documented and is a trivial, straightforward extension of the two-dimensional version.

Additional References

1 Brandt, A., "Multi-Level Adaptive Solutions to Boundary-Value Problems," *Mathematics of Computation*, Vol. 31, No. 133, Apr. 1977, pp. 333–390.

2 Mastin, C. W., and Thompson, J. F., "Transformation of Three-Dimensional Regions onto Rectangular Regions by Elliptic Systems," *Numerische Mathematik*, Vol. 29, 1978, pp. 397–407.

3 Dwoyer, D. L., and Thames, F. C., "Accuracy and Stability of Time-Split Finite Difference Schemes," AIAA Paper No. 81-1005, June 1981.

Authors' Closure

In agreement with Professor Mastin's comment, it has been the authors' experience that the convergence of the multigrid algorithm is improved when the number of points on the finest grid, and consequently the number of available grids, is increased. However due to the excessive memory, and to a lesser extent, computing time requirement this was not feasible for the three-dimensional configurations studied in the present work. The suggestion of using a different relaxation factor for each grid would likely improve the convergence of the overall multigrid scheme and we agree with the reasons given by Professor Mastin. It is our impression that the search for a combination of such factors to yield an "optimum" strategy would be time consuming.

The effectiveness of the multigrid scheme as pointed out by Mr. Thames also depends on the smoothing ability of the basic relaxation scheme and on the particular multigrid cycle used. Both of these have been investigated to some extend. For example line relaxation, and alternating line and column relaxation when applied to the two-dimensional problem were found [1] to improve the convergence. Similar findings resulted when applying cycle C [2]. Both of these improvements result in increased programming complexity and it is difficult to assess the correct trade-off between these additional difficulties and the improvements in the smoothing factor, μ.

In attempting to produce grids with less distortion, nonzero stretching functions P, Q and R were obtained from the values of these evaluated at the boundaries and interpolated within the domain as suggested by reference [3]. A systematic and consistent method could not be found by the present authors and it is suggested a better approach would be to let the boundary nodes where Dirichlet conditions are applied free to move in such a way as to keep the stretching functions zero.

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Non-Newtonian Liquid Blade Coating Process[1]

Brian G. Higgins.[2] The concept of imposing a prescribed pressure drop across a coating applicator to control coating thickness was described by Beguin [8] in a patent nearly 30 years ago. However, the theoretical underpinnings of such a strategy were addressed only quite recently in the open literature (Ruschak [4], Higgins and Scriven [9]). Dr. Hwang takes the concept still further by applying it to blade coaters. His work is thus a welcomed addition to the coating literature, especially since, unlike his predecessors, it is supported in part by experimental data, a rare accomplishment given the proprietary nature of the coating industry.

Dr. Hwang has provided an approximate analysis of a complex free surface flow problem which agrees surprisingly well with experimental data. This is encouraging for others working in the field and the author is to be congratulated for his efforts. There are, however, a number of uncertainties concerning the range of validity of the author's analysis that should be pointed out, especially for those readers who are interested in applying his approximate analysis to related problems.

Equation (22), the basis for the author's pressure drop calculations, is undefined for noninteger values of the power law exponent n when the local gap width of the channel $h(x)$ is less than twice the final film thickness h_0. This deficiency in the analysis arises because of an approximation made by the author in the integration of equation (4) as given in the Appendix. It can be removed, however, by accounting correctly for the absolute sign in equation (9) and subsequent equations given thereafter; the correct procedure for handling the absolute sign is given by Flumerfeldt, et al. [10]. Note in Hwang's experiment $h(x) > 2h_0$ except when the coater was operated at zero or close to zero vacuum [Fig. 3]. Thus for his study the approximation made in the Appendix appears to be adequate.

The applicability of the lubrication approximation for the analysis of coating flows is another area of uncertainty. It follows from the data given in Fig. 3 that the quantity $\overline{BC}/h_{30}$ (blade length/minimum gap width) was never greater than five in Hwang's experiments, and indeed in one experiment it was as low as 2.3. Since the development lengths (inlet port and exit effects) for nearly rectilinear flow beneath the blade are of the order of one to two gap widths (Silliman [11], Silliman and Scriven [12]), it is unlikely that the lubrication approximation can always be justified for experiments of this type. The good agreement between theory and experiment displayed in Fig. 3 for $\overline{BC}/h_{30} = 2.3$ may be fortuitous or it may be supporting evidence for Ruschak's [4] limiting case, i.e., the imposed pressure drop is balanced almost entirely by capillary pressure of the upstream and downstream menisci, viscous pressure drop being unimportant.

When viscous pressure drop is important and the lubrication approximation cannot be justified, numerical simulation is often in order. Numerical simulation of steady Newtonian flow with a free surface is now feasible and in some research laboratories quite routine. For example, Saito and Scriven [13] have recently undertaken a detailed analysis of slot coating, a close relative of blade coating, using the Galerkin finite element method. In that study, approximations of the type made by Hwang and others are examined and their range of validity established.

Additional References

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Author's Closure

The author appreciates the complimentary remarks by the discusser. Criticism can be made to non-Newtonian power law fluids on dimension and sign. The confusion with sign occurs when the shear rate is negative (reverse flow) and the power law exponent n is less than one and noninteger values. Some researchers [7, 14, 15] prefer to use the absolute sign as in equation (9). Others [16] completely ignore the absolute sign. When a reverse flow or $\partial p/\partial x$ exists, it is preferred to use Newtonian flow for determining the signs. Using the absolute sign in equation (9) doesn't alleviate the problem in the Appendix. In fact, without the approximation made in the Appendix, u_3 will not be able to be expressed analytically. Subsequently, we won't be able to represent equation (22) analytically.

According to H. Schlichting [6, p. 170], the fully developed flow is formed at about $x=0.16\ \rho\ u_w h^2 3/\mu$. For the worst case which I have presented in the paper, $\rho=830$ kg/m^3, $u_w=89$ mm/s, $h3=0.28$ mm, $\mu=0.04$ pascal-s; therefore, $x=0.023$ mm. Given the geometry of the device with blade length $\overline{BC}=0.64$ mm, 96 percent of the flow in region 3 are in fact fully developed. Tuntiblarphol and Tallmadge's [2] experimental work shows that the deviations at $\overline{BC}/h_{30}$ of 1 from even the simplest lubrication theory are relatively small. Only at high imposed pressure difference, the pressure drop is balanced almost entirely by capillary pressure. The paper presents the cases with the imposed pressure difference ranging from 0 to 1400 pascal. Ruschak's [4] assumption of the upper and lower free surfaces being pinned on the edges of the applicator does not apply to most of the practical cases. This was also confirmed by Saito and Scriven [13] with finite element method.

As the discusser pointed out, Saito and Scriven [13] have undertaken numerical simulation of steady Newtonian flow only. The author presents an attempt to solve non-Newtonian flow in blade coatings.

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