

## Optimal design of storm water inlets for highway drainage

John W. Nicklow and Anders P. Hellman

### ABSTRACT

A new methodology and computational model are developed for direct evaluation of an optimal storm water inlet design. The optimal design is defined as the least-cost combination of inlet types, sizes and locations that effectively drain a length of pavement. Costs associated with inlets are user-defined and can include those associated with materials, installation and maintenance, as well as other project-related expenses. Effective drainage is defined here as maintaining a spatial distribution of roadway spread, or top width of gutter flow, that is less than the allowable width of spread. The solution methodology is based on the coupling of a genetic algorithm and a hydraulic simulation model. The simulation model follows design guidelines established by the Federal Highway Administration and is used to implicitly solve governing hydraulic constraints that yield gutter discharge, inlet interception capacity and spread according to a design storm. The genetic algorithm is used to select the best combination of design parameters and thus solves the overall optimization problem. Capabilities of the model are successfully demonstrated through application to a hypothetical, yet realistic, highway drainage system. The example reveals that genetic algorithms and the optimal control methodology comprise a comprehensive decision-making mechanism that can be used for cost-effective design of storm water inlets and may lead to a reduced overall cost for highway drainage.

**Key words** | storm water, inlet design, optimal control, genetic algorithm

**John W. Nicklow** (corresponding author)  
Department of Civil and Environmental  
Engineering,  
Southern Illinois University Carbondale,  
Carbondale,  
IL 62901,  
USA  
Tel: +1 618 453 3325;  
Fax: +1 618 453 3044;  
E-mail: [nicklow@engr.siu.edu](mailto:nicklow@engr.siu.edu)

**Anders P. Hellman**  
Grundvattenteknik AB,  
Falun,  
Sweden

### INTRODUCTION

Highways are designed to facilitate traffic movement at specified service levels. The accumulation of water on roadways, however, can pose a significant threat to traffic safety and highway functionality by disrupting traffic flow, increasing vehicular skid distance, raising the potential for hydroplaning and accelerating pavement deterioration (AASHTO 1991; Brown *et al.* 1996). (The term pavement as used herein refers to an asphalt or concrete roadway) As a result, design, installation and maintenance of facilities to remove highway runoff comprise integral parts of the overall roadway design process and can comprise more than 25% of total highway construction costs (Mays 1979). The objective in designing such drainage facilities is ultimately to collect runoff in gutters and intercept flow at storm water inlets in a

manner that provides a high degree of traffic safety while limiting costs.

There are two categories of design variables that determine the effectiveness of storm water inlets in intercepting roadside gutter flow: (1) the type and size of inlets to be utilized, and (2) the required number and corresponding locations of inlets to be installed. The interactions between these variables and the resulting gutter flow characteristics are complicated. First, there are many available sizes of each of the four major inlet types (grates, curb-openings, slotted-drains and combination inlets). Second, the same inlet may operate hydraulically as a weir at shallow water depths and as an orifice when the inlet is submerged. Finally, different gutter geometries and inlet configurations will cause a wide variation in top width of

gutter flow, or roadway spread. Fortunately, generalized design procedures are well documented by many urban drainage criteria and design manuals (Brown *et al.* 1996; Johnson & Chang 1984; IDOT 1989). The real difficulty for the designer, however, lies in finding the combination of these design parameters that solves the following optimization model:

*Minimize*→the material, installation, maintenance and other costs associated with inlets used in design.  
*Subject To*→(i) the physical laws that govern pavement drainage, and (ii) bound constraints on the spread of water onto the pavement.

According to a selected combination of design parameters, the designer traditionally solves the governing equations to evaluate the effectiveness of each inlet and the resulting spatial distribution of spread. If the computed roadway spread exceeds that which is allowable, typically established by local, state or federal guidelines, an alternative set of design parameters must be selected and the process repeated. Consequently, determination of the *most economical* design that effectively drains a section of highway becomes an even more computationally intensive and time-consuming, if not impossible, task. Using this iterative, trial and error technique, every potential design alternative would require evaluation before an optimum could be declared. Furthermore, if one design appears more costly than another, it is nearly impossible to ascertain whether the difference in cost is merely due to poor selection of design variables or due to real differences in drainage effectiveness.

This paper discusses the development of the first optimal control methodology for solving the storm water inlet design problem. The optimal control approach that is implemented is based on a computational interface between hydraulic simulation and optimization techniques. Applications of this interfacing methodology have been increasingly popular in the fields of hydraulics and hydrology and have provided solutions for complex problems in areas of reservoir management (Nicklow & Mays 2000; Unver & Mays 1990; Yeh 1985), bioremediation design and groundwater management (Wanakule *et al.* 1986; Yeh 1992; Minsker & Shoemaker 1998) and the design and operation of water distribution systems (Cunha

& Sousa 2000; Sakarya & Mays 2000). Nicklow (2000) provides a comprehensive review of the benefits of this approach, which include a reduced need for additional simplifying assumptions about the physical system in order to reach an optimal solution and a decrease in size of the overall optimization problem. Such an approach, however, has not been previously applied to solve the inlet design problem. Elimam *et al.* (1989) evaluated the optimal design of gravity sewer networks using linear programming. Templeman & Walters (1979) and Li & Matthew (1990) applied variations of dynamic programming to determine the optimal geometric layout of subsurface drainage networks, while Mays (1979) applied a similar methodological approach for least-cost culvert design. When applied to the inlet design problem, the methodology is formulated to directly evaluate the least-cost design that will effectively drain a roadway section, thus overcoming limitations associated with trial-and-error design approaches, and in turn leads to a reduced total highway construction cost. Furthermore, by incorporating standard hydraulic simulation procedures that have been widely accepted in engineering practice, the optimal control model attempts to integrate existing technologies and improve the practical utility of the new methodology.

## PROBLEM FORMULATION

For the storm water inlet problem, the matrix of decision (design) variables is comprised of inlet types and sizes, as well as the number of inlets required and their location. The state (dependent) vector is the spatial variation of spread along the pavement and the resulting cost of the design. The problem can be expressed mathematically as

$$\text{Min } Z = \sum_{j=1}^I n_j C_j \quad (1)$$

subject to

$$T_{i+1} = f(T_i, U_i) \text{ for } 1 \leq i \leq I-1 \quad (2)$$

$$0 \leq T_i \leq T_{\max} \quad (3)$$

$$j \in J^* \quad (4)$$

where  $Z$  is the total cost of inlets,  $n_j$  is the number of inlets of type  $j$  used in the design,  $C_j$  is the cost associated with inlet type  $j$ ,  $J$  is the number of different types, including sizes, of inlets used in the design,  $T_{i+1}$  is the roadway spread at discrete location  $i+1$ , which depends on the spread at the adjacent upstream location  $i$ ,  $I$  is the total number of discrete locations at which spread is evaluated,  $U_i$  represents the design variables, if any, that are implemented at section  $i$ ,  $T_{\max}$  is the maximum permissible spread, set as a design criterion and  $J^*$  represents the set of predefined inlet types and sizes that are made available for a particular design application.

Equation (1) is the separable objective function to be minimized and represents the total cost of inlets, which can include material, installation (i.e. construction), maintenance and other expenses. Although the current formulation considers only user-defined, fixed unit costs, they could easily be permitted to vary (i.e. volume discounts based on the quantity of inlets or the number of inlet types used). In addition, while other objectives are possible for inlet design, such as minimizing the number of required inlets or minimizing cumulative spread, the function will implicitly depend on the current set of design variables. Equation (2) is the transition, or simulation, constraint that represents the governing hydraulic relationship of highway drainage and describes system behavior in response to the current design alternative. For example, the spread of water at a particular location will depend on the decision of whether to install an inlet immediately upstream of that location, as well as the decision regarding the type and size of inlet to be used. Finally, Equations (3) and (4) are bound constraints used to define a feasible range for roadway spread and inlets selected for the design. The overall problem can therefore be translated as the determination of the types, sizes and locations of particular inlets that both minimize costs and effectively drain the pavement at a level of spread that does not exceed that which is allowable.

The inlet design problem formulation represents a large-scale, nonlinear programming problem for which there are no explicit solution schemes. The formulation can be solved, however, by interfacing a hydraulic

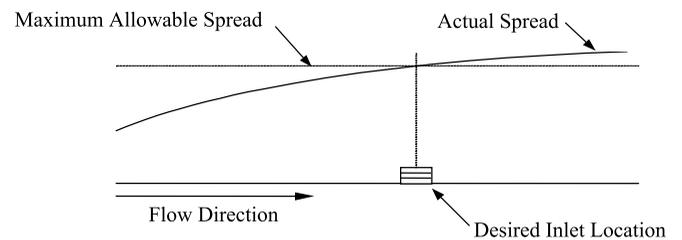


Figure 1 | Desired inlet location

simulation model with a genetic algorithm (GA). The simulation model is used to implicitly solve the transition constraint that yields gutter discharge, interception capacity and spread according to design flow criteria. Subsequently, the GA is used to select the best combination of design parameters to solve the overall optimization problem.

## HIGHWAY DRAINAGE SIMULATION

The simulation model developed for the optimal control methodology was written specifically for this project and is based upon design guidelines established by the Federal Highway Administration (Brown *et al.* 1996) and further explained by Nicklow (2001). While other, more advanced and even more accurate, simulation methods could be integrated to solve the inlet design problem, a large majority of consultant designers and state agencies commonly rely upon these guidelines in some form, whether applied directly or integrated into a prepackaged design model. The simulation model described herein is specifically comprised of preprocessing and drainage computation modules. The overall model handles all hydrologic and hydraulic computations and is capable of evaluating the maximum spacing of a series of predefined inlet types, without exceeding allowable spread criteria. The desired inlet location, shown in Figure 1, thus lies perpendicular to where actual spread intercepts the allowable spread.

To begin, the simulator requires input information that can be divided into the following five categories:

1. Pavement data—roughness, gutter geometry, cross-sectional and longitudinal slopes.

2. Rainfall data—discrete intensity—duration—frequency (IDF) data for a design storm.
3. Longitudinal slope—slope at discrete positions along the pavement.
4. Additional flow—additional offsite runoff added to total gutter flow at discrete positions.
5. Inlet characteristics—type, size and placement order of inlets.

The full length of the pavement considered is divided into a number of evenly distributed shorter segments. Although the length of individual segments can be user-defined, the default length is one unit (e.g. 1 m). Data points, located at the end of each segment, represent locations where flow rate, spread and other hydraulic characteristics are computed. The preprocessor is used to initially place an inlet at the most upstream data point, just downstream of the beginning of the pavement section, and the drainage module is called to compute the spread immediately prior to that location. If the allowable spread has not been reached, the inlet is moved to the next adjacent downstream data point; otherwise, the inlet is kept at this location. The procedure is then repeated for each downstream data point until all desired inlet locations are identified and all inlets have been positioned along the user-defined length of pavement. It should also be noted that inlets are automatically placed at all identified sag locations and immediately upstream of roadway intersections. To position one inlet typically requires 5–15 calls to the drainage module by the preprocessor. These figures depend, of course, on the particular application. However, they would likely be reduced should longer pavement segments be used (i.e. greater than the default), but at the expense of solution accuracy. An alternate method for determining inlet locations was also investigated. This method utilized a GA, along with a penalty function, and will be discussed later in detail.

As noted, the drainage module is used to compute the spread at each data point and thus provides the resulting spatial variation of spread along the roadway. The framework of the drainage module is illustrated in Figure 2. When called, the model first computes the drainage area that contributes flow to the current data point, as shown in Figure 3. The design gutter flow rate at the considered

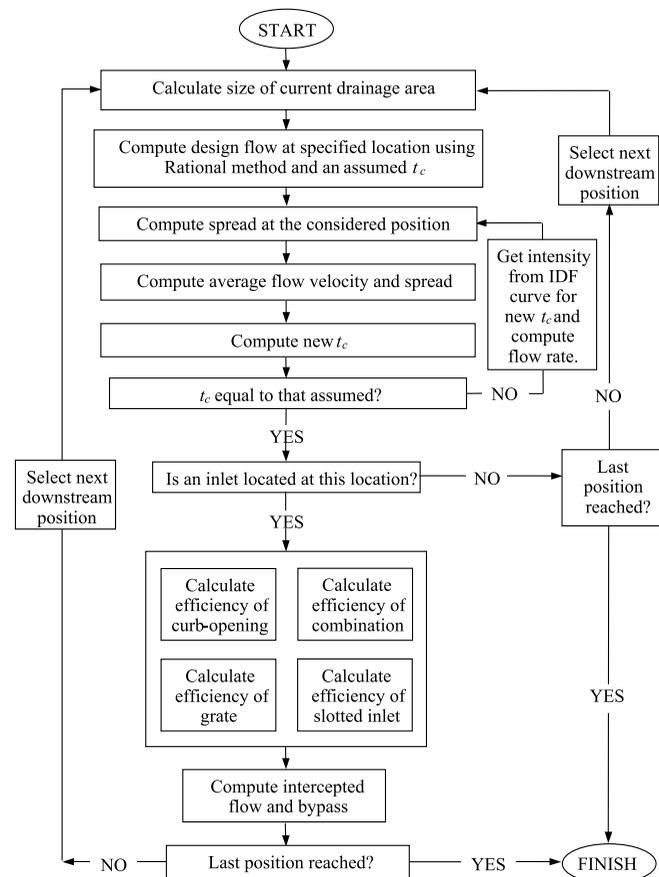


Figure 2 | Computational framework for drainage module

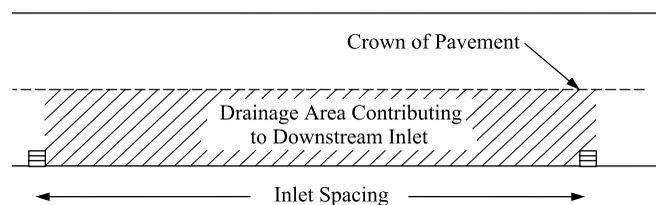


Figure 3 | Size of drainage area

location is then found using the rational equation, using rainfall intensity that is based on an assumed time of concentration ( $t_c$ ) and adding the result to any bypass flow from upstream inlets and any additional offsite runoff. Since contributing pavement areas are typically small, times of concentration are often less than five minutes. For most applications, however, rainfall data for such durations are not available. Therefore, based on this minimum

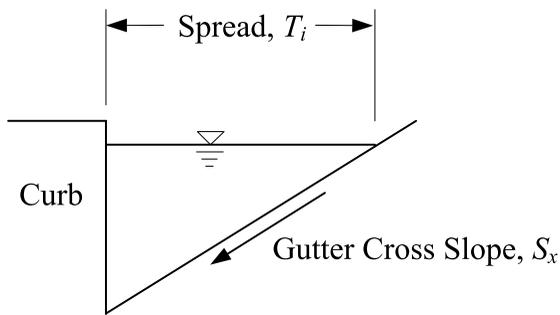


Figure 4 | Triangular gutter section

value recommended by Brown *et al.* (1996) for design applications, a value of  $t_c$  equal to five minutes is initially assumed.

Spread at each data point is computed using established relationships between gutter geometry and flow rate. For triangular gutter sections, shown in Figure 4, the relationship between spread at location  $i$ ,  $T_i$ , in meters and gutter flow,  $Q$ , in  $\text{m}^3/\text{s}$  is expressed as (Izzard 1946)

$$Q = \frac{0.376}{n} S_x^{1.67} S_L^{0.5} T_i^{2.67} \quad (5)$$

where  $n$  is the Manning roughness coefficient,  $S_x$  is the cross-sectional slope of the gutter and  $S_L$  is the longitudinal slope of the pavement. Modified relationships for other gutter configurations, including depressed and parabolic sections, are available in Brown *et al.* (1996) and Nicklow (2001).

Average spread over the drainage segment must be computed in order to obtain an improved estimate of the time of concentration, and thus intensity, for use in the rational equation. The average spread between two data points is defined for a triangular gutter as

$$T_a = (0.65)(T_2) \left[ \frac{1 - (T_1/T_2)^{8/3}}{1 - (T_1/T_2)^2} \right]^{3/2} \quad (6)$$

where  $T_a$  is the average spread,  $T_1$  is the spread at the upstream inlet or starting position and  $T_2$  is the spread at the downstream location.

The average velocity between the upstream and downstream locations is evaluated as the flow rate divided by

the effective flow area at the average spread. Subsequently, the travel time of flow in the gutter section is found by dividing the flow length by the average flow velocity. The result is added to the overland flow time for the pavement section, estimated using the kinematic wave equation, to yield  $t_c$ . The computed time of concentration is compared to the assumed value, and if any variation exists the computed value is used in another iteration of the rational method to obtain a better estimate of gutter flow rate. The exception to invoking another iteration of the rational method is if the time of concentration is less than or equal to the minimum of five minutes.

The interception capacity and corresponding bypass flow for each inlet is evaluated by

$$Q_i = QE \quad (7)$$

and

$$Q_b = Q - Q_i \quad (8)$$

where  $Q$  is the total gutter flow rate at the inlet,  $E$  is the efficiency of the inlet, which varies according to inlet type, as well as gutter geometry,  $Q_i$  is the interception capacity and  $Q_b$  is the flow that bypasses the inlet and continues further downstream to the next inlet. The model then continues by repeating the spread computation at each successive, downstream position, until the entire spatial variation of spread for the roadway is obtained. A final step in the simulation process involves tabulation of the design costs. Based on user-defined costs for each inlet type and size, the total cost is evaluated and stored as the current objective function value. It is important to note that the relationships for efficiency of other inlet types and gutter geometries vary and are numerous. The reader should refer to Brown *et al.* (1996) for a detailed listing of appropriate relationships. In addition, the reader should note that the existence of these multiple relationships imply that a single, continuous simulation equation does not exist.

## APPLIED GENETIC ALGORITHMS

Genetic algorithms (GAs) are a robust, heuristic search procedure that rely on probabilistic search rules.

Developed by Holland (1975), they represent an attempt to adapt the evolution observed in nature to problems in which traditional, deterministic search techniques have difficulties. Although there is no rigorous definition that applies to all GAs, most are at least characterized by the following common elements: ranking and selection of solutions according to fitness, crossover to produce new offspring solutions and random mutation of the new offspring (Mitchell 1996; Haupt & Haupt 1998). Through these elements, GAs simulate survival and generation-based propagation of those solutions that have the fittest objective function values (Belegundu & Chandrupatla 1999).

GAs are different from common gradient-based optimization techniques in that they require no derivative information about the objective function or constraints. Instead, the objective function magnitude, rather than derivative terms, is used to display incrementally better solutions. This characteristic makes GAs amenable for application to nonconvex, highly nonlinear and even discontinuous problems, such as the inlet design problem, for which traditional optimization techniques would fail (Goldberg 1989). The method has thus been increasingly popular for solving optimization and control problems in a wide variety of fields, including water resources engineering (see Wang 1991; Esat & Hall 1994; McKinney & Lin 1994; Ritzel *et al.* 1994; Oliveira & Loucks 1997; Reis *et al.* 1997; Savic & Walters 1997; Wardlaw & Sharif 1999; Hilton & Culver 2000). For a discussion of the detailed framework of genetic algorithms, the reader is also referred to Goldberg (1989), Mitchell (1996) and Haupt & Haupt (1998).

The GA developed for the inlet design problem begins by randomly generating an initial population of chromosomes, consisting of 25 potential design alternatives. The size of the population remains constant throughout each generation of the GA. Although the user is given the flexibility to change the population size, 25 is a default and was selected by testing various population ranges and evaluating the corresponding number of generations required for solution convergence. In addition, the computational time required for convergence was considered. A significantly smaller or larger population resulted in additional generations for convergence without a

corresponding reduction in inlet design cost. Furthermore, larger populations tended to result in excessive, and often wasteful, computational times. Consider that, for each chromosome, the drainage simulation module must be called to evaluate flow characteristics and a resulting objective function value. Since each module run requires approximately 30 seconds to execute, the computational time associated with a larger population quickly increases.

Each member of the initial population is encoded using an array of positive integers, called genes, which represent decision variables. Each integer, or gene, is associated with a particular inlet type and size. The integer coding is established within a separate database, and when the integers are sequenced within a chromosome they describe characteristics of a particular design. For example, after each available inlet type and size has been assigned an integer, the chromosome (1287) would indicate that inlet 1, of unique size and type, is the most upstream inlet. Inlet 2 is the next downstream inlet in the sequence, followed by inlet 8 and inlet 7. The fitness of each chromosome is the associated objective function value (Equation (1)) for the design, retrieved from the drainage simulation model, and thus represents a suitability metric for solving the optimization problem. This value establishes the basis for ranking and selection of the fittest pairs of chromosomes that are mated (i.e. crossover) to produce improved designs.

Based on their objective function values, the chromosomes within the population are ranked in ascending order. The ten best chromosomes, or those with the lowest cost, serve as a mating pool from which parents are selected. Each of these ten designs is first assigned a ranking number according to a Hoerl power function with coefficients fitted so that the ten first ranking numbers sum to unity. The resulting function can be expressed as

$$R_n = 0.4 \times 0.745^C C^{-0.105} \quad (9)$$

where  $R_n$  is the ranking number and  $C$  is a chromosome between 1 and 10. For example, applying the function to the best (i.e. first) chromosome yields a ranking number of 0.298, while for the tenth (i.e. last) chromosome, a ranking of 0.017 is determined. This assigned ranking scheme is shown graphically in Figure 5. Next, to generate offspring

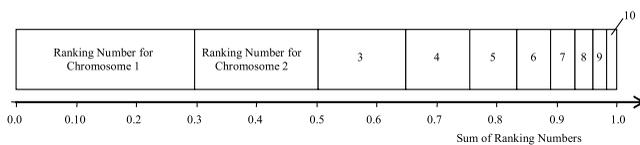


Figure 5 | Sum of ranking numbers

solutions, two parents are chosen from the mating pool according to randomly generated numbers between 0 and 1 and their corresponding ranking. For example, if the random numbers are 0.25 and 0.8, then from Figure 5 the parents will be chromosomes 1 and 5. The actual creation of the offspring occurs through a random, one-point crossover scheme. Consider two chromosomes having 10 genes each, say (4863211144) and (4633312114). If the randomly chosen crossover point falls between the fourth and fifth genes, the two offspring chromosomes are (4863312114) and (4633211144). If the parent chromosomes have different lengths, the crossover position would be a random position within the length of the shorter chromosome, but the length of the offspring is that of the longer chromosome. This operation is carried out to create fifteen new design alternatives, which replace the worst fifteen chromosomes from the initial, or previous, generation. In this way, the fittest chromosomes pass on their genetic characteristics to future generations of solutions. The result is the evolution of design solutions that are better suited to solve the optimization problem than the individuals from which they were created. It should be noted that an elitist strategy is not integrated into the solution technique. As a final step in a given generation, to encourage the entry of new information into each generation, genes are randomly mutated according to a user-defined frequency. The mutation process ensures a continuing, wide search of the solution space and thus essentially discourages convergence upon local optima. The entire operation, including ranking, selection, crossover and mutation, are repeated to form the next generation and a new population of solutions.

Although GAs are particularly useful in solving a range of highly complex, nonlinear optimization problems, there are some disadvantages to their use. The algorithms can be computationally intensive, particularly

in cases where significant computational time and effort is required for objective function evaluation (Hilton & Culver 2000). The structure of the GA, however, is highly suitable for parallel computing, if available. Furthermore, even though GAs search a wide portion of the solution space, they are a heuristic search technique and a globally optimal solution is not guaranteed (Cieniawski *et al.* 1995). This is a common characteristic of most nonlinear optimization methods applied to nonconvex systems. However, reliability in locating global optima can be investigated and possibly improved through repeated sensitivity applications of the GA in which the user varies parameters such as population size and mutation frequency. In fact, the majority of GA literature consistently demonstrates an ability to identify global or very near global optima for a range of complicated problems (Nicklow 2000).

## OPTIMAL CONTROL SOLUTION METHODOLOGY

Two different solution approaches have been investigated for solving the inlet design problem. They vary only in whether a penalty function or the simulation model is used to handle the spread constraint (Equation (3)). The first approach utilizes the GA for evaluating the optimal inlet types, including size, and location of each particular inlet. Thus, chromosomes are modified slightly to include distance (i.e. location) for each inlet, in addition to its type and size. Note that the preprocessing unit is not used in this approach and the drainage module is interfaced directly with the GA. The number of decision variables (i.e. genes) is then two times the number of inlets needed to drain the pavement: (1) type and size and (2) locations and order. Using this approach, the GA at times places inlets with a spacing that causes the actual spread to exceed the allowable width of spread as shown in Figure 6. To clearly indicate that this is an infeasible design, a penalty function is used to assign a poor fitness value, or cost, to any chromosome that violates the spread constraint. For the inlet design problem, this cost is computed using a penalty function that depends on the cumulative area of spread outside the allowable level.

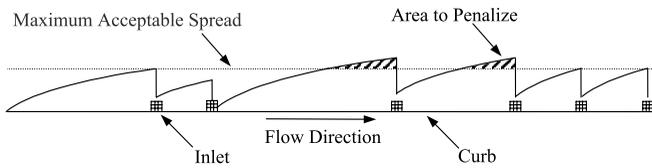


Figure 6 | Exceedance of allowable roadway spread

For this approach, the problem formulation becomes

$$\text{Min } Z = \sum_{j=1}^l (n_j C_j) + C_p \quad (10)$$

subject to the transition constraint

$$T_{i+1} = f(T_i, U_i) \text{ for } 1 \leq i \leq l-1 \quad (11)$$

where  $C_p$  is a penalty function which, in part, is based on a form of the Weibull cumulative distribution function and is given by

$$C_p = 80,000 - 80,000e^{(-0.027 \cdot S_e^{0.65})} \quad (12)$$

and where  $S_e$  is the area of spread exceeding the allowable width of spread. Although other penalty functions could likely be substituted, after preliminary trials and investigation of several types of functions, Equation (12) was selected based on performance in implicitly handling the spread constraint. Note that the constraint describing the pool of available inlets (Equation (4)) is not explicitly included in this formulation since only available inlets are permitted to become part of a chromosome and thus enter any population of solutions. Since the simulation model handles the transition constraint, the GA is now essentially faced with solving a transformed, unconstrained optimization problem.

The total number of possible solutions,  $P$ , both feasible and infeasible, can be computed as

$$P = J^y l^y (l-1)^y (l-2)^y (l-3)^y \dots \quad (13)$$

where  $J$  is the number of predefined inlets,  $y$  is the number of inlets that are needed to drain the pavement and  $l$  is the

length of the pavement, representing the number of discrete possibilities for placement. Note that, if no optimizer is used,  $P$  represents the number of possible solutions that would require investigation in order to find the least-cost design alternative. As a rough example, consider a set of ten predefined inlet types and a pavement length of 1000 meters. If it is arbitrarily assumed that 20 total inlets are needed, there are approximately  $8 \times 10^{19}$  possible solutions that require evaluation, which represents an infeasible task.

Although the overall problem size and difficulty is significantly reduced through use of the optimal control methodology, preliminary tests show that evaluation of an optimal design using the first solution approach is inefficient. The GA effectively searches for the least-cost design, but it tends to require intensive computational effort and computational times on the order of days to find the optimal position of each inlet. This effect is a demonstration of the inability of GAs to efficiently handle constraints, which has been previously noted by Cieniawski *et al.* (1995) and Hilton & Culver (2000).

To overcome the inefficiencies associated with the first approach, a second solution approach is investigated. This approach removes the location of each inlet as a decision variable from the GA, and instead the simulation model is utilized to evaluate the desired inlet locations given a specified order, or sequence, of types and sizes. Here, the GA is interfaced directly with the full simulation model, including both the preprocessing and drainage modules. The preprocessing unit is used to repeatedly call the drainage module and place inlets at the desired inlet location (Figure 1), as described earlier (see the section on highway drainage simulation). With respect to the original problem formulation, the simulator is thus responsible for handling both the transition constraint (Equation (2)) and the spread constraint (Equation (3)).

Since the location of each inlet is no longer solved by the optimization procedure, the GA is now only assigned the task of finding the optimal type, including size, and placement order of inlets. In this way, the size of the overall problem is further reduced and consequently the computational time is significantly decreased. The total number of feasible and infeasible combinations is reduced to

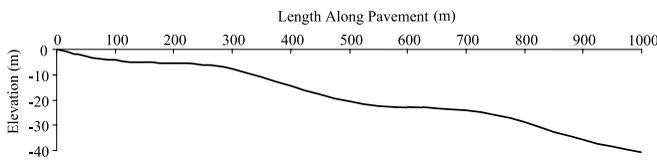


Figure 7 | Roadway profile for example application

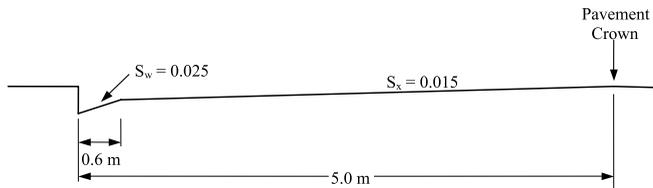


Figure 8 | Cross section for example application

$$P = J^y. \quad (14)$$

For this case, a set of ten predefined inlets and a pavement length that requires 20 total inlets yield  $10^{20}$  possible designs, which represents a significant reduction compared to the first approach. Based on preliminary tests and a comparison of solution approaches, this second approach has been selected for further use in the optimal control model.

## EXAMPLE APPLICATION

To demonstrate the capabilities of the new methodology, the optimal control model is applied to a hypothetical, yet realistic, roadway system that is similar to those found in engineering practice. The profile of this system, shown in Figure 7, illustrates that the total length of pavement is 1000 m. The longitudinal slope is discontinuous and varies from 0.1–7.5% and is provided to the model using 43 discrete elevation points. At 100 m, 200 m, 300 m and 400 m of roadway length, an additional  $0.002 \text{ m}^3/\text{s}$  of off-site runoff is added to the gutter flow rate. Figure 8 illustrates the cross-sectional geometry of this depressed gutter section, including the gutter cross slope ( $S_w$ ) and cross slope of the pavement surface ( $S_x$ ). Manning's roughness coefficient for the pavement surface is set to

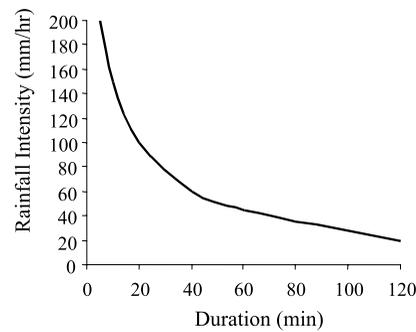


Figure 9 | IDF curve for example application

Discrete data used as  
IDF curve

Duration	Intensity
5	200
10	150
20	100
40	60
60	45
120	20

0.016, which is representative of rough asphalt or broom finished concrete. The runoff coefficient for the rational method is set equal to 0.73 and the rainfall intensity is obtained from the IDF curve shown in Figure 9. The maximum allowable spread, which is the constraining parameter for the number of required inlets, is set to 1.8 m from the curb. The available inlets specified for the example application are those defined in Table 1. Numerous other inlet types and sizes could be specified by the user since a database of inlets has been incorporated into the model.

An optimal solution is investigated by running the optimal control model repeatedly using mutation rates ranging from 5–30%. The reason for running the problem with different mutation rates is that no two problems are identical and each problem and application of the GA will have its own unique convergence characteristics. After a maximum of 250 user-defined generations, each test was terminated and the least-cost design was reported.

The optimal solution for this example was found to be 8620 monetary units and was obtained in the 106th generation for a mutation rate of 10%, as illustrated in a plot of the solution evolution (Figure 10). Note that this solution converges upon a local optimum near the 48th generation, but the associated chromosome is mutated to allow the search for a globally optimal solution to continue. The optimal design is shown in Figure 11, while the resulting spatial variation of spread due to the optimal design is shown in Figure 12. The latter is essentially a plan view of the roadway, with the abscissa being the roadway curb. The small, abrupt changes and 'saw tooth' patterns

**Table 1** | Available inlets for example application

Inlet ID no.	Cost (MU <sup>1</sup> )	Type	Length (m)	Width (m)
1	845	Grate <sup>2</sup>	1.8	0.6
2	795	Grate <sup>2</sup>	1.2	0.6
3	745	Grate <sup>2</sup>	0.6	0.6
4	695	Grate <sup>2</sup>	0.3	0.6
5	1115	Curb opening	5	N/A
6	645	Curb opening	4	N/A
7	575	Curb opening	3	N/A
8	515	Curb opening	2	N/A
9	1249	Combination <sup>3</sup>	4 (grate 1.2)	0.6
10	1199	Combination <sup>2</sup>	3 (grate 0.6)	0.6

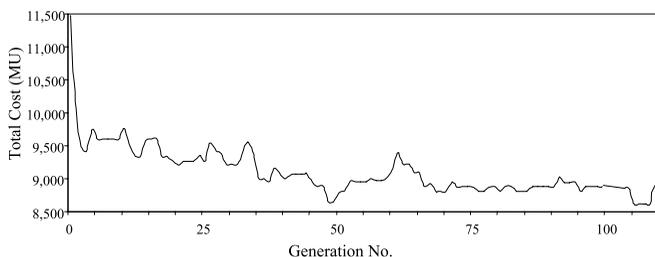
Notes: <sup>1</sup>MU=Monetary Unit.

<sup>2</sup>Grates are 30–85 tilt bar.

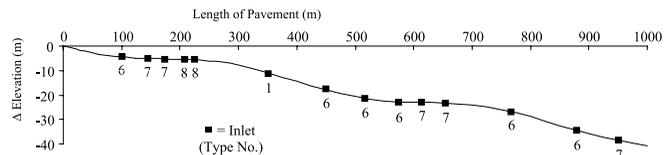
<sup>3</sup>Combination inlets consist of curb opening and grate.

in spread that occur throughout the length of the pavement are due to changes in longitudinal slope. In addition, some of the sudden increases in spread are due to the addition of off-site runoff to the total gutter flow. Tests using other mutation rates converged to values that were approximately 70–560 monetary units (i.e. 0.8–6.5%) greater than the optimal solution. These solutions were very similar to that obtained with a mutation rate of 10%, with the exception of switching one or two of the inlet types within the placement order. It is interesting to note

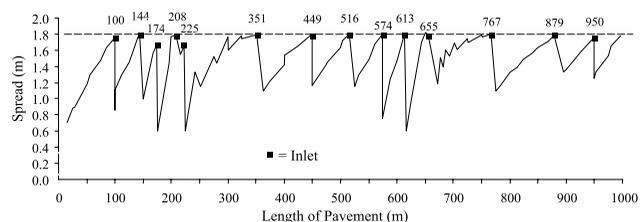
that, from Figure 12, the most economical design does not capture all, or even most, of the gutter flow, but it does maintain a total spread less than that allowable. This is a



**Figure 10** | Evolution of optimal design



**Figure 11** | Optimal inlet design for example application



**Figure 12** | Optimal spatial variation of spread

concept that is frequently used in design practice, following the reasoning that a larger expense may be incurred by using larger capacity inlets and increasing inlet spacing.

Using a Pentium III, 233 MHz personal computer, the application required approximately 20 h to reach the maximum number of generations. The majority of this time is allocated to operation of the simulation model for evaluation of the objective function. This could be overcome, in part, by using a higher-speed computer or parallel computing methods. While this computational time may seem large, one must realize that the user is not required to repeatedly interact with and initiate simulations, as with conventional design procedures.

## SUMMARY AND CONCLUSIONS

A new methodology and computational model have been developed for the optimal design of highway drainage inlets. The optimal control approach is used to directly evaluate the least-cost design alternative that effectively drains a section of roadway. The overall approach is based upon the coupling of a highway drainage simulation model and a GA. The capabilities of the associated model have been confirmed through an application to a hypothetical roadway system.

Conventional simulation models alone are incapable of determining optimal solutions to complex problems. Furthermore, traditional design methodologies require a time-consuming, trial-and-error approach for placing inlets and determining the least-cost design alternative. The current approach overcomes these limitations and yields a complete, optimal inlet design. Since the model does not require user interaction beyond initial data entry, the approach can greatly reduce the work required by the designer for pavement drainage design. In addition, by integrating standard design methodologies (e.g. Federal Highway Administration design procedures) that have been widely accepted in engineering practice, the potential practical utility of the approach is significantly improved.

The GA is an effective optimization algorithm within the optimal control approach. These algorithms are especially attractive for solving such complex, nonlinear

problems since they require no derivative information and search wide portions of the solution space. While two different solution methodologies have been evaluated within the optimal control approach, both overall methods were based on a GA. The first method integrated a penalty function for solving the constraint on roadway spread (i.e. spread must not exceed allowable levels) and relied upon the GA for determining the proposed inlet locations. GAs, however, do not efficiently handle bound constraints and, as a result, this method leads to excessively long computational times. The second method, in which the hydraulic simulation model is used to evaluate the desired inlet locations and the GA is left to evaluate the types, sizes and sequence of inlets, leads to a reduced number of decision variables and is less computationally intensive. In addition, though a global optimum is not guaranteed, reliability in locating optimal designs was improved through preliminary runs in which the population size was investigated and through repeated applications of the GA in which the mutation rate was varied. For the application provided, a mutation rate of 10% yielded the best solution, and different mutation rates between 5–30% resulted in a 0.8–6.5% increase in design cost. The final solution indicates that the most economical overall design and layout is feasible, but does not capture all the gutter flow and completely drain the roadway at any particular location; rather, the optimal design uses smaller inlets at more frequent spacing to simply maintain a spread that is less than that which is allowable.

For smaller projects in which the cost reduction obtained by using the optimization model is marginal, or in cases where very few inlet types are available, the simulator model alone can be a useful design tool. Specifically, the drainage module can be used to examine the flow characteristics associated with different design alternatives. In addition, either the simulation module alone or the overall optimal control model could be used iteratively for sensitivity analysis and for evaluating how different cross-sectional and longitudinal slopes will affect flow and drainage designs prior to construction. The gutter cross slope, in particular, is a critical parameter that significantly affects flow characteristics and thus drainage costs.

Future work will involve expanding the overall optimal control model to directly evaluate sensitivity and

uncertainty effects of model parameters on the least-cost inlet design. This is of particular interest since the hydrologic simulation methods used (i.e. the rational method) are empirical and can be subject to potentially significant uncertainty. Nevertheless, these are the same methods promoted by the US Federal Highway Administration and used by state transportation authorities in nearly all pavement drainage applications today. Subsequently, by integrating current, commonly relied upon design procedures, it is hoped that the optimal control methodology and associated model may be more readily adopted.

Finally, Nicklow (2000) discusses the apparent gap that exists between optimization theory and practice in water resources related disciplines. He notes that component methodologies and models, such as particular optimization methods and hydraulic/hydrologic simulation techniques, are fairly well established, which is not to say that continuous improvement of these components and their broader use is not needed or warranted (e.g. use of improved hydrologic simulation methods for pavement drainage). However, widespread implementation of combined optimization–simulation methods has been extremely limited in typical engineering practice. The application provided here reveals the versatility of the optimal control methodology as a comprehensive decision-making mechanism in handling complex, non-linear control problems. There are a multitude of hydraulic/hydrologic engineering problems for which the posed methodology, in the same or similar form, could likely be used to evaluate optimal designs. It is hoped, therefore, that this study represents a single step in narrowing the gap between theory and practice and sheds light on potential applications in other hydraulic/hydrologic engineering specializations.

## NOTATION

$C$	chromosome between 1 and 10
$C_j$	cost associated with inlet type $j$
$C_p$	penalty function
$E$	inlet interception efficiency
$I$	total number of discrete locations at which spread is evaluated

$J$	number of different inlet types, including sizes, of inlets
$J^*$	set of predefined inlet types
$l$	length of pavement
$n$	Manning roughness coefficient
$n_j$	number of inlets of type $j$
$P$	number of potential solutions
$Q$	gutter flow rate
$Q_i$	inlet interception capacity
$Q_b$	inlet bypass flow
$R_n$	chromosome ranking number
$S_e$	area of spread exceeding allowable spread
$S_L$	longitudinal roadway slope
$S_x$	gutter cross slope
$T_a$	average spread
$T_i$	spread at location $i$
$T_{\max}$	maximum allowable spread
$U_i$	design variables implemented at location $i$
$y$	number of inlets needed to drain a length of pavement
$Z$	total cost of inlets

## REFERENCES

- AASHTO 1991 *Model Drainage Manual*. American Association of State Highway and Transportation Officials, Washington, DC, chapter 13.
- Belegundu, A. D. & Chandrupatla, T. R. 1999 *Optimization Concepts and Applications in Engineering*, Prentice Hall, Englewood Cliffs, NJ.
- Brown, S. A., Stein, S. M. & Warner, J. C. 1996 *Urban Drainage Design Manual, Hydraulic Engineering Circular No. 22*, FHWA-SA-96-078, Federal Highway Administration, US Department of Transportation, Washington, DC.
- Cieniawski, S. E., Eheart, J. W. & Ranjithan, S. 1995 Using genetic algorithms to solve a multiobjective groundwater monitoring problem. *Wat. Res. Res.* **31** (2), 399–409.
- Cunha, M. C. & Sousa, J. 2000 Water distribution network design optimization: simulated annealing approach. *J. Wat. Res. Plann. Mngmnt.*, *ASCE* **125** (4), 215–221.
- Elimam, A. A., Charalambous, C. & Ghobrial, F. H. 1989 Optimum design of large sewer networks. *J. Environ. Engng, ASCE* **115** (6), 1170–1190.
- Esat, V. & Hall, M. J. 1994 Water resources system optimisation using genetic algorithms. *Hydroinformatics '94, Proc. 1st Int. Conf. on Hydroinformatics*. Balkema, Rotterdam, The Netherlands, pp. 225–231.
- Goldberg, D. 1989 *Genetic Algorithms in Search Optimization and Machine Learning*. Addison-Wesley, Reading, MA.

- Haupt, R. L. & Haupt, S. E. 1998 *Practical Genetic Algorithms*. Wiley, New York.
- Hilton, A. C. & Culver, T. B. 2000 Constraint handling for genetic algorithms in optimal remediation design. *J. Wat. Res. Plann. Mngmnt., ASCE* **126** (3), 128–137.
- Holland, J. H. 1975 *Adaptation in Natural and Artificial Systems*. University of Michigan Press, Ann Arbor, MI.
- IDOT. 1989 *Drainage Manual*. Illinois Department of Transportation, Bureau of Bridges and Structures, Springfield, IL.
- Izzard, C. F. 1946 *Hydraulics of Runoff from Developed Surfaces, Proc. Highway Research Board*, vol. 26. Highway Research Board, Washington, DC, pp. 129–150.
- Johnson, F. L. & Chang, F. M. 1984 *Drainage of Highway Pavements, Hydraulic Engineering Circular No. 12*, FHWA-TS-84-202. Federal Highway Administration, US Department of Transportation, Washington, DC.
- Li, G. & Matthew, R. G. S. 1990 New approach for optimization of urban drainage systems. *J. Environ. Engng, ASCE* **116** (5), 927–944.
- Mays, L. W. 1979 Optimal design of culverts under uncertainties. *J. Hydraul. Engng, ASCE* **105** (HY5), 443–460.
- McKinney, D. C. & Lin, M. D. 1994 Genetic algorithm solution of groundwater management models. *Wat. Res. Res.* **30** (6), 1897–1906.
- Minsker, B. S. & Shoemaker, C. A. 1998 Dynamic optimal control of in-situ bioremediation of ground water. *J. Wat. Res. Plann. Mngmnt, ASCE* **124** (3), 149–161.
- Mitchell, M. 1996 *An Introduction to Genetic Algorithms*. MIT Press, Cambridge, MA.
- Nicklow, J. W. 2000 Discrete-time optimal control for water resources engineering and management. *Wat. Int.* **25** (1), 89–95.
- Nicklow, J. W. 2001 Design of storm water inlets. In *Storm Water Collection Systems Handbook* (ed. Mays, L. W.). McGraw-Hill, New York, Chapter 5.
- Nicklow, J. W. & Mays, L. W. 2000 Optimization of multiple reservoir networks for sedimentation control. *J. Hydraul. Engng, ASCE* **126** (4), 232–242.
- Oliveira, R. & Loucks, D. P. 1997 Operating rules for multireservoir systems. *Wat. Res. Res.* **33** (4), 839–852.
- Reis, L. F. R., Porto, R. M. & Chaudhry, F. H. 1997 Optimal location of control valves in pipe networks by genetic algorithm. *J. Wat. Res. Plann. Mngmnt, ASCE* **123** (6), 317–320.
- Ritzel, B. J., Eheart, J. W. & Ranjithan, S. 1994 Using genetic algorithms to solve a multiple objective groundwater pollution problem. *Wat. Res. Res.* **30** (5), 1589–1603.
- Sakarya, A. B. & Mays, L. W. 2000 Optimal operation of water distribution pumps considering water quality. *J. Wat. Res. Plann. Mngmnt., ASCE* **126** (4), 210–220.
- Savic, D. A. & Walters, G. A. 1997 Genetic algorithms for least-cost design of water distribution networks. *J. Wat. Res. Plann. Mngmnt, ASCE* **123** (2), 67–77.
- Templeman, A. B. & Walters, G. A. 1979 Optimal design of stormwater drainage networks for roads. *Proc. ICE, Part I – Design and Construction*, vol 67. Institute of Civil Engineers, New York, pp. 573–587.
- Unver, O. I. & Mays, L. W. 1990 Model for real-time optimal flood control operation of a reservoir system. *Wat. Res. Mngmnt* **4**, 21–46.
- Wanakule, N., Mays, L. W. & Lasdon, L. S. 1986 Optimal management of large-scale aquifers: methodology and application. *Wat. Res. Res.* **22** (4), 447–465.
- Wang, Q. J. 1991 The genetic algorithm and its application to calibration conceptual rainfall-runoff models. *Wat. Res. Res.* **27** (9), 2467–2471.
- Wardlaw, R. & Sharif, M. 1999 Evaluation of genetic algorithms for optimal reservoir system operation. *J. Wat. Res. Plann. Mngmnt, ASCE* **125** (1), 25–33.
- Yeh, W. W.-G. 1985 Reservoir management and operations models: a state-of-the-art review. *Wat. Res. Res.* **21** (12), 1797–1818.
- Yeh, W. W.-G. 1992 Systems analysis in ground-water planning and management. *J. Wat. Res. Plann. Mngmnt, ASCE* **118** (3), 224–237.