Nordic Hydrology, 27 (4), 1996, 247-254

A Graphical Method for Storage Coefficient Determination from Quasi-Steady State Flow Data

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A simple, approximate but practical graphical method is proposed for estimating the storage coefficient independently from the transmissivity value, provided that quasi-steady state flow data are available from a pumping test. In the past, quasi-steady state flow distance-drawdown data have been used for the determination of transmissivity only. The method is applicable to confined and leaky aquifers. The application of the method has been performed for various aquifer test data available in the groundwater literature. The results are within the practical limits of approximation compared with the unsteady state flow solutions.

Introduction

Thiem (1906) obtained a quasi-steady state groundwater movement equation for small diameter wells in confined aquifers as

$$Q = 2\pi T \frac{s_1 - s_2}{\ln(r_2/r_1)} \tag{1}$$

where Q is the constant pump discharge; T is aquifer transmissivity; s_1 and s_2 are drawdown measurements in observation wells which are at radial distances r_1 and r_2, respectively, from the main well center, (see Fig. 1). For the validity of Eq. (1), the aquifer must be confined with infinite areal extent, must be homogeneous, isotropic and of uniform thickness, and the well must have complete penetration.

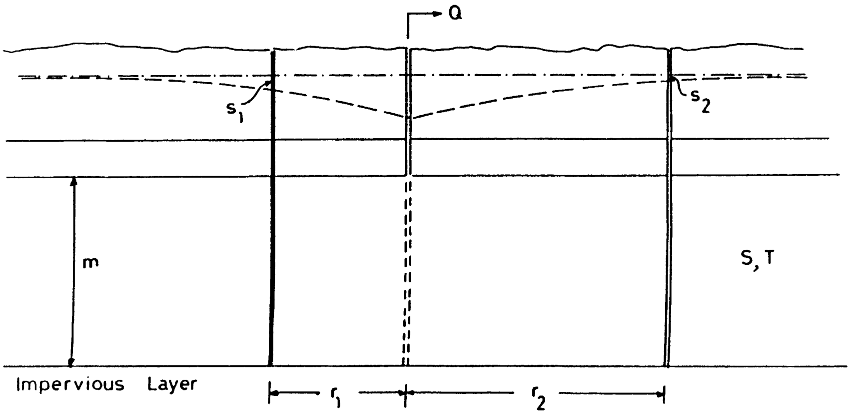

Fig. 1. Confined aquifer configuration with main and observation wells.

On the other hand, in the case of leaky aquifer steady state flow, the valid expression was given first by De Glee (1930) as

$$Q \equiv 2\pi T \frac{s_r}{K_0\,(r/L)} \tag{2}$$

in which s_r is the steady state drawdown in the piezometer at distance r from the main well. $K_0(r/L)$ represents the modified Bessel function of the second kind and zero order, and finally, L is the leakage factor. It is clear from Eqs. (1) and (2) that the drawdown changes with distance, and furthermore, drawdown *versus* the logarithm of the distance plots along a straight line for Eq. (2) only when r/L is small (see remarks to Hantush-Jacob's method in Kruseman and de Ridder 1990; pages 77-80).

In practice, Eqs. (1) and (2) are used for determining the transmissivity only. Kruseman and de Ridder(1990) have presented procedures that can be applied in determining the transmissivity of confined or leaky aquifers. Such procedures do not require complete pump test data, but depend on steady state measurements after long times. The best way to ensure a reliable estimate of the storage coefficient of a confined or leaky aquifer is to analyze time series of drawdown during a pumping test. Time series of drawdown from just one observation well reveals possible positive or negative boundaries within the aquifer and problems with matching type curves indicate heterogeneity, (Şen and Al-Baradi 1991); if time series are available from a number of observation wells distributed around the pumping well then the analysis of aquifer characteristics is further improved. Steady state data cannot be expected to reveal this, especially if data are only available for a few wells.

However, if no time series are available one can use a quasi-steady state drawdown data to roughly estimate the storage coefficient. In the following a simple graphical procedure for estimating the aquifer storage coefficient in addition to the transmissivity determination from a quasi-steady state distance-drawdown record.

Storage Coefficient

The storage coefficient plays a vital role for the transient behavior of groundwater flow and in the estimation of the withdrawal volume of groundwater from an aquifer. Consequently, its precise determination is one of the prerequisites in groundwater resources assessments and management in any region. Unfortunately, some analytical procedures for aquifer test data evaluation by either type curve matching or straight line methods, yield reliable transmissivity values but questionable storage coefficient estimates. For instance, the Cooper and Jacob (1946) recovery straight line method provides estimates of transmissivity only. Furthermore, the reliability of storage coefficient estimations obtained from the Papadopulos and Cooper (1967) and the Şen (1991) methods are highly questionable. The idea of estimating the storage coefficient from the volume of the cone of depression is due to Wenzel (1937), who used drawdown data from a large number of observation wells to calculate the volume of the cone of depression and estimate the specific yield of an unconfined aquifer near Grand Island, Nebraska. Bennett *et al* (1967) applied the method to aquifer tests from the Punjab Region in Pakistan. Attempts to estimate the storage coefficient, S, from quasi-steady state flow measurements without type curve expressions are due to Şen (1987) who defined S as a ratio of two volumes

$$S \equiv \frac{Qt}{V_D} \tag{3}$$

where Qt is the volume of abstracted water during time period t with constant discharge Q and V_D is the depression cone volume. He then obtained theoretically, a rather sophisticated expression for V leading to a complicated expression for S. However, on what follows a straight-forward graphical procedure is suggested which gives firstly an estimate of the depression cone volume from the quasi-steady state flow data and secondly, an estimate of the storage coefficient according to Eq. (3).

Quasi-Steady-State Flow

Theoretically, Eqs. (1) and (2) represent a straight line on a semi-logarithmic paper with drawdown *versus* distance of each observation well from the main well center. Consequently, two or preferably more observation wells will be used in the practical application for the construction of such an experimental straight line. It is well known that the slope of the line is related to the transmissivity. It is suggested herein that the revolution volume of the same straight line about the drawdown axis is related to the depression cone volume, V_D. If this volume is known, then the storage coefficient can be estimated from Eq. (3). The necessary steps for the suggested procedure are as follows:

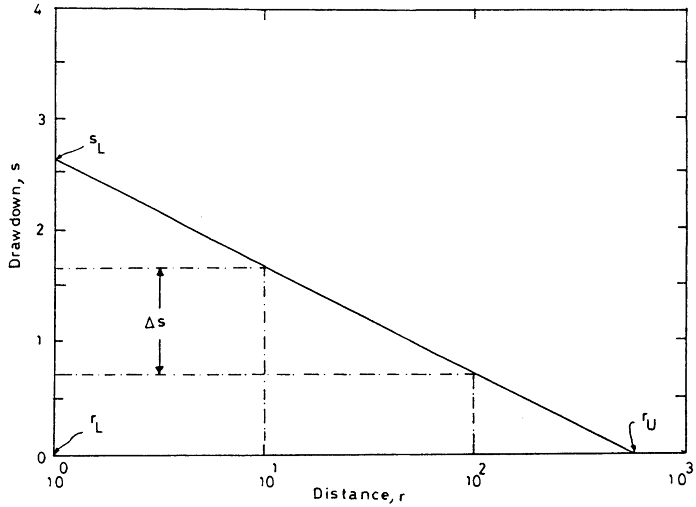

Fig. 2. Schematic distance-drawdown plot.

a) Plot the observed late drawdown data from observation wells located at various distances on a semi-log paper with drawdowns on the vertical linear axis and the corresponding distances on the horizontal logarithmic axis as in Fig. 2. This is one of the major steps in the Jacob straight line method.
b) Draw the best-fit straight line through the scatter points of data.
c) Extend this line in both directions until the points of intersection with the vertical and horizontal axes are obtained.
d) Determine the slope, Δs, and then the intersection values, s_L and r_U of this line on the drawdown and the distance axes, respectively. Clearly, r_U corresponds physically to the radius of influence and the drawdown value beyond this distance is approximately zero everywhere within the aquifer. Besides, the drawdown intersection has the distance, r_L from the main well center (see Fig. 2).
e) It is well known that the slope of the fitted line can be found theoretically from Eqs. (1) and (2) as

$$\Delta s \equiv \frac{2.3\, Q}{2\pi T} \tag{4}$$

which has been used so far in the literature for the transmissivity estimation.
f) Calculate the area, A, of the triangle constituted by the two axes and the fitted straight line in Fig. 2. Hence,

$$A \equiv \frac{1}{2}\,(r_U - r_L)\, s_L \tag{5}$$

which is logically related to the depression cone volume
g) Consider the revolution of the area below the fitted line about the drawdown axis but with back transformation of distances to linear scale which yields the depression cone volume estimate as

$$D \equiv \pi \frac{(r_U - r_L)^2}{\ln(r_U/r_L)} s_L \tag{6}$$

h) The substitution of this volume expression into Eq.(3) gives the storage coefficient estimate as

$$S = \frac{Qt}{\pi(r_U - r_L)^2 s_L} \ln(r_U/r_L) \tag{7}$$

Field Applications

The validity of the methodology developed here for the identification of storage coefficient from quasi-steady state flow in confined aquifers and for perfectly steady state flow in leaky aquifers is presented through applications to field data already presented by Kruseman and de Ridder (1990; Tables 3.1 and 4.1). First of all results of the application of Theim's method to data from pumping test »Oude Korendijk« are reproduced herein in Table 1 from Table 3.2 given by the same authors. The pumping discharge is $Q \equiv 788$ m^3/day and the quasi-steady state flow is reached after $t = 830$ min.

Table 1 – Aquifer Test Data and Results

r_1 (m)	r_2 (m)	s_1 (m)	s_2 (m)	Δs (m)	r_u (m)	r_L (m)	s_L (m)	V_D ($\times 10^3$m^3)	T (m^2/day)	S ($\times 10^{-4}$)
30	90	1.088	0.716	0.64	810	0.1	3.00	687	370	6.69
0.8	30	2.236	1.088	0.73	1000	0.1	2.87	979	396	4.86
0.8	90	2.236	0.716	0.68	850	0.1	2.88	722	389	6.28
								Averages	385	5.94

The plots of distance-drawdown data on a semi-logarithmic paper as shown in Fig. 3 give two points which are connected by straight lines, and the procedure in the previous section gives the relevant quantities as shown in the second part of the table. Although the storage coefficient estimates of the proposed method are somewhat larger than the estimates from Theis and Jacob methods, in their order of magnitude they show very good agreement with Theis (1.6×10^{-4}), and Jacob (1.7×10^{-4}, 4.1×10^{-4} and 1.7×10^{-4}) method estimates, (Kruseman and de Ridder 1990).

The method is also applied to the transient Qude Korendijkk data given by Kruseman and de Ridder(1990; Table 3.1). The plots of distance-drawdown values from two piezometers at distances 30 m and 90 m after 140 min, 300 min, 600 min, and 830 min from the aquifer test start are shown in Fig. 4, with four parallel straight lines. The implementation of the steps in the previous section gives rise to relevant data as shown in Table 2 for the various cases.

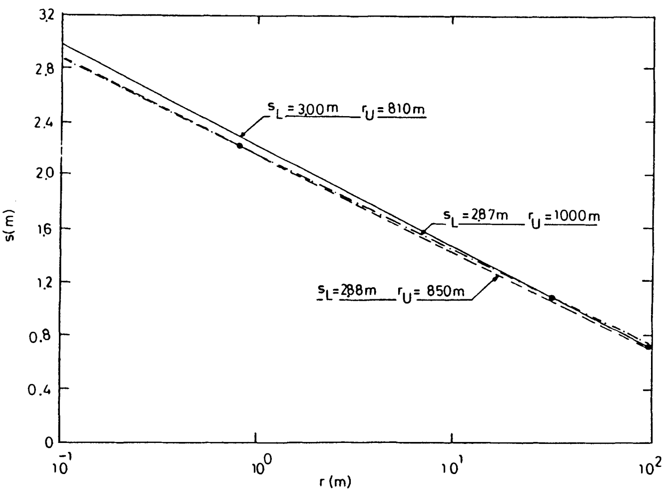

Fig. 3. Distance-drawdown plot for 'Qude Koredij' data.

Table 2 – Storage Coefficient Determination.

Time (min)	Δs (m)	r_u (m)	r_L (m)	s_L (m)	S ($\times 10^{-4}$)	r_L (m)	S ($\times 10^{-4}$)
140	0.76	450	1.0	2.04	3.62	10.0	3.76
300	0.76	590	1.0	2.12	4.53	10.0	2.97
600	0.76	710	1.0	2.19	6.23	10.0	4.13
830	0.76	785	1.0	2.20	7.12	10.0	4.78
				Averages	5.35	10.0	3.91

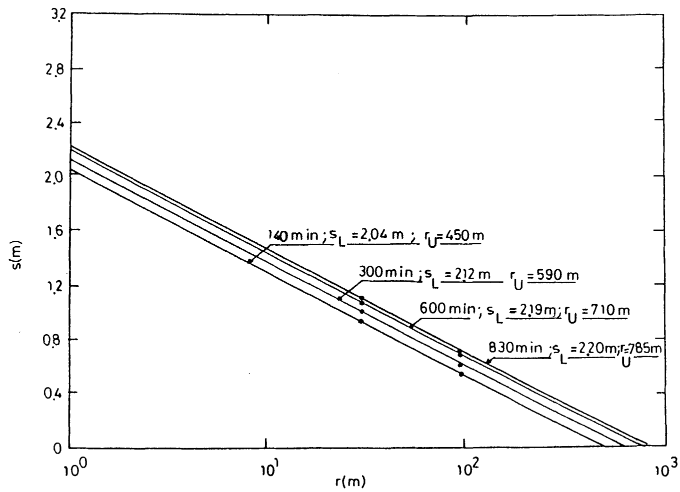

Fig. 4. Distance-drawdown plot for Jacob method with 'Qude Korendijk' data.

It is interesting to notice in this table that changes in the time duration lead to storage coefficient values with the same order of magnitude. Moreover, they also compare very well with the results obtained from other techniques. It seems from the sixth column that the estimated storage coefficient changes with time and such changes are due to the heterogeneities of the aquifer portion within the depression cone and explained in detail by Şen (1994). However, according to Kruseman and de Ridder (1990, page 70) no heterogeneities can be detected from analysis of transient data. According to them the apparent increase of S with time may be due to leakage. A critical point in the application of the method appears in choosing the lower distance value. Logically, it might be taken as equal to the well diameter. However, the choice of any small distance has no significant practical effect on the final storativity estimation. To illustrate this, r_L, is chosen as equal to 1.0, and the corresponding drawdown as well as the storativity calculations are presented in the last two columns. It is obvious that in spite of the significant difference in the r_L values, the storativities do not deviate significantly from one another, *i.e.* not by order of magnitude. The model assumptions that can explain the relative significance and systematic deviations between the coefficients estimated by various methods including Theis and Jacob procedures are already presented by Şen (1988).

Finally, the leaky aquifer steady state flow data Dalem are also adopted from Kruseman and de Ridder (1990; Table 4.1). The distance-drawdown plot is reproduced in Fig. 5. The pump discharge is 761 m^3/day and the steady state is reached after about 0.4 days from the pump start. The substitution of the slope value from Fig. 5 into Eq. (4) yields that $T \equiv 2{,}020$ m^2/day. Additionally, the necessary quantities for the storage coefficient calculation are $r_U = 1{,}100$ m; $s_L = 0.32$ m; and $r_L = 6$ m. The substitution of these values into Eq. (6) yields that $S = 1.4\times10^{-3}$. This value compares very well with the storage coefficient estimates obtained from the application of other techniques such as the Hantush inflection point and the Hantush type curve methods which gave 1.7×10^{-3}, and 1.5×10^{-3}, respectively, (Kruseman and de Ridder 1990).

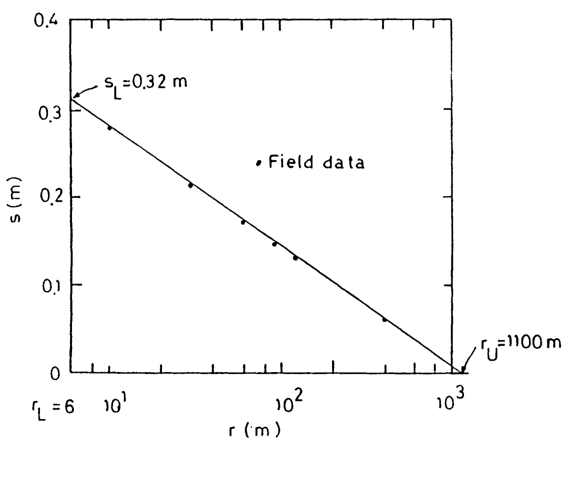

Fig. 5.
Leaky aquifer distance-drawdown plot for Dalem data.

Conclusions

An approximate method has been developed for identifying the storage from quasi-steady state flow in confined and steady state flow data in leaky aquifers. The basis concept is related to obtaining the depression cone volume estimation from a distance-drawdown plot on a semi-logarithmic paper. Definition of storativity as the ratio of abstracted water volume to the depression cone volume leads to a simple formula for calculating its value. The storage coefficient procedure results from the methodology presented in this paper conform practically very well with the results obtained from traditional methods on the basis of order of magnitude. The use of the proposed method is recommended if no time series are available showing the complete change of drawdown with time during a complete aquifer test.

References

Bennett, G. D., Rehman, Sheikh, I. A., and Sabir, A. (1967) Analysis of aquifer tests in the Punjab region of west Pakistan, U.S. Geological Survey, Water Supply Paper 1608-G.

Cooper, H. H., and Jacob, C. E. (1946) A generalized graphical method for evaluating formation constants and summarizing well field history, *Am. Geophys. Union Trans., Vol. 27*, pp. 526-534.

De Glee, G. J. (1930) Over groundwaterstromingen bij teronttrekking door middel van putten, Thesis, J. Waltman, Delft (The Netherlands), 175 pp.

Kruseman, G. P., and de Ridder, N. A. (1990) Analysis and evaluation of pumping test data, Bulletin 11, Institute for Land Reclamation and Improvement, Wageningen, pp. 41-69.

Papadopulos, I. S., and H. H., Cooper (1967) Drawdown in a well of large diameter, *Water Resour. Res., Vol. 3*, pp. 241-244.

Şen, Z. (1987) Storage coefficient determination from quasi-steady state flow, *Nordic Hydrology, Vol. 18*, pp. 101-110.

Şen, Z. (1988) Dimensionless time drawdown plots of late aquifer test data, *Ground Water, Vol. 26*, No. 5, pp. 615-624.

Şen, Z. (1991) Recovery type curves for large diameter wells, *Nordic Hydrology, Vol. 22*, pp. 253-261.

Şen, Z. (1994) Hydrogeophysical concepts in aquifer test analysis, *Nordic Hydrology, Vol. 25*, pp. 183-192.

Şen, Z., and Al-Baradi, A. (1991) Sample functions as indicators of aquifer heterogeneities, *Nordic Hydrology, Vol. 22*, pp. 37-45.

Thiem, G. (1906) *Hydrologische Methoden*, Gebhardt, Leipzig, 56 pp.

Wenzel, L. K. (1937) The Thiem method for determining the permeability of water-bearing materials and its application for determination of specific yield. U.S. Geological Survey, Water Supply Paper 679.

Received: 1 December, 1994
Revised version received: 12 October, 1995
Accepted: 30 November, 1995

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