backwash operation. At a relatively high $\kappa$ value of 0.015, the rate of particle removal from filter media is faster, as characterised by a higher peak effluent concentration and a sharper reduction in effluent concentration with backwash time. This is due to the fact that a higher $\kappa$ value means less strong interaction between particles and filter grains, which makes particles more easily dislodged from surfaces of filter grains.

Figure 3 illustrates suspended particle profiles along the filter with backwash time. It can be clearly demonstrated that suspended particle concentrations sharply decline from the top to the bottom of the filter in the backwash. This is because most particles are retained at the top of filter during the filtration. Therefore, more particles are dislodged from the top of filter during the backwash.

The axial mixing can be significant when combined air scour and water backwashing mode is applied. Figure 4 shows the effect of axial dispersion coefficients on suspended particle concentrations in the filter during backwash.

Table 1 | The values of model parameters for filtration model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$</td>
<td>0.8 m</td>
</tr>
<tr>
<td>$c_{in}$</td>
<td>5 mg l$^{-1}$</td>
</tr>
<tr>
<td>$u$</td>
<td>5 m hr$^{-1}$</td>
</tr>
<tr>
<td>$d_c$</td>
<td>0.8 mm</td>
</tr>
<tr>
<td>$d_p$</td>
<td>4 $\mu$m</td>
</tr>
<tr>
<td>$\gamma_0$</td>
<td>30</td>
</tr>
</tbody>
</table>

Figure 2 | Effluent concentration profiles in the backwash operation $D_z = 0.0001$. 

Input – inlet concentration, bed depth, flow rate, etc

Rapid gravity filtration model

Backwash model

Outputs – Effluent concentrations, head loss, etc

Figure 1 | Diagram of simulation on combined filtration and backwash models.
backwash operation. The comparison of suspended particle concentrations demonstrates that stronger axial mixing will bring more suspended particles from the upper layers to the lower layers of the filter during backwash. These results suggest that axial mixing will reduce particle removal efficiency in the backwash. However, the reason why the air scour is introduced in the backwash operation is to improve particle removal efficiency. As experimentally observed by Fitzpatrick (1993), the introduction of air will enhance fluid shear forces and other forces causing dislodgement of particles from filter grains. Therefore, it can be concluded that the introduction of air scour has two contradictory effects on particle removal efficiency. This suggests that an optimum air flow rate exists to maximise particle removal efficiency in the backwash operation for a certain backwash system.

Simulation of combined filtration and backwash models

Figure 5 shows effluent concentration profiles for three filtration and backwash cycles. It can be seen that there is little difference in effluent concentration profile for three consecutive filtration/backwash cycles. The same simulated results are generated for head loss profiles. This is due to the fact that the filter is almost completely cleaned up after backwash operation with a \( \kappa \) value of 0.01. The simulated results show that specific deposits along the filter are negligible after backwash.

The simulation is also conducted to investigate how the particles still retained on filter grains after backwash affect the following filtration run. Figure 6 illustrates the change in effluent concentration profile for three consecutive filtration/backwash cycles.
consecutive filtration/backwash cycles at a \( k \) value of 0.005. Compared to the first filtration run, the filter is ripened faster and the base line at working stage is lower in the second filtration run. However, the breakthrough occurs earlier in the second filtration run. This can be explained by the fact that there are a significant amount of particles still attached on filter grains after backwash operation at a \( k \) value of 0.005. These particles serve as additional collectors to catch suspended particles in the following filtration run. However, it can also be seen from Figure 6 that there is little difference in effluent concentration profile for filtration runs 2 and 3. With a constant \( k \) value of 0.005, simulated specific deposits along the filter maintain unchanged after each simulation on backwash. This gives a constant pre-condition for filtration runs after the first one.

The above simulation discusses two artificial scenarios of complete clean up and constant particle removal efficiency in consecutive backwash operations. In reality, the backwash operation may become gradually less efficient with consecutive filtration/backwash cycles, causing continuous accumulation of particles on filter grains. In this case, \( k \) value in the backwash model should be become smaller with consecutive filtration/backwash cycles. The simulated results in Figure 6 suggest that in the case with decrease in \( k \) value with consecutive filtration/backwash cycles, the filter can be continuously ripened faster and effluent concentrations at working stage can continuously become lower with filtration runs. However, the breakthrough will take place earlier and earlier and eventually the filter loses separation capability after a number of filtration/backwash cycles. The simulation on combined rapid gravity filtration and backwash models suggests that the filter should not be completely cleaned up in the backwash and a certain amount of particles retained on filter grains after backwash can be beneficial for the next filtration runs. This is consistent with the experimental results in the literature.

**CONCLUSIONS**

For combined air-scour and water backwash operation, axial mixing can be significant. The simulated results from backwash model show that axial mixing reduces particle removal efficiency in the backwash. However, the introduction of air enhances fluid shear forces and other forces causing dislodgement of particles from filter grains. This suggests that an optimum air flow rate exists to maximise particle removal efficiency in the backwash operation for a certain backwash system.

In the case with decrease in detachment coefficient with a sequence of filtration/backwash cycles, the filter can be continuously ripened faster and effluent concentrations at working stage can continuously become lower with filtration runs. However, the breakthrough will take place earlier and earlier and eventually the filter loses separation capability after a number of filtration/backwash cycles. The simulation on combined rapid gravity filtration and backwash models suggests that the filter should not be completely cleaned up in the backwash and a certain amount of particles retained on filter grains after backwash can be beneficial for the next filtration runs. This is consistent with the experimental results in the literature.

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**REFERENCES**


