Gerrard [9] by using three different methods to determine the length of the vortex formation region. The drag coefficient increased from 0.9 to 1.15 within the same Reynolds number range [14]. However, the reduction of the formation length and the increase of drag coefficient were also promoted at lower Reynolds numbers by attaching thin trip wires at 90 deg from the stagnation point [14]. It may be argued that the rise in Strouhal numbers observed by the present authors was produced by the shrink of the nearwake as triggered by the wall proximity. The confirmation of this argument appeared in the last statement of the conclusions by the authors.

The authors tried to compare their results with those reported by Goktun [15] and measured at $Re = 1.53 \times 10^5$. The smaller increase in Strouhal number, $f/f_0 = 1.04$, found by Goktun [15] at $\delta/D = 0.5$ had different physical origin. The upper end of the subcritical range led to the precritical beginning of transition to turbulence in the boundary layers. The latter started to displace the separation points on the cylinder in the downstream direction and the resulting narrower nearwake produced slight increase in the Strouhal number.

Finally I have two questions to the authors:

1. What was the actual value of the Strouhal numbers measured on isolated cylinders at the three Reynolds numbers tested?
2. Whether the departure between curves *b* and *c* in Fig. 10 was more pronounced when δ/D was decreased below one.

Additional References

13 Shaw, T. L., "Wake Dynamics of Two-Dimensional Structures in Confined Flow," *Proceedings 14th Congress IAUR*, 1971. Vol. 2, pp. 41-48.

14 Schiller, L., and Linke, W., "Pressure and Friction Drag of Cylinders at Reynolds Numbers 5,000 to 40,000 (in German)," *Zeitschrift fur Flugtechnik and Motorluftschiffahrt*, Vol. 24, No. 7, 1933, pp. 143-148.

15 Goktun, S., "The Drag and Lift Characteristics of a Cylinder placed near a Plane Surface," MSc thesis, Naval Postgraduate School, Monterey, Calif., 1975.

Authors' Closure

The authors wish to thank G. Buresti and M. M. Zdravkovich for their discussions.

The questions which have been formulated are mostly related to some aspects of the experimental procedure, so that the response will try to clarify each point separately.

- No appreciable variation of the Strouhal number with Re was detected for the isolated cylinders. In fact the differences were in the range of uncertainty of the velocity (±4 percent) and frequency measurements (0.7 ÷ 2 percent depending on the frequency value) and the values varied from 0.202 to 0.216.
- The upstream water velocity was controlled and no detectable variation could be found as a function of the gap size.
- The number of spectra to be averaged was chosen to be 16 because this was the minimum in order to achieve perfect repeatibility in relation to the amplitude and frequency resolution of the F.F.T. analyzer. The probe was positioned around $y/D = 0.5$ because this location was found to be the one where the peaks in the frequency spectra were the sharpest.
- No systematic variation of the bandwidth of the peaks was found (see Fig. 4 for an example).
- The authors did not perform systematic measurements on the maximum value of θ at values of d/D different from one. This was because the determination of each point was very time consuming, requiring the processing of data obtained after about 120 minutes of data acquisition.

Effects of Surface Solidification on the Stability of Multilayered Liquid Films[1]

F. I. P. Smith.[2] This paper extends the work of earlier papers the author had written conjointly. Here, the uppermost layer now has a solidified liquid-air interface which has the effect of stabilizing interfacial shear waves or destabilizing gravity-capillary waves associated with top-heavy stratification.

It would be of interest if the author had compared his result by taking $\beta = 0$ with that of Yih who found instability generated by viscosity variation in superposed layers flowing horizontally. Yih has a section on moving upper boundaries and concludes with a fairly detailed discussion on the various effects arising. Yih, C. S., "Instability due, to Variation of Viscosity," *Journal of Fluid Mechanics*, Vol. 27, 1967, p. 337.

(In the preprint examined an obvious error has crept into the second last sentence in the paragraph containing equation (6).)

Author's Closure

The author is grateful to Dr. Smith for pointing out an obvious error in the second to last sentence in the paragraph containing equation (6) of the original manuscript. The right sides of the inequality signs in this sentence should have been 1 instead of 0. As to the suggestion that we compare the results for $\beta = 0$ and $\gamma_2 = 1$ with that of Yih, we point out that there will be no flow when $\beta \rightarrow 0$ in our problem. While the present flow is driven by gravity the flow studied by Yih was driven by external shear force or pressure gradient.

Simple and Explicit Formulas for the Friction Factor in Turbulent Pipe Flow[1]

Don J. Wood.[2] It appears that this paper incorrectly claims the presentation of a significantly improved explicit friction factor formula. The statement that all the existing explicit equations are either simple and not accurate or accurate and not simple is refuted by the author himself in the paper. The equation presented by Swamee and Jain in reference [4] is certainly equivalent in simplicity to the one offered by the author and the claim of superior accuracy for the author's relation is simply not significant. The author states that his relation has maximum error of 1.5 percent compared to the Colebrook-White formula while the Swamee-Jain relation has a maximum error of nearly 3 percent. The author further states that Colebrook-White formula may be 3-5 percent or more in error compared to experimental results. Based on this reasoning it does not appear that the author contribution is of any great significance, since the Swamee-Jain formula is a more than adequate relationship. The author does present a formula for the smooth to rough transition which appears to represent the transitions well. However, it includes a factor *n*

[1] By S. P. Lin, published in the March, 1983 issue of the JOURNAL OF FLUIDS ENGINEERING, Vol. 105, pp. 119-121.

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[1] By S. E. Haaland, published in the March, 1983 issue of the JOURNAL OF FLUIDS ENGINEERING, Vol. 105, pp. 89-90.

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which apparently varies from situation to situation and therefore limits the value of this formula.

Author's Closure

The author agrees with Mr. Wood that the present work is a minute contribution to knowledge – but to give a significant contribution to knowledge was not the purpose of the note. Rather the formulas given are offered more as a service to the engineering community which use formulas for the friction factor in their work. It is simply a contribution in trying to make simple, explicit formulas more widely known because they may be of value to a large number of people.

I agree with Mr. Wood that the formula given by Swamee and Jain is also a simple and useful formula, but it is less accurate than the present one in the domain of small Reynolds numbers and large values of the relative roughness. I disagree with Mr. Wood's statement about the generalized formula. It is the freedom of choice of the exponent n which makes it useful. For the practical application mentioned, to model the recommendation given by AGA which accomodates newer experimental results for very large Reynolds numbers and very small relative roughness, the choice of $n=3$ was made, but, for example, $n=4$ would serve just as well.

I regret that I did not know about the work of Swamee and Jain before the second revision of the note, although I knew of the formula in question since I had found it myself. Instead of changing the note completely it was decided to make a reference to the work of Swamee and Jain in a single paragraph. However, it seems like a waste to bicker about "who made the greater contribution to knowledge" when there is so little to speak of in the first place. The important thing seems to be to make these simple, explicit formulas known to the mechanical engineering community.

Three-Dimensional Body-Fitted Coordinates for Turbomachine Applications[1]

C. Wayne Mastin.[2] The authors have applied the multigrid algorithm to the three-dimensional grid generation equations with only a modest increase in convergence rates. Since the equations to be solved are nonlinear, it was noted that the determination of optimal relaxation parameters and convergence rates is not feasible. However, a few general remarks on the iterative solution of elliptic equations will most likely clarify the numerical results of the authors and indicate cases where the multigrid method would be more effective. First of all, even the finest grid has a relatively large grid spacing when compared with the two-dimensional problem in reference [5]. The grids which are plotted also indicate that the forcing functions were not very large. Under these conditions no acceleration procedure would produce drastic increases in rates of convergence. Indeed, this can be observed by noting the small changes in convergence rates of the point SOR when the relaxation factor is changed. The multigrid method would be more appropriate on grids with more points and on grids generated by using larger forcing functions, since both of these conditions will decrease the convergence rate of point SOR. The efficiency of the scheme might also be improved by selecting the relaxation factor based on the size of the grid rather than use a constant factor throughout the multigrid cycle. Recall that the optimal point SOR relaxation factor for the solution of Laplace's equation varies from slightly over 1 on a coarse grid to almost 2 on a fine grid. Thus it might be profitable to use a smaller relaxation factor on the coarser grids.

Frank C. Thames.[3] The authors applied the multigrid technique (reference [1]) to accelerate the solution of the three-dimensional grid generation equations (reference [2]). These equations are coupled, quasilinear, and, usually stiff. In short, they are difficult to solve efficiently. Although the authors' purpose was laudable, their effort did not succeed. This is easily seen by a brief analysis of Fig. 7 in which the multigrid and SOR convergence histories are compared when optimum acceleration parameters, W, are used for each method. These data show that the rate of residual reduction (as implied by the slope of the Residual Norm versus Work Units curve) is the same for both methods. That is, the SOR method with optimum W is just as "fast" as the multigrid scheme with optimum W. The convergence history comparisons given in Figs. 4 through 6 are irrelevant since nonoptimum W's are used for one or the other method.

There are two reasons why the authors were unsuccessful in applying multigrid to the grid problem. First, the cycling algorithm they chose to implement (designated cycle A in reference [1]) is not as efficient as cycle C for nonlinear problems. Details of cycle C may be found in reference [1]. Second, the authors totally neglected perhaps the most important aspect of the multigrid algorithm - the smoothing ability of the relaxation method used to drive the multigrid cycle. This ability is given quantitative measure by the smoothing factor, μ, defined in reference [1] as

$$\mu \equiv \max\{g(\theta_x,\theta_y\theta_z)\},\ \frac{\pi}{2} \leqslant |\theta_x,\theta_y,\theta_z| \leqslant \pi$$

where $g(\theta_x,\theta_y,\theta_z)$ is the von Neumann damping ratio of the relaxation method being used. Note that μ measures the relaxation scheme's ability to damp (smooth) high frequency error components in the solution. Based upon this writer's calculations, a μ of 0.8 is the best SOR can attain for the three-dimensional grid problem. This value will rapidly deteriorate toward 1.0 as the number of unknowns is increased or if a severe grid stretch is involved. Moreover, SOR becomes unstable in the low frequencies for a somewhat moderately skewed grid (cross derivative terms in equations (4) through (6) become large). Note that the grids in the paper were rather badly skewed (Figs. 8 and 9). The authors avoided, to some extent, the poor smoothing factor problems of SOR in two ways: (1) Only very small grids - 1377 points (17 × 9 × 9) - were generated. Typical, "small" three-dimensional grids have about 30,000 points. (2) The authors used to stretching functions (P, Q, and R in equations (4) through (6)). Thus, the adverse effects of stretching and first derivatives on μ were avoided. In summary, the poor performance of the multigrid scheme seen by the authors does not result" . . .from the poor interpolation scheme as well as the small number of grids." Rather, the poor performance is due to the inadequate smoothing factor of the SOR method. The most probable value of μ for their results is 0.95 - which is poor. A value of μ this large causes multigrid to run as if only the fine grid was being relaxed which is consistent with the authors' observation: " . . .it was found that the only important factor is the number of sweepts on the finest grid." The three-dimensional ADI method [3] on the other hand, is capable of attaining values of $\mu=0.6$. The method is well

[1]By R. Camarero and M. Reggio, published in the March, 1983 issue of the JOURNAL OF FLUIDS ENGINEERING, Vol. 105, pp. 76–82.

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