which apparently varies from situation to situation and therefore limits the value of this formula.

Author's Closure

The author agrees with Mr. Wood that the present work is a minute contribution to knowledge – but to give a significant contribution to knowledge was not the purpose of the note. Rather the formulas given are offered more as a service to the engineering community which use formulas for the friction factor in their work. It is simply a contribution in trying to make simple, explicit formulas more widely known because they may be of value to a large number of people.

I agree with Mr. Wood that the formula given by Swamee and Jain is also a simple and useful formula, but it is less accurate than the present one in the domain of small Reynolds numbers and large values of the relative roughness. I disagree with Mr. Wood's statement about the generalized formula. It is the freedom of choice of the exponent n which makes it useful. For the practical application mentioned, to model the recommendation given by AGA which accomodates newer experimental results for very large Reynolds numbers and very small relative roughness, the choice of $n=3$ was made, but, for example, $n=4$ would serve just as well.

I regret that I did not know about the work of Swamee and Jain before the second revision of the note, although I knew of the formula in question since I had found it myself. Instead of changing the note completely it was decided to make a reference to the work of Swamee and Jain in a single paragraph. However, it seems like a waste to bicker about "who made the greater contribution to knowledge" when there is so little to speak of in the first place. The important thing seems to be to make these simple, explicit formulas known to the mechanical engineering community.

Three-Dimensional Body-Fitted Coordinates for Turbomachine Applications[1]

C. Wayne Mastin.[2] The authors have applied the multigrid algorithm to the three-dimensional grid generation equations with only a modest increase in convergence rates. Since the equations to be solved are nonlinear, it was noted that the determination of optimal relaxation parameters and convergence rates is not feasible. However, a few general remarks on the iterative solution of elliptic equations will most likely clarify the numerical results of the authors and indicate cases where the multigrid method would be more effective. First of all, even the finest grid has a relatively large grid spacing when compared with the two-dimensional problem in reference [5]. The grids which are plotted also indicate that the forcing functions were not very large. Under these conditions no acceleration procedure would produce drastic increases in rates of convergence. Indeed, this can be observed by noting the small changes in convergence rates of the point SOR when the relaxation factor is changed. The multigrid method would be more appropriate on grids with more points and on grids generated by using larger forcing functions, since both of these conditions will decrease the convergence rate of point SOR. The efficiency of the scheme might also be improved by selecting the relaxation factor based on the size of the grid rather than use a constant factor throughout the multigrid cycle. Recall that the optimal point SOR relaxation factor for the solution of Laplace's equation varies from slightly over 1 on a coarse grid to almost 2 on a fine grid. Thus it might be profitable to use a smaller relaxation factor on the coarser grids.

Frank C. Thames.[3] The authors applied the multigrid technique (reference [1]) to accelerate the solution of the three-dimensional grid generation equations (reference [2]). These equations are coupled, quasilinear, and, usually stiff. In short, they are difficult to solve efficiently. Although the authors' purpose was laudable, their effort did not succeed. This is easily seen by a brief analysis of Fig. 7 in which the multigrid and SOR convergence histories are compared when optimum acceleration parameters, W, are used for each method. These data show that the rate of residual reduction (as implied by the slope of the Residual Norm versus Work Units curve) is the same for both methods. That is, the SOR method with optimum W is just as "fast" as the multigrid scheme with optimum W. The convergence history comparisons given in Figs. 4 through 6 are irrelevant since nonoptimum W's are used for one or the other method.

There are two reasons why the authors were unsuccessful in applying multigrid to the grid problem. First, the cycling algorithm they chose to implement (designated cycle A in reference [1]) is not as efficient as cycle C for nonlinear problems. Details of cycle C may be found in reference [1]. Second, the authors totally neglected perhaps the most important aspect of the multigrid algorithm - the smoothing ability of the relaxation method used to drive the multigrid cycle. This ability is given quantitative measure by the smoothing factor, μ, defined in reference [1] as

$$\mu \equiv \max\{g(\theta_x,\theta_y\theta_z)\},\ \frac{\pi}{2} \leqslant |\theta_x,\theta_y,\theta_z| \leqslant \pi$$

where $g(\theta_x,\theta_y,\theta_z)$ is the von Neumann damping ratio of the relaxation method being used. Note that μ measures the relaxation scheme's ability to damp (smooth) high frequency error components in the solution. Based upon this writer's calculations, a μ of 0.8 is the best SOR can attain for the three-dimensional grid problem. This value will rapidly deteriorate toward 1.0 as the number of unknowns is increased or if a severe grid stretch is involved. Moreover, SOR becomes unstable in the low frequencies for a somewhat moderately skewed grid (cross derivative terms in equations (4) through (6) become large). Note that the grids in the paper were rather badly skewed (Figs. 8 and 9). The authors avoided, to some extent, the poor smoothing factor problems of SOR in two ways: (1) Only very small grids - 1377 points (17 × 9 × 9) - were generated. Typical, "small" three-dimensional grids have about 30,000 points. (2) The authors used to stretching functions (P, Q, and R in equations (4) through (6)). Thus, the adverse effects of stretching and first derivatives on μ were avoided. In summary, the poor performance of the multigrid scheme seen by the authors does not result" . . .from the poor interpolation scheme as well as the small number of grids." Rather, the poor performance is due to the inadequate smoothing factor of the SOR method. The most probable value of μ for their results is 0.95 - which is poor. A value of μ this large causes multigrid to run as if only the fine grid was being relaxed which is consistent with the authors' observation: " . . .it was found that the only important factor is the number of sweepts on the finest grid." The three-dimensional ADI method [3] on the other hand, is capable of attaining values of $\mu=0.6$. The method is well

[1] By R. Camarero and M. Reggio, published in the March, 1983 issue of the Journal of Fluids Engineering, Vol. 105, pp. 76–82.

[2] Department of Mathematics and Statistics, Mississippi State University, Miss. 39726.

[3] NASA Langley Research Center, Hampton, VA.

documented and is a trivial, straightforward extension of the two-dimensional version.

Additional References

1 Brandt, A., "Multi-Level Adaptive Solutions to Boundary-Value Problems," *Mathematics of Computation*, Vol. 31, No. 133, Apr. 1977, pp. 333–390.

2 Mastin, C. W., and Thompson, J. F., "Transformation of Three-Dimensional Regions onto Rectangular Regions by Elliptic Systems," *Numerische Mathematik*, Vol. 29, 1978, pp. 397–407.

3 Dwoyer, D. L., and Thames, F. C., "Accuracy and Stability of Time-Split Finite Difference Schemes," AIAA Paper No. 81-1005, June 1981.

Authors' Closure

In agreement with Professor Mastin's comment, it has been the authors' experience that the convergence of the multigrid algorithm is improved when the number of points on the finest grid, and consequently the number of available grids, is increased. However due to the excessive memory, and to a lesser extent, computing time requirement this was not feasible for the three-dimensional configurations studied in the present work. The suggestion of using a different relaxation factor for each grid would likely improve the convergence of the overall multigrid scheme and we agree with the reasons given by Professor Mastin. It is our impression that the search for a combination of such factors to yield an "optimum" strategy would be time consuming.

The effectiveness of the multigrid scheme as pointed out by Mr. Thames also depends on the smoothing ability of the basic relaxation scheme and on the particular multigrid cycle used. Both of these have been investigated to some extend. For example line relaxation, and alternating line and column relaxation when applied to the two-dimensional problem were found [1] to improve the convergence. Similar findings resulted when applying cycle C [2]. Both of these improvements result in increased programming complexity and it is difficult to assess the correct trade-off between these additional difficulties and the improvements in the smoothing factor, μ.

In attempting to produce grids with less distortion, nonzero stretching functions P, Q and R were obtained from the values of these evaluated at the boundaries and interpolated within the domain as suggested by reference [3]. A systematic and consistent method could not be found by the present authors and it is suggested a better approach would be to let the boundary nodes where Dirichlet conditions are applied free to move in such a way as to keep the stretching functions zero.

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3 Thomas, P. D., "Composite Three-Dimensional Grids Generated by Elliptic Systems," *AIAA Jour.*, Vol. 20, 1982, pp. 1195–1202.

Non-Newtonian Liquid Blade Coating Process[1]

Brian G. Higgins.[2] The concept of imposing a prescribed pressure drop across a coating applicator to control coating thickness was described by Beguin [8] in a patent nearly 30 years ago. However, the theoretical underpinnings of such a strategy were addressed only quite recently in the open literature (Ruschak [4], Higgins and Scriven [9]). Dr. Hwang takes the concept still further by applying it to blade coaters. His work is thus a welcomed addition to the coating literature, especially since, unlike his predecessors, it is supported in part by experimental data, a rare accomplishment given the proprietary nature of the coating industry.

Dr. Hwang has provided an approximate analysis of a complex free surface flow problem which agrees surprisingly well with experimental data. This is encouraging for others working in the field and the author is to be congratulated for his efforts. There are, however, a number of uncertainties concerning the range of validity of the author's analysis that should be pointed out, especially for those readers who are interested in applying his approximate analysis to related problems.

Equation (22), the basis for the author's pressure drop calculations, is undefined for noninteger values of the power law exponent n when the local gap width of the channel $h(x)$ is less than twice the final film thickness h_0. This deficiency in the analysis arises because of an approximation made by the author in the integration of equation (4) as given in the Appendix. It can be removed, however, by accounting correctly for the absolute sign in equation (9) and subsequent equations given thereafter; the correct procedure for handling the absolute sign is given by Flumerfeldt, et al. [10]. Note in Hwang's experiment $h(x) > 2h_0$ except when the coater was operated at zero or close to zero vacuum [Fig. 3]. Thus for his study the approximation made in the Appendix appears to be adequate.

The applicability of the lubrication approximation for the analysis of coating flows is another area of uncertainty. It follows from the data given in Fig. 3 that the quantity $\overline{BC}/h_{30}$ (blade length/minimum gap width) was never greater than five in Hwang's experiments, and indeed in one experiment it was as low as 2.3. Since the development lengths (inlet port and exit effects) for nearly rectilinear flow beneath the blade are of the order of one to two gap widths (Silliman [11], Silliman and Scriven [12]), it is unlikely that the lubrication approximation can always be justified for experiments of this type. The good agreement between theory and experiment displayed in Fig. 3 for $\overline{BC}/h_{30} = 2.3$ may be fortuitous or it may be supporting evidence for Ruschak's [4] limiting case, i.e., the imposed pressure drop is balanced almost entirely by capillary pressure of the upstream and downstream menisci, viscous pressure drop being unimportant.

When viscous pressure drop is important and the lubrication approximation cannot be justified, numerical simulation is often in order. Numerical simulation of steady Newtonian flow with a free surface is now feasible and in some research laboratories quite routine. For example, Saito and Scriven [13] have recently undertaken a detailed analysis of slot coating, a close relative of blade coating, using the Galerkin finite element method. In that study, approximations of the type made by Hwang and others are examined and their range of validity established.

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[2] The Institute of Paper Chemistry, Appleton, WI 54912.