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Author's Closure

The author appreciates the additional information and references provided by Professor Ioakimidis. In the journal article under discussion there was insufficient space to provide a review of the development of the theory of finite-part integrals and hypersingular integral equations. If this had been done, the publications of Professor Ioakimidis would have figured prominently. Since several of the formulas used for numerical computation were taken directly from Kaya and Erdogan (1984, 1985), these references were cited, with the knowledge that Professor Ioakimidis' publications were in turn adequately referenced in those sources.

In addition to the references provide by Professor Ioakimidis, the following references are of historical importance concerning the development and application of finite-part integrals in mechanics: Hadamard (1923), Mangler (1951), and Kutt (1975a, 1975b).

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A Sixth-Order Plate Theory—Derivation and Error Estimates[3]

C. W. Bert.[4] The author is to be congratulated for making three advances in the theory of small deflections of plates: (1) improvement on Levinson's theory by incorporation of thickness normal strain and transverse isotropy; (2) provision of an *à priori* error estimate for the resulting theory; and (3) reducing the coupled equations of the theory to a single sixth-order equation in the midplane deflection.

In connection with the error estimate, the author attributed the improvement in the error estimate for transverse shear stress over that of the Reissner (1944, 1945) theory to *both* the z^4 term in the deflection and the z^3 term in the in-plane normal stresses. It would be interesting to determine the relative importance of each of the two effects. Also, one would expect that, if the z^4 term in the deflection plays a role, then the z^2 term would also.

The author's theory still suffers from one small deficiency, from a physical viewpoint, in that the thickness normal stress is neglected. This could be remedied in the case of normal-pressure loaded plates in the way that has been used by Reissner (1944, 1945) and Baluch et al. (1984), for instance.

In the absence of normal pressure, apparently all sixth-order uniform-thickness plate theories, when reduced to a single equation in the midplane deflection, are found to have the following form, using the author's geometry (total plate thickness = $2h$) and other notation:

$$D(1+Ch^2\Delta)\Delta\Delta w=0 \tag{1}$$

The only difference among them is the expression for C, as listed in Table 1.

Table 1 Coefficient C for isotropic plates according to various sixth-order theories

Theory	Reduced by	C
Reissner (1944, 1945)	Speare and Kemp (1977)	$(2/5)(2-\nu)/(1-\nu)$
	Shirakawa (1983)	$-2/5$
	Discusser (Bert, 1986)	$-2/5$
Ambartsumyan (1970)	Shirakawa (1983)	$-1/(3K)$
Schmidt (1969)	Schmidt (1969)	$(2/3)/[(1-\nu)K]$
Levinson (1980)	Discusser (Bert, 1986)	$-2/5$
Rychter (1987)	Rychter (1987)	$(4/5)/(1-\nu)$

It is interesting to note that, if one takes the shear correction factor K in the Schmidt *ad hoc* theory to be 5/6, the result is a value exactly equal to that of the present author. However, as first enunciated by Shirakawa (1983) for the Reissner and Ambartsumyan plate theories and recently verified by the discusser for the Reissner and the Levinson theories, the value of C is identical to the coefficient of the second term in Reissner's (1945) second fundamental equation, his equation (22), for an auxiliary stress function ψ,

$$[1-Ch^2\Delta](\psi)=0 \tag{2}$$

If the shear correction K for Ambartsumyan's theory is taken to be 5/6, then the value of C derived by Shirakawa for that theory and for Reissner's (1945) theory coincides with that obtained for Levinson's theory by the discusser. Also, if K in Schmidt's theory is taken to be 5/6, the resulting C value coincides with that of Rychter.

It is noted that the degrees of polynomials in z for the in-plane displacements and the normal deflection used in the author's theory has been derived by the discusser (Bert, 1986) using a Saint-Venant inverse approach that is a generalization of the 1883 work of Clebsch, as presented by Timoshenko and Woinowsky-Krieger (1959).

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[3]By Z. Rychter and published in the June, 1987, issue of the ASME Journal of Applied Mechanics, Vol. 54, pp. 275–279.

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A Sixth-Order Plate Theory—Derivation and Error Estimates[3]

R. Schmidt.[5] The theory of plates to which the author refers to as "Levinson's theory" and cites Levinson (1980) as his reference is nothing else but a linearized version of the nonlinear theory published by Schmidt (1977). A statement to that effect can be found in *Applied Mechanics Reviews*, Vol. 34, No. 7, July 1981, Review 6280, p. 952. Moreover, Dr. Levinson has acknowledged the identicalness of the two theories in a private letter to the discusser.

The theory of Schmidt (1977) has not been completely unknown. It has been referred to in several journals, including this one (e.g., Sathyamoorthy and Chia, 1980).

In the name of fairness, this oversight should be acknowledged and corrected.

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Levinson, M., 1980, "An Accurate, Simle Theory of the Statics and Dynamics of Elastic Plates," *Mechanics Research Communications*, Vol. 7, No. 6, pp. 343-350.

Sathyamoorthy, M., and Chia, C. Y., 1980, "Effect of Transverse Shear and Rotatory Inertia on Large Amplitude Vibration of Anisotropic Skew Plates; Part 1—Theory," ASME JOURNAL OF APPLIED MECHANICS, Vol. 47, No. 1, pp. 128-132.

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A Sixth-Order Plate Theory—Derivation and Error Estimates[3]

E. Reissner.[6] The unqualified acceptance of results in Speare and Kemp (1977), in conjunction with the author's derivation of analogous results based on an analysis in Levinson (1980), suggests the following observation.

The essence of the results in Speare and Kemp (1977) is that is should be appropriate to reduce the sixth order theory of shear deformable plates in Reissner (1945) to one equivalent sixth order equation

$$D\left\{1+\frac{2-\nu}{1-\nu}\frac{h^2}{10}\right\}\nabla^4 w=q, \tag{1}$$

for the deflection w of a homogeneous isotropic plate, with h designating plate thickness and ∇^2 for the Laplace operator, in place of the author's $2h$ and Δ. Unfortunately, the derivation of equation (1) depended on a fundamental oversight which invalidates this equation as well as its consequences.

To describe what is involved in this matter we depart, as in Spear and Kemp (1977), from the following three equations for shear stress resultants Q_x and Q_y and a load intensity function q,

$$\frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}=-q, \tag{2}$$

$$\left[1-\frac{h^2}{10}\nabla^2\right][Q_x,\ Q_y]=-\left[\frac{\partial}{\partial x},\ \frac{\partial}{\partial y}\right]\left[D\nabla^2 w+\frac{h^2 q}{10(1-\nu)}\right]. \tag{3}$$

In order to deduce equation (1) from equations (2) and (3), the assumption was made that the terms $h^2\nabla^2(Q_x,\ Q_y)$ and $h^2 q$ in equation (3) were small compared with the terms $(Q_x,\ Q_y)$ and $D\nabla^2 w$, respectively, in such a way that it was appropriate to set, *in these terms*, $(Q_x,\ Q_y)=-D(\partial\nabla^2 w/\partial x,\ \partial\nabla^2\partial y)$, with q as in equation (2), so as to have, in place of equation (3)

$$(Q_x,\ Q_y)=-D\left[\frac{\partial}{\partial x},\ \frac{\partial}{\partial y}\right]\left[\nabla^2 w+\frac{h^2}{10}\left[1+\frac{1}{1-\nu}\right]\nabla^4 w\right], \tag{4}$$

with the introduction of equation (4) into (2) leading to equation (1).

While it should be evident that the validity of equation (1) should be suspect in view of the facts that (*i*) the indicated reduction procedure necessarily ceases to be valid for plates acted upon by edge loads only, and (*ii*) the positiveness of the coefficient of the term $h^2\nabla^2$ is associated with physically unreasonable behavior of portions of the solution function w, one can, going beyond this, state the reason for these unacceptable results as follows. It is the essence of shear-deformable plate theory that portions of the terms $h^2\nabla^2(Q_x,\ Q_y)$ in equation (3) are *not* small compared to the corresponding portions of the terms $(Q_x,\ Q_y)$, consistent with the fact that the solution of the sixth order theory of shear deformable plates involves a boundary layer solution contribution which is absent in the classical fourth order Kirchhoff theory.

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A Method of Eliminating Lagrangian Multipliers from the Equation of Motion of Interconnected Mechanical Systems[7]

J. G. Papastavridis.[8] Although its final result, i.e., equations (14) are correct, the Note contains a number of erroneous statements:

(*i*) The imposition of the *N catastatic* nonholonomic constraints (1) on the system does *not* affect the number of its (independent or unconstrained or minimal) generalized coordinates; the latter, as we infer from equations (10), are still M in number and not M-N as the Note states.

(*ii*) The constraint reactions, in an ideal system, produce no work for any *virtual* displacement, i.e., displacement compatible with the instantaneous/frozen constraints (in the form (1), but with $\dot{q}$ replaced with δq), and not just ". . . any displacement compatible with the constraints," as that author states (near the top of p. 236). Thus the following "zero reaction power" condition (between his equations (7′) and (8)) holds *only for catastatic* (i.e., homogeneous) nonholonomic constraints *such as* (*1*), but *not for general acatastatic* (i.e., nonhomogeneous) ones; for the relevant definitions see Rosenberg (1977).

[3]See footnote 3, p. 249.

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[7]By S. Vlase, published in the March, 1987, issue of ASME JOURNAL OF APPLIED MECHANICS, Vol. 54, pp. 235-237.

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