A Shell Model Calculation for Scattering of Electrons by Be⁹

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Elastic and inelastic scattering of 190 Mev electrons by ground and 2.43 Mev states of Be⁹ is computed by using an intermediate coupling shell model. The discrepancy between the calculated and the observed inelastic cross sections reported earlier is now removed by taking the correct spin value \( J=\frac{5}{2}^- \) for the 2.43 Mev state. It is shown that a good fit to the observed data requires that different radial extension parameters be chosen for the \( s \)- and the \( p \)-shell nucleons. The best values obtained are \( a_s=1.23 \) f for the \( s \)-shell and 2.0 f for the \( p \)-shell.

§ 1. Introduction

The charge distributions of nuclei in \( p \)-shell have been investigated in considerable detail by means of electron scattering experiments. For Be⁹, calculations for elastic and inelastic scattering cross sections have been made using a number of different nuclear models. In particular, Pal and Mukherjee have applied the intermediate coupling shell model to obtain the wave functions of the various states of Be⁹, and calculated from these the scattering cross sections. Elastic scattering from the ground state and inelastic scattering from 2.43 Mev and 6.8 Mev states of Be⁹ has been observed for electrons of 190 Mev energy. The results of the calculations of Pal and Mukherjee for this case agree well with the observed results for the elastic scattering, but there is a pronounced disagreement between the calculated and the observed results for inelastic scattering for the magnitude and also for the angular distribution. These authors have used a mixture of central, tensor and spin-orbit forces with suitable parameters to obtain the wave functions and energy levels in the intermediate coupling. No attempt has been made to obtain a precise agreement with observed energy level scheme of Be⁹. Besides, it appears that they identify the 2.43 Mev level as \( J=1/2^- \). It is now well known that this level is \( J=5/2^- \). This has prompted us to make a reevaluation of the scattering cross sections in the intermediate coupling model, and to seek a better agreement with the observed results. Our calculations show that not only an improved agreement between theory and experiment can be obtained by taking into account the correct spin value \( J=5/2^- \) of the 2.43 Mev level, but a satisfactory fit with the experimental results for elastic as well as inelastic scattering can be obtained.
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only by taking different values of the radial parameter \( a_0 \) in the harmonic oscillator wave functions for the s-shell and the p-shell. Some information on the value of the intermediate coupling parameter \( \zeta \) is also obtained.

§ 2. Calculations

French, Halbert and Pandya\(^6\) have made an intermediate coupling calculation for the energy levels of Be\(^9\). We choose for the wave functions of the ground and 2.43 Mev states the results given by them. For small values of the intermediate coupling parameter \( \zeta \) (as defined by French, Halbert and Pandya), the components in the wave functions with symmetry other than \([41]\) are very small, and these are neglected by Pal and Mukherjee. We have verified by explicit evaluation that the effect of these components on the scattering cross sections is indeed quite insignificant.

The scattering cross section is given by equations (1) and (9) of Tassie's paper.\(^2\) The matrix element \( f_{nn'} \) occurring in the expression is given by

\[
f_{nn'} = \langle JM, TM_T | \sum_f e^{-i (K \cdot R_f)} \cdot \frac{1}{2} (1 - \tau_f^{(f)}) | J'M', T'M_T' \rangle,
\]

where \( JM \) and \( J'M' \) denote the final and the initial states of the nucleus, and the other symbols are defined in references 2). Here contributions to the scattering by the current terms (shown to be small by Pal and Mukherjee) and the magnetic moments of the nucleons are neglected. The wave functions \( |JM\rangle \) are written in the intermediate coupling shell model as superposition of the suitable L-S coupling terms,

\[
|JM, TM_T\rangle = \sum K'(JT, \alpha LS) |\alpha LS, JM, TM_T\rangle.
\]

The coefficients \( K'(JT, \alpha LS) \) are tabulated by French, Halbert and Pandya, for different values of the parameter \( \zeta \). The matrix element \( f_{nn'} \) then contains terms of the type

\[
\langle (l)^n \alpha LS, JM, TM_T | \sum_f e^{-i (K \cdot R_f)} \cdot \frac{1}{2} (1 - \tau_f^{(f)}) | (l)^n \alpha'L'S', J'M', T'M_T' \rangle
\]

\[
= \sum (-i)^k (2k+1) \langle j_k(KR) \rangle \cdot \frac{1}{2} n \cdot \delta_{\alpha\alpha'} \delta_{M_T M_T'} \delta_{M M'}
\]

\[
\times [J' LL']^2 \cdot D_{J'k} \langle J'M' k0 |JM\rangle \sum \langle (l)^n \alpha LS, T | \rangle \langle (l)^{n-1} \alpha'L'S', T' \rangle \cdot (-1)^{L'+S-J} \cdot W(LJL'; J') \cdot W(LJL; Lk)
\]

\[
\times W(lLlL; Lk) \cdot \{ \delta_{TT'} \delta_{M_T M_T'} - \delta_{M_T M_T'} \cdot \sqrt{6} (2T'+1) \}
\]

\[
\times (-1)^{\frac{3}{2}-Tr} \cdot \langle T'M_T' |10 |TM_T\rangle \cdot W\left( \frac{1}{2} T \frac{1}{2} T'; \tilde{T}1 \right)
\]

where,
\[ \langle j_b(KR) \rangle_i = \int_0^\infty \psi_i^* j_b(KR) \psi_i \rho^2 d\rho, \quad \rho = \frac{R}{a_0}, \]

\( \psi_i \) is the radial harmonic oscillator wave function and involves the parameter \( a_0 \) as \( \exp(-r^2/a_0^2) \) and \( j_b(KR) \) are the spherical Bessel functions, the other notations being as follows:

\[ [abc]^{1/2} = [(2a+1)(2b+1)(2c+1)]^{1/2}, \]

\[ D_{ll',kk} = [l'l'k]^{1/2} \langle 10l'0|k0 \rangle. \]

\( \langle J'M'JM|k0 \rangle \) are the well-known Clebsch-Gordan coefficients, \( \langle \cdots|\cdots \rangle \) are the coefficients of fractional parentage, and are listed by Jahn and Van Wieringen, \( W(abcd;ef) \) are the Racah coefficients. The derivation of Eq. (3) follows standard tensor algebraic procedures developed by Racah. It is now easy to see that for scattering from the closed s-shell, the matrix element takes the simple value 2\( \langle j_0(KR) \rangle_i = 0 \), where

\[ \langle j_0(KR) \rangle_i \psi_i \int_0^\infty \psi_i^* j_b(KR) \psi_i(s) \rho^2 d\rho = \mathcal{C}. \]

We also write:

\[ \langle j_0(KR) \rangle_i = \int_0^\infty \psi_i^* (s) j_b(KR) \psi_i(s) \rho^2 d\rho = \mathcal{C}, \]

\[ \langle j_0(KR) \rangle_i = \int_0^\infty \psi_i^* (p) j_b(KR) \psi_i(p) \rho^2 d\rho = \mathcal{C}. \]

The above result for s-shell scattering also follows easily from Pal and Mukherjee's equation (3a), as on carrying out the summation over \( T \), the second term in brackets vanishes.

\[ \S 3. \quad \text{Results and Discussion} \]

Figs. 1 and 2 show the results of the calculations as compared with the observed results for inelastic and elastic scattering for 2.43 Mev and the ground states respectively. First we discuss the inelastic scattering (Fig. 1) as this depends only on the radial integral \( \mathcal{C} \) which involves the wave functions of the p-shell nucleons, and enables us to determine directly the parameter \( a_0 \) for the p-shell wave functions. It can be seen that a good agreement with the observed results is obtained for \( \zeta = 1.4 \) and \( a_0 = 2.0 \). Increase in \( \zeta \) leads to larger calculated cross sections, as may be seen from the curve for \( \zeta = 2.8 \). Also shown in this figure is a curve for the energy level \( J = 1/2^- \), for \( \zeta = 1.4 \) and \( a_0 = 1.67 \). This agrees well with the curve obtained by Pal and Mukherjee.
for this state, and shows that the lack of agreement with the experimental results in their case is due to an incorrect assumption for the spin (and consequently the wave function) of the 2.43 Mev state.

In Fig. 2 is shown the calculated curve for elastic scattering using the same parameters as obtained from analysis of the inelastic scattering, i.e. $a_0=2.0 \, f$, $\zeta=1.4$ and the same value of $a_0$ for both s-shell and p-shell harmonic oscillator wave functions. The agreement with the experimental results is now seen to be rather poor in this case, and it may also be seen that variation of $a_0$ does not improve the matters; although with $a_0=1.7 \, f$, the agreement is not too bad, it is still not quite satisfactory. We have checked that for elastic scattering, variation of $\zeta$ from 1.4 to 2.8 does not alter the cross sections substantially. We therefore attempted to choose different values for the parameter $a_0$ in the s-shell and in the p-shell, i.e. the value of $a_0$ used in the radial integral $C$ is different from that used in the integrals $D$ and $E$. It is then found that for the choice $a_0=2.0 \, f$ in p-shell and $a_0=1.23 \, f$ in s-shell, it is possible to obtain a satisfactory agreement with the observed results.

The results obtained above are similar to those of Burleson and Hofstadter,\textsuperscript{7} and of Elton\textsuperscript{8} for Li\textsuperscript{9}, where they also find it necessary to assume different oscillator pa:\ -

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**Fig. 1.** Inelastic scattering of 190 Mev electrons by Be\textsuperscript{9} 2.43 Mev state ($J=5/2^-$) of Be\textsuperscript{9}. The dashed lines represent the curves for $a_0=1.67 \, f$, $\zeta=1.4$ and 2.8; the solid line represents the cross section for $a_0=2.0 \, f$, $\zeta=1.4$. Also shown in the figure is a curve for $J=1/2^-$, $a_0=1.67 \, f$, $\zeta=1.4$. The dots represent the experimental values.

**Fig. 2.** Elastic scattering of 190 Mev electrons by Be\textsuperscript{9}. The dashed lines represent the curves for $a_0=1.7 \, f$ and $a_0=2.0 \, f$ with $\zeta=1.4$. The solid curve represents the cross section for $a_0(s)=1.23 \, f$, and $a_0(p)=2.0 \, f$ with $\zeta=1.4$; the dots represent the experimental values.
meters $a_0$ for $s$-shell and $p$-shell nucleons. Unlike Burleson and Hofstadter, but agreeing with Elton, our results show a larger value of $a_0$ for $p$-shell than that for $s$-shell, which apparently signifies a more extended potential well for $p$-nucleons than for $s$-nucleons. It is also interesting to note from Figs. 1 and 2, that for an intermediate value of $a_0$ viz. $a_0=1.7 f$, it is possible to obtain a not too bad fit with experimental results for elastic scattering, but the discrepancy in inelastic scattering would be considerable in this case. In this connexion we also mention the recent results of Jackson\(^9\) who has investigated the scattering of electrons from Li\(^6\) using a modified form of the harmonic oscillator potential, but with the same parameters for $s$- and $p$-shell nucleons. One may conclude from all these results that a simple single-parameter harmonic oscillator shell model for describing the charge-distributions of $p$-shell nuclei seems to be inadequate, and at least a two-parameter model is needed.

The elastic scattering cross section is found to be insensitive to changes in value of $\zeta$ from 1.4 to 2.8. However, the inelastic cross section changes considerably with $\zeta$, and gives the best fit for $\zeta=1.4$. This agrees with the earlier suggestion of French, Halbert and Pandya, but disagrees with Pinkston\(^10\) who finds $\zeta=2.8$ to be the better value from an analysis of inelastic scattering of $\alpha$-particles from Be\(^9\).

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**References**

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