

The Physics of Long Term Drawdowns in Pumping from Artesian Aquifers

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The applicability of the Theis-formula (1935) and the Jacob-Hantush leakage-formula (1946 and 1956) as tools for the description of water-level variations due to pumping from artesian aquifers, is shown to be rather limited, especially in long term pumping situations. The limitations are discussed and illustrated through a review of some mathematical models with more reliable and physical realistic preconditions, and a general perception of the physics governing the drawdowns in pumping from leaky artesian aquifers is hereby established. Data from 10 years of heavy pumping from a leaky artesian multi-aquifer at Egebjerg near Horsens, Denmark, are analysed, and the result turns out to be in good agreement with the theoretical discussion.

Introduction

In this paper all of the aquifer/aquitard systems are supposed to be homogeneous and of large extent in the horizontal directions. These limitations are present because the aim is to establish realistic vertical boundary conditions, which is a physical problem, while horizontal boundaries merely give rise to problems of geometrical nature. Physically realistic upper boundary conditions are, for example, necessary if a sound connection between the ground-water and surface-water systems is to be established in a hydrological model.

The vertical boundary condition in the Theis formula is the presence of completely impermeable confining layers, Fig. 1. This precondition is generally not fulfilled in nature, and therefore some models, describing the leakage of water

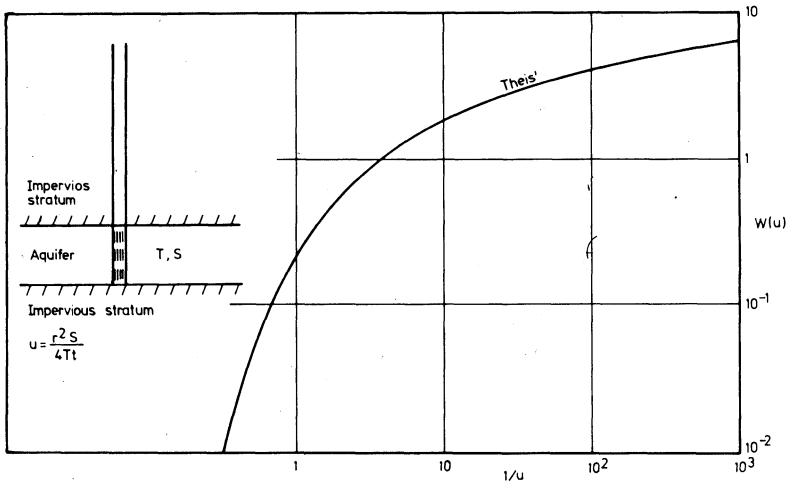


Fig. 1. Theis' solution for the drawdowns in an aquifer with impervious top and bottom.

into the aquifer from surrounding layers have been evaluated. The most well-known of these models is the Jacob-Hantush formula. In this formula the leakage takes place through a non-elastic upper aquitard, which is physically described by the leakage-factor P'/m' , P' being the vertical permeability of the aquitard and m' its thickness. The base of the aquifer is assumed impermeable, and the leakage is originated in a zone with constant head (for example an upper aquifer) situated above the aquitard, Fig. 2.

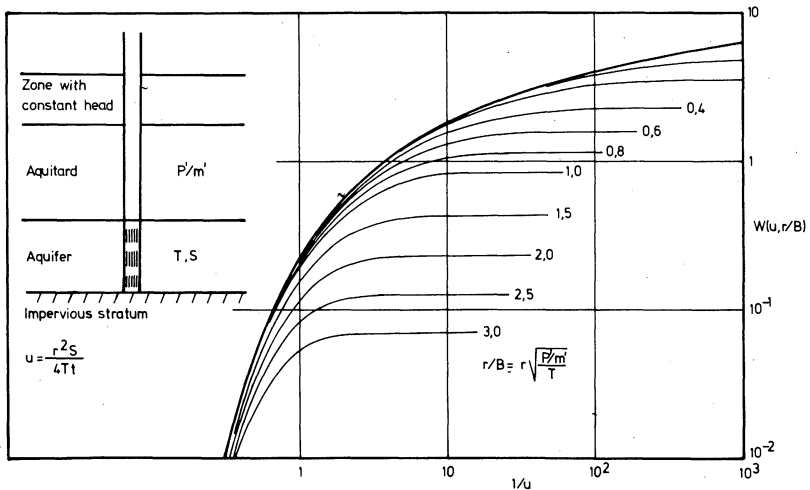


Fig. 2. The Hantush - Jacob leakage - solution. The leakage into the aquifer is originated in the zone with constant head and passes through the rigid aquitard (no storage). The constant head in the upper zone results in a final steady-state after a certain period of pumping.

Discussion of the Preconditions in the Jacob-Hantush Leakage Formula

The impermeable base of the aquifer and the lack of storage in the aquitard are conditions, that might be tolerably fulfilled in many cases. The constant head in the upper zone, however, is in many cases physically absurd. This is clear from the fact, that the well-known final steady-state solution of the model assumes, that all the water pumped from the aquifer is replaced by water from the upper zone. Therefore this zone cannot possibly maintain a constant potential, unless the zone consists of a lake or river or the like with a total water circulation much greater than the groundwater withdrawal from the artesian aquifer. As the constant head cannot normally be maintained, the leakage-steady-state of the drawdowns in the aquifer cannot either, and the drawdowns will continue beyond this »pseudo«-steady-state.

However, in lots of pumping tests in Denmark, data curves matching the Jacob-Hantush type curves ending up with the described steady-state, have been observed. It must therefore be concluded, that zones, which can maintain at least approximately constant head in a certain period during pumping in fact exist. It is a widespread perception, that these zones consist of upper or lower aquifers with potentials above that of the pumped aquifer. This is not correct. First the relative potentials between the pumped aquifer and the one assumed yielding the leakage is not important, according to the model. Furthermore, the constant head in the leakage-yielding aquifer (if it is an artesian aquifer) can only be approximately maintained if this aquifer has a much greater transmissivity and a much greater extent than the drawdown-cone in the pumped aquifer. This follows from the fact that the leakage-yielding aquifer, in order to maintain an approximately constant potential, must be able to withdraw water from a much greater area, and with much smaller drawdowns than the pumped aquifer. As these conditions will almost never be fulfilled, it must be concluded, that upper or lower artesian aquifers generally cannot create a steady state in the pumped aquifer, their contribution to the pumpage being too small, as they cannot maintain a constant head, even in a short period of withdrawal.

The statement made above is further confirmed from the fact, that data following the Jacob-Hantush type curves very closely have been observed during pumping tests in artesian aquifers, where no upper or lower aquifers were present. The only way to explain this, is to assume the presence of a zone with a storage large enough to maintain an approximately constant head in a certain period. Such a zone will often be present at the water table. Whether this is situated in sand or in clay, a specific yield much larger than the elastic storage in the artesian aquifer will be present here. As the drawdowns from the pumping in the artesian aquifer reaches this zone, they will therefore be damped considerably, and if the specific yield at the water table is large enough, a period of steady state will occur in the aquifer. This period will last until the drawdowns at the water table become significant, and the drawdowns will then continue in the

whole system, making use of the total storage in the saturated zone of the area (specific yield at the water table, elastic storage in artesian aquifer(s) and storage from consolidation of the aquitard(s)). These matters have been described mathematically by Boulton (1954), Hantush (1967) and Cooley and Case (1973).

It follows from above, that the water induced in an artesian aquifer from »leakage« in a pumping situation, is generally originated from the groundwater storage in the area, and that the »leakage- steady-state« is a pseudo-steady-state of limited duration. In some special cases, however, the leakage situation may be combined with an increase in infiltration and thereby produce a final steady-state. This may be the case if the water table is situated very close to, or at the land surface. Then a lowering of the water table may produce a decrease in evapotranspiration and surface-runoff and thereby increase the infiltration sufficiently for a final steady-state to be reached. If, on the other hand, the water table is situated a certain (rather small) distance below the land surface, lowering of the water table will not produce increased infiltration, and the leakage will therefore, as mentioned, consist of ground water storage only. This will produce continued drawdowns in the area until one or more constant-head-boundaries (streams, lakes, shores) are reached by the drawdown-cone.

Review of Mathematical Models with Alternative Vertical Boundary Conditions

In 1960, Hantush published a model accounting for storage in an aquitard on top of (or/and below) the aquifer. This model assumes, that the drawdowns never reach the top of the aquitard (small values of P'/m') and is therefore normally valid in only a rather limited period of pumping. In this period the model should replace the Theis formula, as the release of water from consolidation of aquitards at base and top of the pumped aquifer will be of significance from the very start of pumping, Fig 3.

As pointed out by Neumann and Witherspoon (1969b), the use of a Theis type curve analyzing pumping tests in aquifers, where the consolidation of aquitards is of significant importance, will lead to serious errors in the estimates of transmissivity and leakage factor. This is important to notice, especially because the type curves of the Hantush (1960) model are very close to the Theis curve in shape. However, in most localities in Denmark, the effect of consolidation of aquitards does not seem to be very significant during aquifer tests. This is probably partly because most extensive aquitards in Denmark are heavily preconsolidated by the ice, and partly because most artesian aquifers are rather shallow, meaning that the drawdowns often reaches the water table (top of aquitard) after rather short terms of pumping, thus limiting the period of validity of the model. It must be noted

On Long Term Pumping from Artesian Aquifers

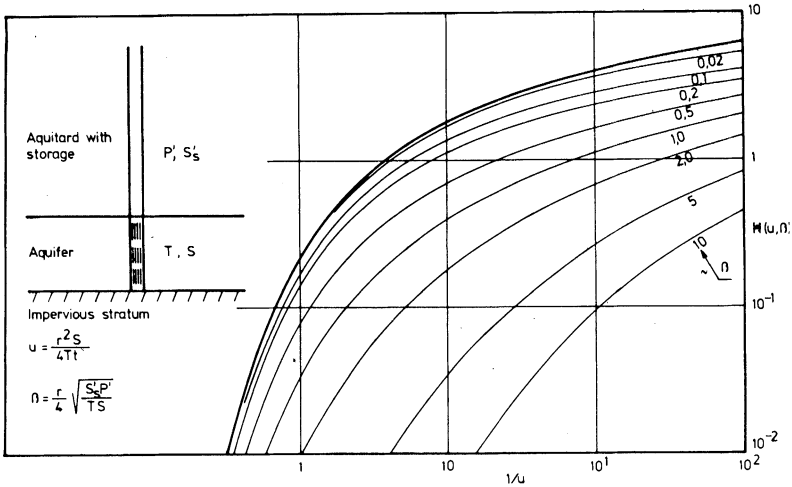


Fig. 3. Hantush' model of drawdowns in an aquifer, accounting for storage in an upper (or lower, or both) aquitard of infinite thickness (valid as long as the drawdowns have not reached the top of the aquitard).

though, that even if the consolidation storage in aquitards is not significant during short terms of pumping, it may very well be a significant part of the total ground water storage, utilized in long term pumping situations.

A model accounting for drawdowns in an upper, non-pumped aquifer was first published by Hantush (1967). A complete solution of the case is not given, but special and asymptotic solutions are presented. Storage in aquitards is neglected. It is shown that for sufficient long terms of pumping the rate of drawdown in the pumped and the unpumped aquifer is the same, the pumping then utilizing the storage in both aquifers. A difference in head between the two aquifers is present and makes possible the transportation of water from the unpumped to the pumped aquifer (through the aquitard).

A complete solution for a confined two-aquifer system (pumped and unpumped aquifer, separated by an aquitard, impermeable base and top of the system) was presented by Neumann and Witherspoon in 1969 (a). The storage in the aquitard is accounted for, and formulas are presented for the drawdowns in both aquifers and the aquitard. The formulas are too complicated to be used directly in engineering practice, but is probably the most general set of formulas available in groundwater hydraulics today (a set of formulas for a multi-aquifer system have been evaluated by the authors too).

The case of an artesian aquifer overlain by a water table aquitard is extremely important in practice in Denmark, because it covers the very frequent practical case of a regional artesian aquifer overlain by deposits of clay and/or silt in which some more or less local (but not regional) aquifers are embedded. This case may

be formulated as a special solution to the Neumann-Witherspoon model (1969a), but is explicitly treated by Boulton (1954) and Cooley and Case (1973), both with directly practically useable results. In both treatments the storage at the water table (in the aquitard or in the aquifer in the non-artesian case) is used as the upper boundary condition. Delayed yield from storage at the water table in fine-grained sediments is accounted for. Consolidation storage according to Hantush (1960) in the aquitard is incorporated by Cooley and Case.

The Cooley and Case model may shortly be described as follows: At first the drawdowns in the aquifer follow the Hantush (1960) type curves, taking the aquitard consolidation into account. As the drawdowns in the upper aquitard reaches the water table, the pseudo-steady-state (Jacob 1946) occurs in the aquifer if the specific yield is large enough to maintain approximately constant head at the water table in a certain period. After the drawdowns at the water table have become significant, the drawdowns in the aquifer continue following the second part of the delayed yield type curve (Boulton 1954). The asymptotic solution for long terms is a Theis curve with the parameters T (transmissivity of the pumped aquifer) and S_{Tot} (total storage of the geologic profile in the area). This solution will be valid for the drawdowns as long as no constant head (run-off) boundaries are reached by the drawdown-cone.

The Cooley and Case model does not present formulas for computing the drawdowns in the aquitard or, which is more important, at the water table. To accomplish this, the special or asymptotic formulae of Hantush (1967) or the general solution of Neumann and Witherspoon (1969a) must be used. These drawdowns in the upper zone of an area, induced from pumping in an artesian aquifer, are very important to know if the influence of the pumping on the ground water run-off to streams in the area is to be judged. This ground water run-off will often mainly be generated in secondary, upper aquifers and the influence of the pumpage on the head in these aquifers will therefore determine the stream depletion. Stream depletion induced this way must be supposed to take place in many localities in Denmark today, and is a lot more difficult to forecast with reasonable accuracy than the simple type of stream depletion induced from pumping in an aquifer with direct hydraulic connection to the stream (Jenkins 1968 et al.). This is because the description of the problem necessitates the use of three dimensional ground-water models, as a two-dimensional model in many cases will not be able to account for aquitard and water table storage in a satisfactory way. Only a three-dimensional groundwater model can provide information about drawdowns in the upper zone and the resulting effect on the surface-runoff.

In the following though, a two-dimensional modelling of an artesian reservoir-area, according for the total storage, will be shown. This simple modelling is possible, because no considerable stream-depletion takes place in the area, and therefore only the drawdowns in the aquifer is of interest (for the time being).

On Long Term Pumping from Artesian Aquifers

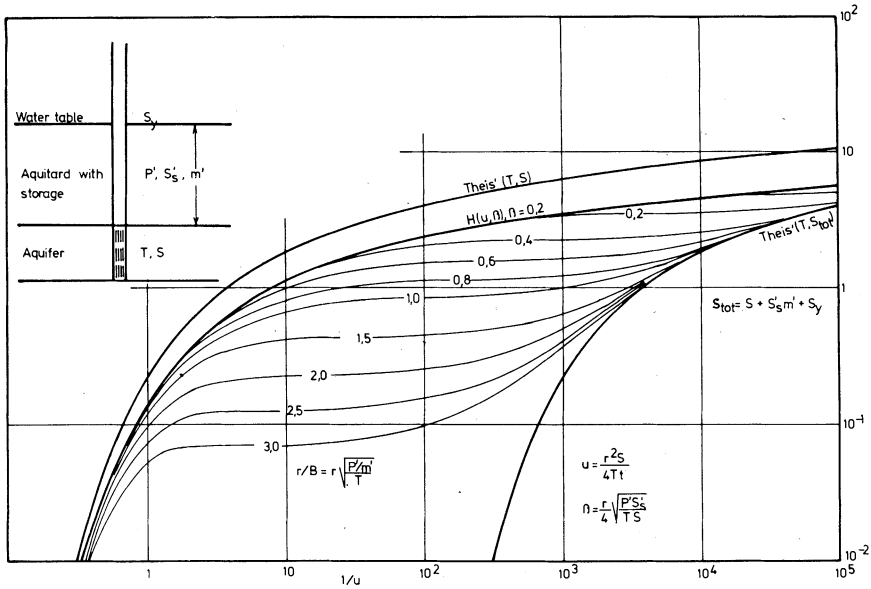


Fig. 4. Examples of drawdown-curves in aquifers overlain by water-table aquitards with $\beta \equiv 0.2$. The pseudo-steady-state occurs, because $S_{tot}/S > \text{ca. } 100$. Curves like these were obtained by Cooley and Case (1973) by using Boulton's delayed-yield-boundary condition at the water table in the aquitard.

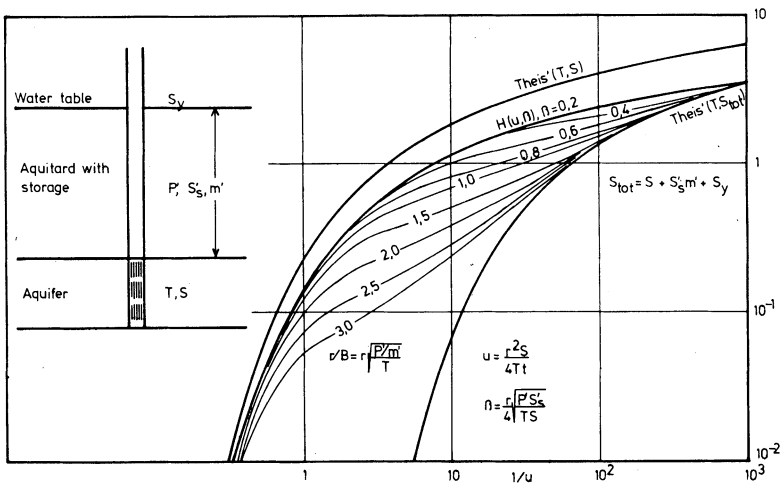


Fig. 5. Example of type-curves similar to those in Fig. 4, but with $S_{tot}/S < \text{ca. } 100$. The curves are graphically interpolated on the basis of Fig. 4. Note, that the leakage-pseudo-steady-state does not occur, and that the transition between the $H(u, \beta)$ -curve and final Theis curve is hardly noticeable for small values of r/B .

The Long Term Drawdowns at Egebjerg Waterworks, Horsens Municipal Water Supply

The geologic circumstances in this area are described in detail elsewhere (Andersen and Haman 1970). Only the following, rather brief description of the region will be given here:

The well field is established in glacial and interglacial deposits, situated in a valley in the tertiary sediments. The direction of the valley is approximately NNE-SSW, the width may be judged to be about 5 km, and the bottom of the valley is about 100 m below sea level. The glacial and interglacial deposits at the well field forms a multi-aquifer with 1 – 3 artesian and 1 semi-artesian, partly unconfined aquifer. The aquifers are separated by aquitards of drift clay, diluvial clay/silt and, on top of the inter glacial sand-aquifer, diatomaceous earth.

Some of the aquifers are probably of rather limited extension, but the multi-aquifer as a whole must be expected to cover a considerable area. The tertiary deposits forming the bottom and the eastern wall of the valley about 3 – 4 km from the well field center, must be expected to be practically impermeable (marine deposits of clay from the miocene, oligocene and eocene periods). The western wall however, is very likely to be permeable, as the miocene deposits here are largely composed of sand. It is therefore probable, that the hydraulic connection to the west is reaching very far, so that the aquifer system is practically unlimited in this direction. To the north there is no information about the extension of the aquifer system, because of lack of sufficiently deep wells, a considerable extent must be expected however, as this is the approximate direction of the tertiary valley. To the south a hydraulic connection with Horsens Fjord is possible in a distance from the well field center of about 6 km.

The well field has been pumped since 1966 and the monthly withdrawal until January 1976 is shown in Fig. 6. The 10-year period is divided into 4 periods, each corresponding to a certain mean value of the monthly withdrawal. These mean values and periods are used in the computations as the monthly fluctuations are of minor importance in the total response from the aquifer. This is illustrated in Fig. 7, where some drawdowns in observation wells during the 10 year period are shown. The trend is absolutely dominating, and the fluctuations, both natural and from monthly variations in the pumpage, are not confusing the picture significantly. The plots shown are from wells situated in distances of about 300 (58) – 2000 (202) m from the well field center.

The analysis is carried out as follows:

Distance-drawdown plots of all observation wells at the ends of the four periods in Fig. 6 were prepared, Fig. 8. The difficulties involved in this are briefly resumed:

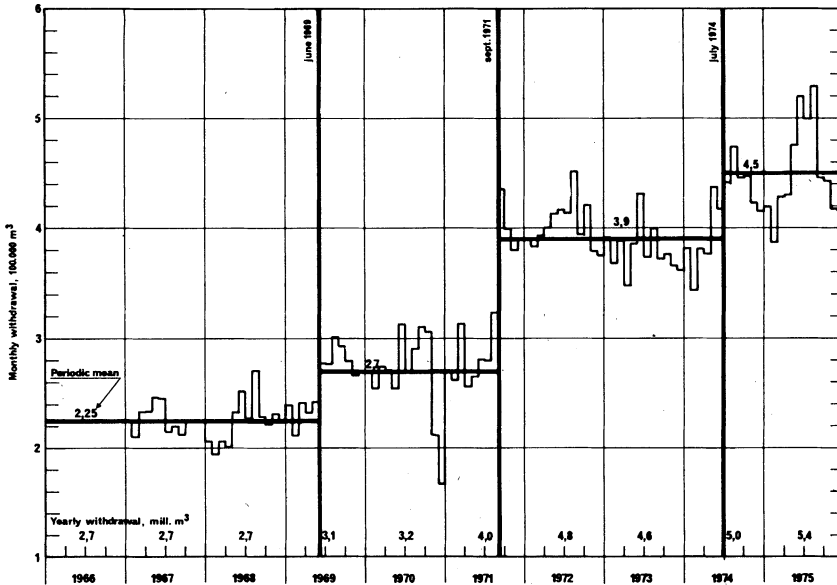


Fig. 6. Monthly withdrawal from the Egebjerg Waterworks, Horsens municipal Water Supply, in the period 1966-1976. This period is divided into intervals, where the mean withdrawal has been approximately constant.

- 1) An original water table for each observation well had to be chosen on the basis of the rather few water level observations from the period before 1968. This could be done with reasonable accuracy in most cases.
- 2) The periodic fluctuations in the water levels had to be evened out. This was done satisfactorily by eye.
- 3) A locality for the well field center had to be decided for each period, new wells being taken into use and old ones abandoned from time to time. As the groups of points in Fig. 8 do not change significantly with minor changes in the locality of the well field center, this has not been a serious problem.

The next task is to choose a basic model of the aquifer system. As it is a multi-aquifer a multi-aquifer model could be used. However, the information on the different aquifers in the system is not detailed enough to justify the use of such a complicated model. Instead a model of the Cooley and Case type described above is used, considering the whole aquifer system as one single aquifer. Then it will not be possible to account in details for the scatter of the points in the 4 distance-drawdown plots, as this scatter is the result of the vertical and horizontal inhomogeneities of the system, which are now neglected in the model. Furthermore, it will not be possible to account for changes in the drawdown distribution in the 10-year period, caused by changes in the distribution of the pumpage on the

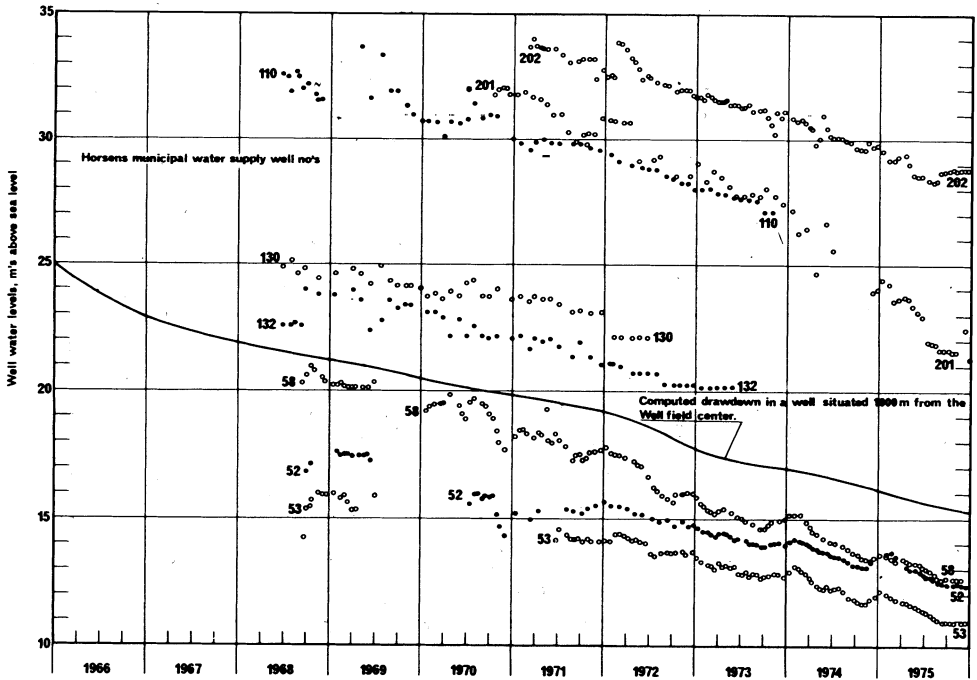


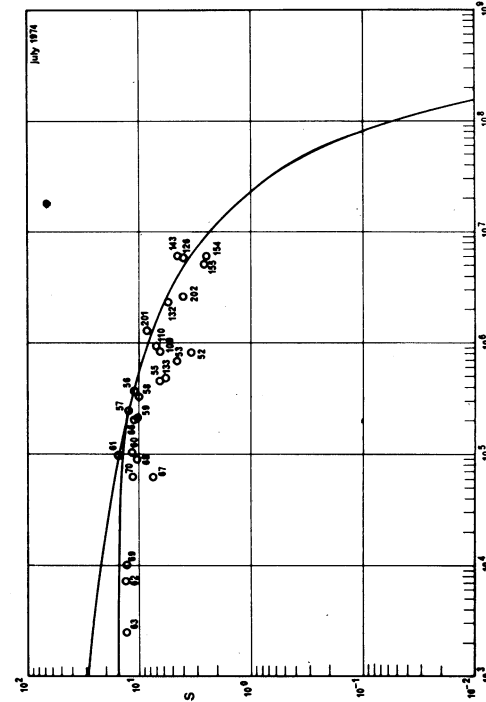
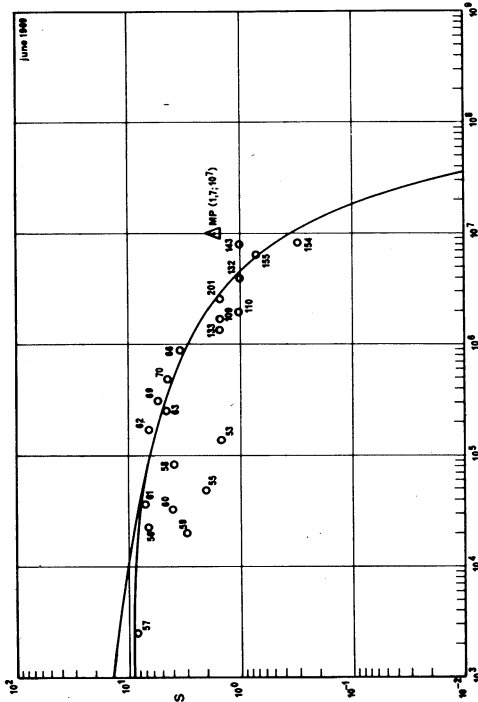
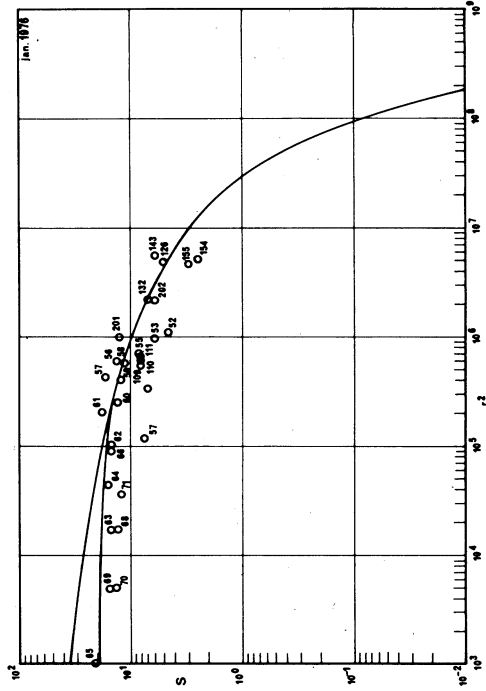
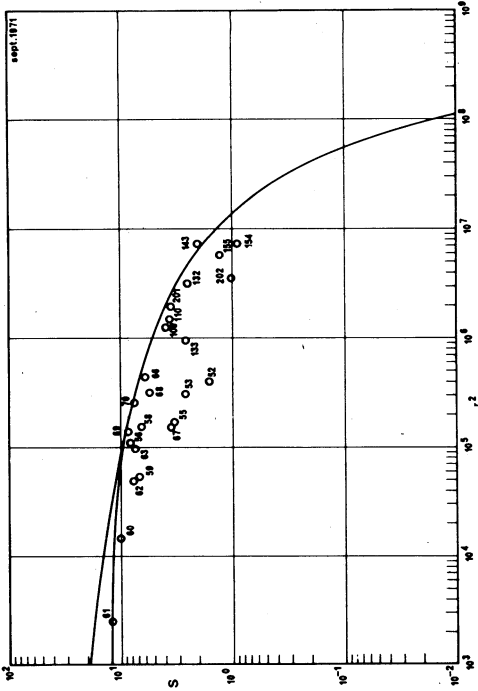
Fig. 7. Water level observations in a number of observation wells inside the well field area in the period 1968-1976. The calculated water-level variations in a hypothetical well inside this area are also shown. The calculation is based on the simple, two-dimensional model of the aquifer, as described in the text.

various aquifers. On the other hand, the model becomes very simple, and the physical main features governing the long term behaviour of the system (average transmissivity and total storage over a very large area) can still be unveiled, though the effects of local inhomogeneities are neglected.

It is furthermore necessary to know at which state of the chosen model the system has been in the 4 situations shown in Fig. 8. To judge this, 6 pumping tests, carried out in the area in 1968 by the Geological Survey of Denmark (Andersen and Haman 1970) are reviewed. Even if these few pumping tests are naturally not

Fig. 8. Distance-drawdown plots at the end of each period with constant mean withdrawal as shown in Fig. 6. All observation wells in the area are included, except some dug wells with no contact to the regional water table. The curves drawn correspond to the mean withdrawal distribution in Fig. 6, and the described simple, two-dimensional model of the aquifer. The curve »Jan.1976« is thus a superposition of four Theis-curves, one for each period of constant mean withdrawal. A correction for the distribution of the pumpage on several wells is made (the lower left branch of the curves on the four plots).

On Long Term Pumping from Artesian Aquifers



representative of a large area, it can be concluded, that the late time Theis curve is reached after a rather short time of pumping in the area, and that this part of the model therefore will be sufficient for the description of the long term drawdowns. A Theis curve is now matched in the first plot in Fig 8, and the parameters found are $T = 4 \times 10^{-3} \text{ m}^2/\text{sec}$ and $S_{tot} = 0.10$. On the basis of these parameters and the simplified withdrawal-distribution in Fig. 6, the corresponding curves on the other three plots in Fig. 8 are constructed. An empirical correction for the distribution of the pumpage on several wells is made in all 4 cases according to the actual shape of the well-field. The curves are found to match the 4 groups of points in a rather uniform way. As a further illustration a time-drawdown expiration in a hypothetical well 1000 m from the center is computed on the basis of the model, and shown in Fig. 7. It is seen to fit the general trend very good. The drawdowns in well 58 are somewhat steeper, because this well is situated closer (300 – 500 m) to the well field center. The drawdowns in 201 are steeper in the last 3 years of the period, which is partly because the well field center has moved in the direction of this well, partly because it is screened in a deep part of the aquifer, which was not directly producing until 1972.

No boundaries of the aquifer system have been taken into consideration so far. As mentioned the closest regional impermeable boundary must be expected to be about 3 – 4 km away to the east. A mirror well corresponding to this boundary will therefore be situated 6 – 8 km away, and will in January 1976 have produced drawdowns less than 1 m in the area where most of the observation wells are placed. This effect would be so small in Fig. 8, that it would hardly be noticeable. The same can be said about the eventual positive boundary at Horsens Fjord in a distance of about 6 km from the well field. An effect of stream depletion in the area is not noticeable in the drawdowns and must, so far, be expected to amount to only a small part of the withdrawal. No reliable run-off data from streams in the area exist, covering the period, but there is no evidence that a serious depletion should have taken place. It must be realised, however, that the effect of impermeable boundaries and of stream depletion on the drawdowns might, to some extent, outbalance each other.

The transmissivity value found in the analysis is considerably smaller than some of the values found in the earlier pumping tests, even though it is supposed to represent the whole aquifer system. This is probably because the various aquifers in the system are of limited extent and often connected through aquitards only. In this way the local transmissivity of a certain aquifer is likely to be greater than the average transmissivity of the whole system. It must be noted too, that the values of the determined model parameters, to some extent, depend on how the model curves are placed in the groups of points in Fig. 8.

The found total storage in the area is relatively large, meaning that the water table in a considerable part of the area of influence must be situated in deposits of sand.

Conclusion

It has been shown that the Theis formula (1935) and the Jacob-Hantush leakage formula are only fit for the description of drawdowns in artesian aquifers during rather limited terms of pumping. This is due to the preconditions on which these formulas are stated. A number of models with more realistic preconditions, especially in long term pumping situations have been reviewed with the following result:

The long term drawdowns of an artesian aquifer are governed by the transmissivity of the aquifer and the ground water storage in the area, especially the consolidation storage of aquitards and the specific yield at the water table. The effect of the transmissivity of a non-pumped aquifer (upper or lower) on the drawdowns in the area will be of minor importance, unless this transmissivity is very large and the non-pumped aquifer is of great extent compared to the extent of the drawdown-cone in the pumped aquifer.

This results has been used in the analysis of the drawdowns from 10 years of heavy withdrawal in a semi-artesian – artesian multi-aquifer at Egebjerg, near Horsens, Denmark. The result of the analysis is briefly resumed: The multi-aquifer is of large extent and has a total transmissivity of about $4 \times 10^{-3} \text{ m}^2/\text{sec}$. The total storage in the area is about 10%. The effect of negative or positive hydrological boundaries has not been significant (or has outbalanced each other) in the 10-year period, thus confirming the assumption of the closest regional impermeable boundary being situated about 3 – 4 km to the east. Stream depletion in the area has probably so far been small compared to the pumpage, which has mostly been extracted from the ground water storage.

Acknowledgments

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