Development of an effluent discharge policy for the Tay Estuary based on a finite element model

S. Passone, D.B. Das and V. Nassehi
Chemical Engineering Department Loughborough University, Loughborough LE11 3TU, Leicestershire, UK

Abstract: The tidal hydrodynamics and effluent distribution in estuaries involve a complicated range of solute transport phenomena modelled by partial differential equations. Therefore, the quantitative estimation of the risks of water and soil contamination of coastal areas as a result of polluted estuary flows, or effects of the effluent input on the chemical loads, involves the solution of these equations. Generally, the pollutants load in an estuary is determined by the nature of land use which by altering the watershed hydrology or chemical detention/release in the river banks affect the water quality of the estuaries. The present modelling work aims to investigate the solute transport behaviour in the Tay Estuary in Scotland. Based on this study, an attempt to devise an estuary specific discharge strategy for the Tay has been made. The numerical calculations are based on using 2D Galerkin finite element discretisation of the governing equations in an Eulerian co-ordinate system. The flexibility of the formulation allows it to be extended to moving boundary situations encountered in most tidal water systems.

Keywords: Discharge policy; finite element method; Tay Estuary; tidal dynamics

Introduction

Estuaries are complex hydrodynamic systems often with high concentration of human and industrial activities nearby supporting growing economies. With the downstream flow of water, the estuarine system frequently involves mixing of discharges from wastewater treatment plants or other agricultural and industrial uses (direct sources) and seepage from polluted groundwater (indirect sources). Maintenance and improvement of water quality in estuaries are therefore a major concern for many national environmental control agencies and local governing bodies. The problems arise due to the necessity of developing reliable and cost-effective waste discharge schedules, avoiding future soil and water contamination and meeting the increasingly stringent environmental standards. In most indirect release of pollutants to the estuaries, the discharge mechanisms follow a blind migration pattern, particularly with respect to the time and amount. The locations of the sources are also uncertain in general. As a result, in theory, precise discharge policies for such probabilities, or any other hypothetical situations, cannot be developed. Alternatively, in the case of known indirect sources a secondary transport model (e.g., pollutants’ mobility model for soil if they are discharged from estuary banks) needs to be combined with the hydrodynamic model of the estuary for predictive calculations. For direct injection of chemicals, the task is simplified since most significant quantities can be known and manipulated. It is, therefore, necessary that estuarine-specific discharge policy to be made available by taking care of the important hydrodynamic and boundary conditions in any given estuary.

The hydrodynamic and the pollutant transport behaviours are defined mathematically by the widely used shallow water and convection-diffusion equations, respectively (Kawahara and Umetsu, 1986; Leclerc et al., 1990; Petera and Nassehi, 1996; Nassehi and Kafai, 1999). However, for the purpose of controlling and managing estuarine water quality a much broader perspective, based on a multi-disciplinary (Thomass, 1998) approach, needs to be adopted. Due to the complexities of the estuarine flow (see Leclerc et al., 1990), the predictive calculations used in the water quality management rely on powerful
computer techniques with the main emphasis on the accuracy and minimum costs of the computation. This paper deals with sample environmental evaluations (calculations) based on a finite element scheme for pollutants’ transport specific to the Tay Estuary in Scotland. It is shown that these results can be used to design discharge policy for this estuary. The discharge optimisation methodology described in this paper is a flexible and general technique which can be used in many other estuaries. However, the hydrodynamic conditions of any given estuary should be taken into account. Thus in practice, individual discharge policies for watercourses should be formulated and no claims regarding the generality of conclusions drawn from a single study can be made.

Statement of the problem and objectives of the present study

The Tay Estuary has been described recently by Nassehi and Kafai (1999) and therefore a repeat detailed description of the estuary is avoided here. Because of its typical flow behaviour, it has also been subjected to many other hydrological and statistical analysis in the last three decades (e.g., McManus, 1968; Buller et al., 1972; Williams and West, 1973; Charlton and McNicoll, 1975; Pontin and Reid, 1975; Nassehi and Williams, 1987). The Tay Estuary is quite meandering. The dominant flow channel on its southern side is deep and narrow having several sand point-bars (Figure 1). The floodplain on the northern side is wide, shallow and includes a number of sand over-banks. It is not surprising, therefore, that variation in the quantity and the velocity of water is observed in the channel along the width of the estuary. In addition, because of very shallow depth in the northern side the pollutant concentration could be high for a long time in this region before re-entrainment of the pollutants’ cloud and flowing into the main channel.

The intricacy and the varying nature of the flow pattern of an estuary in general, or the Tay Estuary in specific, can only be described by multi-parameter equations. They incorporate factors such as flow resistance, fixed or moving (temporal and spatial) domain conditions, flow channel geometry, flooding and ebbing at a point with relation to the tidal cycle and, convective and diffusive forces. Physical models, though sometimes recommended for use (McDowell and O’Connor, 1977), suffer from serious scaling effects, which reduce the accuracy of the results. They are also limited by their inability to add adequate features into the model structures and the high cost of model construction. For success of predictive discharge policies, numerical modelling therefore provides a very reliable approach. This work adopts a finite element based Galerkin scheme for computer modelling and simulation of estuarine region. The justification on the selection of the finite element technique for the Tay Estuary has been given by Petera and Nassehi (1996) and, Nassehi and Kafai (1999). It is assumed in the present study that the sources of pollutants are man-made (direct) sources, hence known in the numerical analysis. The calculations are done for fixed flow domain boundaries. This assumption introduces an approximation to the simulated results. The hypothesis, however, ensures that the velocity and the water level can be simulated according to the features of dominant flow channel in the southern side of the Tay Estuary where the flow domain is almost constant. The large longitudinal dispersion of pollutants introduced into the Tay is also generally determined by the flow of water within the dominant channel. Fixed boundary, therefore, provides a reasonable way of simulating the Tay Estuary particularly because one of the major management objectives, i.e., fast computation, can be achieved via its use. By design of discharge strategies, this work aims to develop guidelines for different release mechanisms; for example, for deciding how the effluents need to be injected (continuously throughout a tidal cycle, continuously but only at some periods in a tidal cycle or, in batches), at which location and, at what time. Typically, such goals may be reached through a number of routes. But for conservation of estuarine resources, the selection of only the best policy is to lead to the achievement of the
specified goals. It is expected in ideal conditions that the ebb tides carry the pollutants away to the sea. Similarly, during flood tides, inflow of seawater is anticipated. However, for all practical purposes any concentration below the prescribed toxic limits is welcome.

This study has carried out accurate numerical calculations for the phase-lag at different places of importance and the timing of the effluent injection with respect to the nature of ebb and flood tides in the Tay Estuary. Numerically simulated velocity of water and the known distance of a location from the estuary mouth have been used to obtain the phase lags. The initial dilution and the subsequent dispersion of pollutants, which are influenced by the main flow channel morphology, momentum of pollutants’ jet, turbulent mixing beyond the initial mixing zone and the tidal currents, are also taken into account in selecting the optimum discharge strategies. Other interrelated issues such as the exchange mechanisms of the pollutants’ cloud with the sea, with particular attention to the tidal flow asymmetry, and the possible undesirable effluent re-entrainment are also considered. Finally, it should be mentioned that this work hypothesises use of a “conservative approach” or “worst possible scenario calculation” by way of over-design for the discharge policies so that the environment is surely protected. All results presented in this paper are for 2D surface water flows. As illustrated by Nassehi and Kafai (1999), the flexibility of the finite element formulation allows it to be extended easily for moving boundary situations encountered in the Tay Estuary.

The mathematical formulation
The physical system is represented by a mathematical model based on the numerical solution of the following depth averaged shallow water equations of continuity (1) and momentum (2), and the advection-diffusion Eq. (3).

\[
\begin{align*}
\frac{\partial H}{\partial t} + \frac{\partial (Hu_i)}{\partial x_i} & = 0 \\
\frac{\partial Hu_j}{\partial t} + u_j \frac{\partial (Hu_i)}{\partial x_j} & + gH \frac{\partial (\eta)}{\partial x_i} + g \frac{u_i}{CH^2} & = 0 \\
H \frac{\partial c}{\partial t} + u_j \frac{\partial (Hu_i c)}{\partial x_j} & = \frac{\partial}{\partial x_i} \left( \frac{HD_i}{\partial x_i} \right)
\end{align*}
\]

where \(H, u_i, g, \eta, C\) and \(D_i\) are the total water depth, water velocity components, acceleration due to gravity, relative water surface elevation with the respect to a datum, Chezy friction coefficient and dispersion coefficient, respectively for \(i,j=1,2\). The mathematical formulation involves the application of the Finite Element Method due to the ability of this technique in coping with complex domain geometry and boundary conditions. The Galerkin solution scheme with isoparametric quadrilateral elements and Lagrangian shape
functions for the spatial approximation and the time discretisation based on the Taylor-Galerkin (Donea, 1984) method have been utilised. The effects of the wind and the Coriolis forces can be ignored as explained by Falconer (1992). The formulation provides depth-averaged velocity components and water elevations at the nodal points of the finite element grid and, the corresponding concentration values. The Dirichlet boundary conditions of water elevation at the estuary mouth and the fresh water flows at the estuary head and, the arbitrary initial conditions are used to solve equations (1) and (2). Boundary conditions equal to the concentrations of the discharged pollutants are defined at particular sampling locations. The time step for the temporal simulation is selected in view of the desired accuracy and the length of the simulation time. The Chezy coefficient \( C \) and, the dispersion coefficient \( D \) are related to the water elevation and the turbulence, respectively, expressed as functions of the depth and the velocity (Nassehi and Kafai, 1999). Therefore, through optimisation procedures these parameters are updated at any time step as new velocity and water elevation values are generated.

Results and discussions
The computer model developed in this study has been utilised to simulate typical estuarine flow parameters such as tidal elevations, water velocities and pollutant concentrations available, which are necessary for devising the aimed discharge schedule for the Tay Estuary. The model has been verified by comparing the simulated results for water level with monitored values available in the literature (Figure 2). The comparison shows good agreement and confirms the accuracy of the simulations. Though this work is primarily based on the numerical analysis of the hydrodynamic aspects of the Tay, the importance of qualitative judgements for engineering designs is not denied here. In fact, as illustrated below, this work does rely on both numerical and qualitative arguments for the statement of an effluent discharge problem and its possible remediation.

The simulated results for water elevation show the progressive deformation of the tidal wave, and the elevation peaks at different locations (Figure 2). Also, the numerical model has provided the phase lag values, increasing with the distance from the mouth. These phase lags are taken into account to establish the timing of pollutants’ release. The present model is most suitable for the main flow channel. The simulation of very shallow water depth in the northern side can be difficult because unstable or unrealistic solutions are often encountered. Initially, the simulator was used to obtain profiles of flow variables at five selected locations, namely, Broughty Ferry, Newport, Flisk, Newburgh and Inchyra, according to their increasing distances from the estuary mouth as shown in Table 1. The table also presents the minimum and maximum water levels, maximum velocity during ebb and flood tide and, the phase lag values during low and high water at these locations based

![Figure 2](https://iwaponline.com/wst/article-pdf/43/7/247/429975/247.pdf)
on the numerical model. Inchyra, located on the Tay River, above its confluence with the River Earn (Figure 1), is influenced by the combined effects of incoming and outgoing tides. As such, different hydrodynamic behaviour is observed there and, contrary to other locations, higher maximum flood tide velocity than ebb tide velocity is found. This clearly affects the pollutants’ dispersive and convective behaviour at this location and is noted for the inclusion in the future development of the numerical scheme.

However, the main emphasis here is on the description of a discharge policy for the Tay Estuary based on the calculations for the other four locations. The velocity profiles for the estuary, as shown by Petera and Nassehi (1996) have been found to be most prevalent in the southern side (i.e., main flow channel). During the flood tides, velocities in the southern side are generally below 1 m/s; though, with the ebb tides, the velocities may rise (Table 1, Figure 3). For the design of a discharge policy, this work adopts 0.8 m/s as the conservative ebb tide velocity based on the average speeds along the Estuary. In case of convection dominated flow processes, as in the present case, a low water velocity increases the time for transportation of the pollutants to cover a given distance.

The development of a conscious pollutant discharge policy relies on careful evaluation of a number of parameters such as the release mechanisms of pollutants, timings of discharges relative to the tidal cycle, surface water velocity, and the dispersive capacity of the estuarine water. Along with these functions, the conveying of the pollutants to the sea also depends on the distance of the discharge location from the mouth (L), transportation distance (S) and the available time (T). In some real situations the pollutant sources may behave as “continuous”, particularly if the time gap between two consecutive discharges is less than the pollutant dilution time. They result in build up of pollutant loads in the estuary which makes management of the water quality more difficult. Similarly, a contaminated river, combined with relatively continuous (intermittent) or instantaneous discharge, may create an artificial continuous source in the estuary. However, the selection of inlet

Table 1 Hydrodynamic data for the Tay Estuary

<table>
<thead>
<tr>
<th>Location</th>
<th>Distance (km)</th>
<th>Water level (m)</th>
<th>Maximum velocity (m/s)</th>
<th>Phase lag (minute)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Minimum</td>
<td>Maximum</td>
<td>Flood tide</td>
</tr>
<tr>
<td>Broughty Ferry</td>
<td>9.0</td>
<td>0.002</td>
<td>4.95</td>
<td>1.20</td>
</tr>
<tr>
<td>Newport</td>
<td>15.75</td>
<td>0.250</td>
<td>5.10</td>
<td>0.89</td>
</tr>
<tr>
<td>Flisk</td>
<td>24.75</td>
<td>0.130</td>
<td>5.25</td>
<td>0.79</td>
</tr>
<tr>
<td>Newburgh</td>
<td>33.75</td>
<td>1.160</td>
<td>5.50</td>
<td>0.78</td>
</tr>
<tr>
<td>Inchyra</td>
<td>38.25</td>
<td>1.750</td>
<td>5.70</td>
<td>0.80</td>
</tr>
</tbody>
</table>

Figure 3 Velocity profiles during the flood and ebb tide
boundary conditions for numerical calculations eliminates such probabilities in this work. In this study the concentration distributions with specified initial and boundary conditions for an assumed continuous release in the Tay Estuary are simulated. Figure 4 presents one typical example of the simulations where a distribution for the initial four hours of the discharge in Newburgh is shown. As apparent, during the ebb tide, the concentration values increase while, with the increase in the water flux during the flood tide, concentration values start to fall. The pollutant cloud does not show a very fast dilution.

On the other hand, relatively continuous releases are followed by a well-dispersed concentration profile as presented in Figure 5. The hypothetical process that was started towards the end of an ebb tide in Newburgh was continued for 1.2 hours. This results in the
increase of the concentration during the course of the discharge; but disperses to give lower levels as it is stopped. The initial decline in the concentration profile may have been due to the combined effects of diffusion and convection. However, the concentration continues to decrease even when velocity is low, implying less convective effects, which indicates the good diffusive capacity of the Tay. The possible increase of pollutant concentration because of returning water to the sampling point during the flood tides also does not seem to influence the process.

The change of concentration values at different stations during the course of the imposed discharge mode is monitored to find out any possible implications. Figure 6 illustrates such a profile. As evident, the pollutants concentration distribution follows a complicated process, which might be hard to explain through commonly used Gaussian models.

The main drawback of the imposed intermittent discharge modes is that they require a very cautious selection of all relevant parameters. For example, if the pollutants are injected at the beginning or during the ebb tide, their transportation time is reduced by the discharge period, which is undesirable in all situations. Similarly, if the pollutants were released

<table>
<thead>
<tr>
<th>Location</th>
<th>L (km)</th>
<th>T (hr)</th>
<th>S (km)</th>
<th>S≥L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Newburgh</td>
<td>33.75</td>
<td>5.1</td>
<td>14.7</td>
<td>x</td>
</tr>
<tr>
<td>Fisk</td>
<td>24.75</td>
<td>5.55</td>
<td>15.9</td>
<td>x</td>
</tr>
<tr>
<td>Newport</td>
<td>15.75</td>
<td>5.83</td>
<td>16.8</td>
<td>✓</td>
</tr>
<tr>
<td>Broughty Ferry</td>
<td>9.0</td>
<td>5.95</td>
<td>17.2</td>
<td>✓</td>
</tr>
</tbody>
</table>

Figure 7 Proposed discharge policy in the form of an algorithm
during flood tide, they would move upstream initially; hence, during ebb tides, the necessary transportation distance to the sea might increase drastically. Contrary to continuous and intermittent processes, a batch injection is characterised by a number of flexible possibilities; for example, with respect to the choice of the discharge time. As mentioned earlier, the timing of discharge in relation to the tidal cycle is a very important factor for the development of a discharge policy. This determines the pollutant loads in the estuary by way of allowing, or preventing, the pollutants to move to the sea during the ebb tides. For precise calculation of the available transportation time, therefore, the phase lag and the duration of discharge \( T_d \) must be taken into account. Also, in every situation, the time difference between the two tidal cycles (the slack water) should also be considered. Figure 7 presents the important steps necessary for designing a general discharge policy in the form of an algorithm for relatively continuous and batch releases. The design might necessitate adjustments of some parameters especially the duration of discharge and the distance of the location from the estuary mouth. An instantaneous process hardly allows any change in the release time. So, the only possible way to formulate a better management strategy is by the selection of a more appropriate release site. In case of relatively continuous processes, if necessary both discharge time and location might be altered. However, release time is easier to adjust than the location of the discharge and initially it should be opted for any modification. Due to the presence of a large number of uncertainties involved in the calculation only a conservative approach, therefore, provides the safest option for the required task.

It has been discussed above that the release during flood tides may take any material upstream, which is undesirable for pollutant transportation to the sea. A probable way of discharging the effluents is at the beginning of an ebb tide (Webb and Tomlinson, 1992). However, what seems to be the best option is to discharge during the slack water. This work analyses the use of batch modes to formulate a conservative discharge policy for the Tay Estuary because of the negligible release time considered in this case. It is often argued that such releases are not cost-effective since it might be necessary to incur huge expenditures for construction of storage tanks (Bikangaga and Nassehi, 1995). However, if instantaneous releases permit better management of water quality, it certainly justifies their selection for the future protection of the environment. The releases are done during the slack water so that the entire ebb period can be utilised for pollutants’ transportation while avoiding any possibility of the effluents moving upstream. A conservative water velocity has been selected (0.8 m/s) which makes the predictions safe. The necessary transportation distances of the release points from the mouth are known. The pollutants are assumed to be non-reactive and they are released from single point sources. Possibilities of reactive pollutants and multiple sources are being explored and are to be presented in a forthcoming paper. Table 2 shows the results of the proposed discharge mode. The release of pollutants has been done instantaneously and, the necessary transportation distances \( L \) for different locations have been compared to the distances of pollutants’ carriage by the estuary during the ebb tide \( S \). One important assumption that has been made is that a pollutant cloud is not influenced by any other source coming from the other locations. Hence, effluents are released at a single point at a time. Only the effluents originated at Newport and Broughty Ferry are carried away to the sea. Newburgh and Flisk, hence, cannot be considered as suitable locations for batch discharges of effluents. The waste products generated at these locations, therefore, should ideally be transported towards the mouth, possibly to Newport, before they are released into the Tay.

Conclusions
The necessity of formulating a suitable discharge policy for the conservation of the Tay Estuary in Scotland has been re-emphasised here. Although numerical models provide a
very practical way of simulating the hydrodynamic behaviour of the estuary, for designing
discharge schedules qualitative judgements should also be used. Simple conservative
estimates, accounting for the uncertainties in the hydrodynamic variables have shown that
Newport is the most suitable location for effluent discharge in this estuary. Complex hydro-
dynamic aspects such as moving boundaries, flow in narrow channels, presence of reactive
pollutants in the estuary and release from multiple sources need to be incorporated in the
developed model. To achieve these tasks, without ignoring the multi-disciplinary point of
view, a computational system is being developed to satisfy the user’s practical require-
ments reflecting the expert knowledge involved in the definition of the problem. The pres-
et study is an important step towards the validation of the developed computer based
methodology for formulating discharge policies under realistic conditions of a complex
estuary.

References
determination of optimum effluent discharge policies in tidal water systems. Wat. Res., 29(10),
2367–2375.
Buller, A.T., Charlton, J.A and McManus, J. (1972). Data from physical and chemical measurements in the
Tay Estuary for Neap and Spring tides. Tay Estuary Research Report, No.: 2, University of Dundee,
Dundee.
147–157.
London.
McManus, J. (1968). The hydrology of the Tay basin in Dundee and District. S.D. Jones (ed.). British
Association for the Advancement of Science, Dundee, pp.107–124.
Methods Fluids, 39, 4159–4182.
Thomass, R.V. (1998). The future “Golden Age” of the predictive models for the surface water quality and