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Application of a Cell Model to the Bellebeek Watershed

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A new version of a semi-distributed routing model has been applied to the Bellebeek watershed in Belgium, where runoff data were available for a number of internal gauging stations in addition to the outlet runoff data. The new version has provisions for separately routing the rainfall excess input and the channel inflow input to each unit of area represented in the model by a cell. The cells are interconnected in a tree-like structure reflecting the main drainage pattern of the watershed. The semi-distributed nature of the model makes it possible to simulate spatially variable rainfalls or moving storms. The cell model was calibrated by adjusting its three parameters to reproduce, as nearly as possible, the outflow at the main gauging station of the watershed. The direct surface runoff hydrographs for internal points of the model were then compared to the measured runoff at the internal gauging stations.

Introduction

The conversion of rainfall excess into direct surface runoff is traditionally analyzed by means of lumped models. In these models the areal mean rainfall, estimated from a limited number of point observations, is related to the discharge measured at a fixed location on the river, which serves as the outlet of the watershed. The conversion process is described by the familiar convolution integral and

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an instantaneous unit hydrograph if the surface runoff system is assumed to be linear. Otherwise, if a nonlinear system is assumed, the lumped system is represented by some nonlinear model with its corresponding parameters or kernel functions. In either case the relevance of the lumped approach to the surface runoff system for points upstream of the discharge recording station, especially those defining smaller subwatersheds, is very limited. Also the ability of the lumped approach to describe spatially distributed phenomena, like non uniform rainfall or moving storms is rather poor. The effects of changes in urbanization and drainage patterns in the watershed are also difficult to predict with lumped models.

The obvious solution to the above deficiencies of the lumped approach is the use of a distributed model. Two alternatives are available for such models. One is to use a completely distributed model in which continuity and flow equations are written for each small element of area and for each short section of the channel network. The second possibility, adopted in this study, is to use a model based on dividing the total watershed area into a relatively small number of fairly large subwatershed units of area. The boundaries of the subwatershed units are based on the drainage and the topography of the watershed. Each of these units is represented in the model by a lumped model unit identified herein as a cell. The interaction between the cells provides a sufficient description of the spatial processes taking place in the watershed.

The input to each cell in the model is composed of the rainfall excess of the area represented by the cell and the outflow from the adjacent cells upstream of the cell considered. The output of each cell is the direct surface runoff, as derived from the two types of input by the operation of the cell. The resulting cell model, which provides a semi-distributed description of the real watershed processes, makes it possible to study the influence of spatially distributed rainfall and of moving storms. It makes it also possible to estimate the flow at intermediate points in the watershed, such as outlets of smaller subwatersheds. The effects of some local interventions, such as the construction of a detention reservoir of the changes in the degree of urbanization in some localities could also be studied by the cell model.

Development of Cell Models

The cell model, which is essentially a spatially distributed routing model, is a logical development of earlier models incorporating linear or nonlinear reservoirs for representing the instantaneous unit hydrograph. The first such models were lumped in the sense that the rainfall excess input for the entire watershed was routed through a number of equal or unequal reservoirs connected in series. Later models have, however, taken into account the fact that not all parts of the rainfall excess are undergoing the same amount of routing. The rainfall excess near the watershed outlet must have less routing than rainfall excess deposited on the upper reaches of the watershed.

The idea of representing the conversion of rainfall excess to surface runoff by a distributed model can be traced to Dooge (1959), who proposed a model consisting of a single cascade of linear reservoirs and linear channels. In contrast to earlier models, the input to Dooge's model was distributed to all the reservoirs instead of being concentrated at the uppermost reservoir. Dooge used his model, however, to derive an expression for the instantaneous unit hydrograph, which was then convoluted with the rainfall excess hyetograph for the entire watershed to produce its direct surface runoff hydrograph. The derivation outlined by Dooge is based on an assumption that the rainfall excess has an unchanging distribution over the area of the watershed. This assumption thus rules out the possible use of Dooge's model as a distributed cell model, accepting inputs of various spatial distributions.

The first really distributed cell model can be attributed to Laurenson (1964). His model consisted also of a single cascade of unequal reservoirs. However, each of these reservoirs represented a definite portion of the watershed and received an input equal to the rainfall excess over the area it represented. The rainfall excess input for each cell was added to the output of the upstream cell and the sum was routed through the reservoir at the outlet of the unit of area considered. The boundaries of the area units were determined by the location of the isochrones for the watershed.

Other models based on, or derived from the Laurenson model are branched network cell models in which the arrangements of the routing reservoirs is based on the topology of the stream network in the watershed. Examples of such models are given by Mein, Laurenson and McMahon (1974), by Boyd (1978, 1981), by Diskin and Simpson (1978) and by Boyd, Pilgrim and Cordery (1979). The common feature of these models is that each unit of area in the watershed is represented in the model by a single reservoir. In some models the reservoirs are linear, while in others nonlinear reservoirs were used as the routing elements.

The Cell Model

The cell model used in the present study is a modification of the model described by Diskin and Simpson (1978). In this model, as in other models described above, the area of the watershed is devided into a number of smaller units according to the drainage pattern and the topographic features of the watershed. These area units are represented in the model by cells that are interconnected to form a tree-like structure reflecting the topology of the main drainage pattern of the watershed. Two types of cells are recognized, exterior cells, which have only rainfall

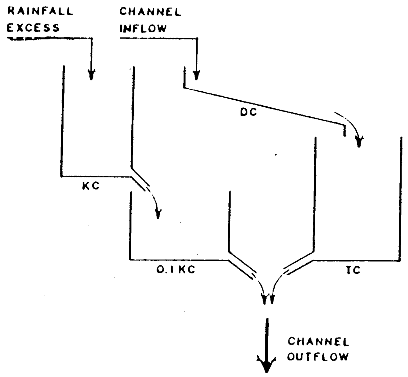

Fig. 1. Schematic structure of a single cell unit.

excess as input, and interior cells having both rainfall excess and channel inflow as inputs. Each cell simulates the transformation of rainfall excess and channel inflow (for interior cells) taking place in the natural unit to produce the outflow at the outlet of the unit considered.

The main modification between the new version of the cell model and the previous model is the distinction between the two types of inputs to interior cells. The two inputs are (i) the rainfall excess over the area represented by the cell, and (ii) the channel inflow at the upstream end of the stream through this area. The channel inflow is obviously zero for exterior cells that do not have any other cells upstream. For an interior cell with more than one upstream cell, the channel inflow is taken to be the sum of the outputs of these upstream cells. Different transformations were used in the new version to obtain the output due to these two inputs. The two outputs of each cell are added to form the channel inflow input for the next downstram cell.

The rainfall excess input to each cell is transformed into the corresponding output by routing it through a pair of linear reservoirs in series (Fig. 1). The two reservoirs represent the overland and channel flow in the watershed. To keep the number of parameters to a minimum, the reservoir constant of one reservoir was arbitrarily taken to be 0.1 of the value of the constant for the second reservoir. Further, this constant was assumed to be proportional to the square root of the area represented by the cell. Assuming that the area of a cell of average size is *A*, and that the reservoir constant for this cell is *KA*, the value of the constant *(KC)* for any cell is

$$KC = KA*\left(\frac{a}{A}\right)^{\frac{1}{2}} \qquad (1)$$

where *a* is the area represented by the cell considered. The impulse response function for transforming the rainfall excess input to each cell is given by the following expression

$$h(t) = \frac{\exp(-\frac{t}{KC}) - \exp(-\frac{10t}{KC})}{0.9\,KC} \qquad (2)$$

where t is the time from start to input.

The total channel inflow input to each interior cell is routed through the channel in the area represented by the cell by a simple lag and route procedure based on a model consisting of a linear channel and a linear reservoir in series (Fig. 1). The channel input is first delayed by a time delay DC and then routed through the linear reservoir having a reservoir constant TC. Again for the sake of keeping the number of parameters to a minimum these two constants were assumed to be proportional to the length of the channel of each cell, in relation to the average of the channel lengths of all cells.

$$DC = DA*(\frac{\ell}{L}) \qquad (3)$$

$$TC = TA*(\frac{\ell}{L}) \qquad (4)$$

where DA and TA are the corresponding values of the two parameters for a cell representing an area with average channel length L, and where l is the length of the channel for the individual cell. The impulse response function for transforming the channel input of each cell is given by

$$g(t) = \frac{\exp(-\frac{t-DC}{TC})}{TC} \qquad \text{for} \quad t > DC \qquad (5a)$$

$$g(t) = 0 \qquad \text{for} \quad t < DC \qquad (5b)$$

The total output of each cell is equal to the sum of the two outputs, derived from the rainfall excess function and from the channel inflow function by convolution with their impulse response functions, given by Eqs. (2) and (5), respectively. A schematic representation of the structure of a single cell is given in Fig. 1. The drawing shows the various components of each cell and the parameters associated with these components.

Watershed and Data Sources

The Bellebeek watershed, chosen for the present study, is located in the central part of Belgium to the west of Brussels (Fig. 2). The watershed (to the gauging station) has an area of 92.0 sq.km and a main stream length of 15.5 km. The Bellebeek river is a tributary of the Dender river joining it some 3.0 km downstream of the gauging station. The elevation at the gauging station is about 12.0 m

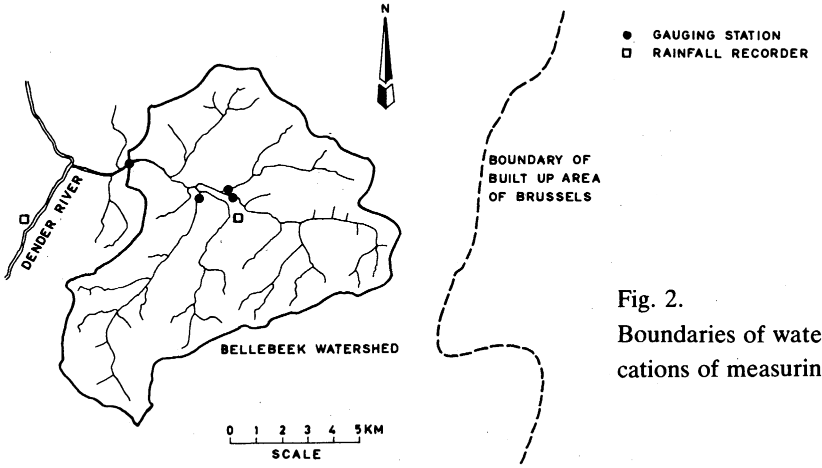

Fig. 2.
Boundaries of watershed and locations of measuring stations.

and the elevation at the junction with the Dender river is about 8.0 m above sea level. The highest point in the watershed is at an elevation of 100 m above sea level. The Bellebeek watershed has a hilly landscape which is characterized by relatively high upper regions, sloping surfaces covered by colluvium material, and valleys filled with alluvium (Wyseure *et al.* 1982). The proximity of Brussels has induced intermittent dispersed urbanization in the area which was originally only agricultural land.

The suitability of this watershed for the present study of the cell model stems from the fact that it has a number of gauging stations on its main tributaries in addition to the main gauging station at the outlet. The main gauging station was established in 1967 in the lower reaches of the Bellebeek where initially only daily observations of water levels were carried out. Since July 1972 the water level measurement is done continuously with a water level recorder of the Hydraulic Service of the Ministry of Agriculture. The additional gauging stations on the main tributaries of the watershed were installed by the Hydrological Services of the Free University of Brussels (VUB). Synchronized measurements in all stations have been conducted by the VUB since October 1979 (Van der Beken and VanLishout 1980). The locations of the main gauging station and the three auxiliary stations are shown in Fig. 2. Some characteristics of the subwatersheds defined by these gauging stations are listed in Table 1.

Table 1 – Main divisions of the Bellebeek watershed

Subwatershed	Area (sq.km)	Main Stream Length (km)	Paved Area Ratio (%)	Cells in Model
Steenvoordebeek	24.7	7.6	6.9	1-2-3
Nieuwe Molenbeek	12.9	5.8	5.3	6-7
Hunselbeek	22.8	9.5	5.7	8-9-10

The continuous recording of rainfall in the watershed was started at Ternat, near the center of the watershed, only in October 1979. An additional recorder in Denderleeuw, near the confluence of the Bellebeek and Dender rivers, is operated by the Hydrological Service of the Public Works Department. The nearest information about rainfall time distribution in the initial period of the water level recording was the main meteorological station of the Royal Meteorological Institute at Ukkel. The location of this well equipped station is about 10 km to the east of the boundary of the Bellebeek watershed.

Outline of Study

Using the drainage pattern plan and the topographic map of the Bellebeek watershed, the area of the watershed was divided into 15 small units of area represented in the model by a cell (Fig. 3a). The boundaries of the area units were chosen so that the internal gauging stations coincided with the output points of some cells. The cells were interconnected according to the drainage pattern of the watershed (Fig. 3b). The areas represented by individual cells ranged from 1.8 sq.km to 9.9 sq.km with an average value of 6.1 sq.km. The lengths of the channels in these areas ranged from 0.8 km til 4.4 km. The average channel length was 3.1 km.

The next step was to calibrate the cell model using the measured ruoff at the outlet of the entire watershed. The calibration of the model consisted of determining a set of optimal values for the parameters of the model that minimized an objective function, defined as the mean absolute deviation between computed values of direct surface runoff and the values derived from the observed total runoff. The three parameters, all expressed in units of time (hours), are:

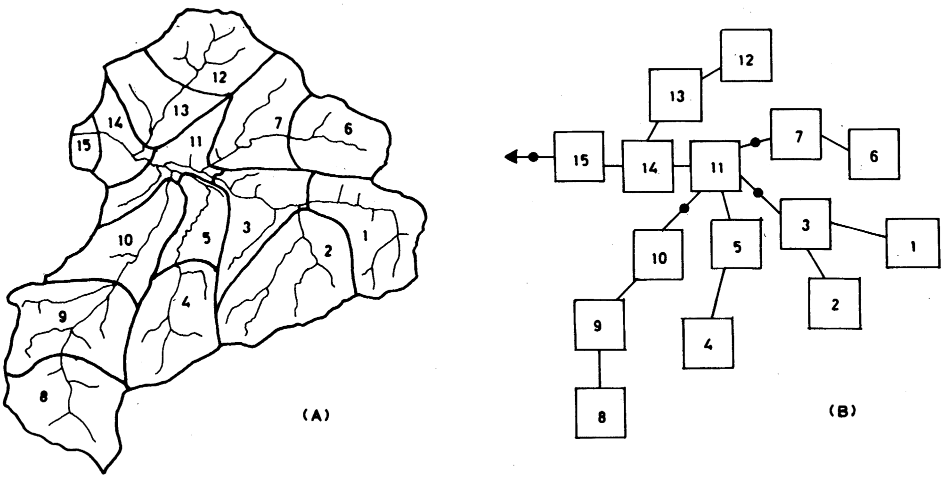

Fig. 3. Division of watershed into cell units and corresponding cell model.

1) The value of the reservoir constant *(KA)* for a cell of average size.
2) The time delay *(DA)* for a cell with a channel of average length.
3) The value of the routing coefficient *(TA)*, again for a cell with a channel of average length.

The final step in the study was to compare the hydrographs of direct surface runoff derived from measurement at the internal gauging stations with those produced by the model at the corresponding internal points of the cell model. Using the optimal values of the parameters, the model was used to produce hydrographs of direct surface runoff for the outlets of cell No. 3, cell No. 7 and cell No. 10. The hydrographs were compared to the direct surface hydrographs derived from observations at the outlets of the Steenvoordebeek watershed, the Nieuwe Molenbeek watershed, and Hunselbeek watershed, respectively.

Choice of Data

A fairly large number of hydrographs were observed at the main gauging station of the Bellebeek watershed in the period of measurements starting from 1972. Examination of the records revealed that the observations were often affected by the closing and opening of the gates of a watermill located a short distance upstream of the gauging station. This interference was in some cases masking the exact starting time or peak of a hydrograph, and in others disturbing its recession curve. In the latter case it was found possible to make a suitable correction and restore the natural recession curve of the hydrographs concerned.

The study of the available records also pointed out another problem related to the Bellebeek watershed. The problem was expressed by the fact that two basically different recession curves were observed. One type of recession curves had a time constant of about 35 hours and the second a time constant of about 22 hours. The former type of recession curves was found to be associated with hydrographs with peaks that did not exceed the maximum discharge capacity of the channel network of the watershed. The second type of recession curves, having a lower value of recession constant, was found to be associated with runoff hydrographs with a flat peak, indicating a nearly constant discharge for a prolonged time, lasting in some cases several hours.

The flat prolonged peak of some of the hydrographs was taken to indicated that, in these cases, the runoff exceeded the discharge capacity of the channels and flooded the banks of the channels. The constant prolonged peak flow was assumed to be approximately equal to the discharge capacity of the channel upstream of the gauging station. To avoid complications, the present study was constrained to representing only the first type of runoff hydrographs, those in which the peak flow did not exceed the discharge capacity of the channels in the watershed.

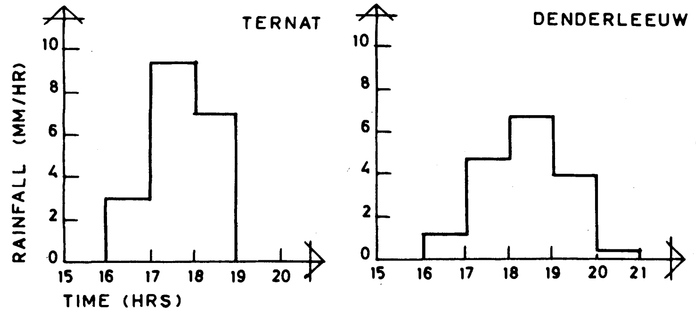

Fig. 4.
Total rainfall hyetographs for storm of March 13, 1980.

The data selected for the calibration and the testing of the model were the direct surface runoff hydrographs derived from discharge measurements at various gauging stations for the storm event of March 13, 1980. This storm had a fairly high peak flow that did not exceed the discharge capacity of the channels at the four gauging stations in the watershed.

In the calibration phase, the direct surface runoff hydrograph for the outlet of the entire watershed was compared to the output of the model at the outlet of cell No. 15. The parameters of the model were adjusted using a standard optimization procedure to obtain a minimum value for the objective function defined above. The testing phase of the study consisted of comparing direct surface runoff hydrographs generated at three internal points with those derived from measurements for the same storm of March 13, 1980.

The input to the model consisted of rainfall excess values, at time intervals of one hour, individually derived for each group of cells representing a subwatershed. The basic data were the two hyetographs of total rainfall recorded at Ternat near the center of the watershed and at Denderleeuw to the west of the watershed. The recorded-two hyetographs are shown in Fig. 4. The locations of the rain measuring stations are shown in Fig. 2. Using the Thiessen method it appears that only the inputs to cells Nos. 8 and 9 are represented by the Denderleeuw station, while the inputs to all other cells are derived from the rainfall measured at Ternat.

Rainfall excess values were derived from the total rainfall hyetographs by assuming a uniform rate of loss (ϕ-index) for each group of cells representing the area contributing to an existing gauging station. The value of this rate was adjusted to produce the volumes of direct surface runoff, as derived from measurements at the various gauging stations. The values of the direct surface runoff volumes (rainfall excess) and the corresponding values of the mean rates of loss are listed in Table 2 for the various subwatersheds as well as for the entire watershed. The resulting rainfall excess hyetographs are given in Fig. 5. Table 2 lists also the percentage paved area for various groups of cells and adjusted values of the loss rate for the unpaved areas. The adjusted values of loss rates were derived by assuming that there are no losses for the paved areas.

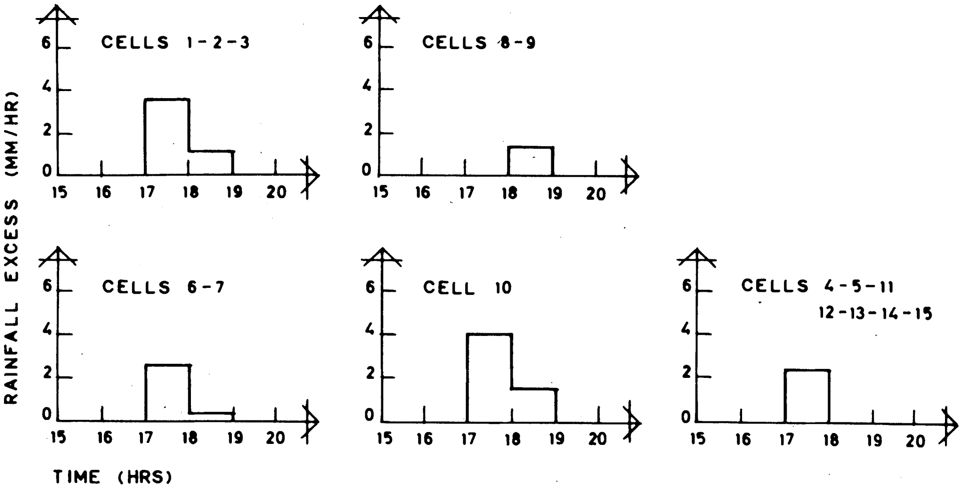

Fig. 5. Rainfall excess hyetographs for various groups of cells.

Table 2 – Loss rates for various cell groups

Cells No.	Rainfall Excess (mm)	Mean Loss Rate (mm/hr)	Paved Area (%)	Loss Rate for Unpaved Area (mm/hr)
1-2-3	4.56	5.97	6.9	6.53
6-7	2.84	6.83	5.3	7.60
8-9-10	2.71	5.48	5.7	6.18
4-5	2.33	7.17	5.8	8.23
12-13	2.33	7.17	4.5	7.99
11-14-15	2.33	7.17	8.2	8.71
Entire watershed	3.09	6.30	6.1	6.82

Results and Comparisons

As expected, the result of the calibration phase was a direct surface runoff hydrograph which agreed quite well with the hydrograph derived by baseflow separation from the observed field data. The comparison between the two curves is presented in Fig. 6. The mean absolute deviation between the two hydrographs, which served as the objective function in the calibration process, is 0.28 cu.m/sec or about 3.6 % of the observed peak flow. The good agreement is, of course, due to the use of the observed curve in the calibration process.

The optimal values of the three parameters were:

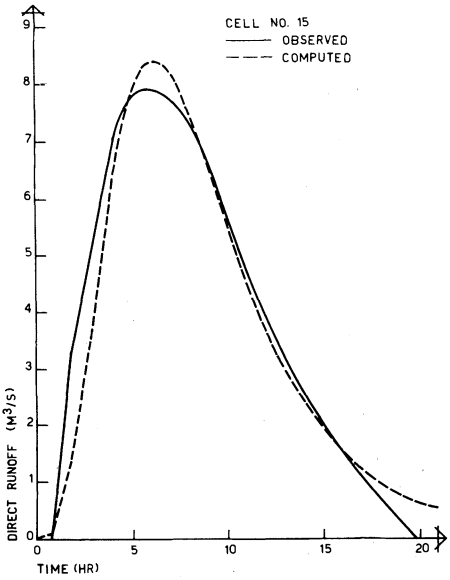

Fig. 6. Observed and computed hydrographs for the main gauging station, storm of March 13, 1980.

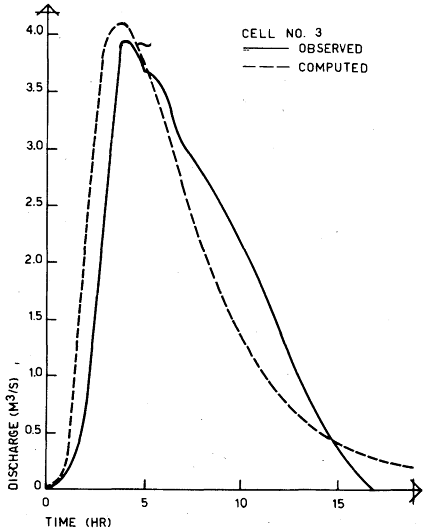

Fig. 7. Observed and computed hydrographs for the Steenvoordebeek subwatershed (cell No. 3).

Reservoir routing coefficient: $KA = 3.73$ hrs.
Linear channel lag constant: $DA = 0.00$ hrs.
Channel routing coefficient: $TA = 1.31$ hrs.

These values were kept constant in the course of the comparisons carried out to verify the model. The verification consisted of comparing the direct surface runoff hydrographs generated by the model for the locations of the internal gauging stations with the hydrographs derived from measured field data at the same points.

Figs. 7, 8 and 9 show the simulated and observed direct surface runoff hydrographs for the internal gauging stations for the runoff event of March 13, 1980. The simulated hydrographs were derived by the cell model using the rainfall excess hyetographs shown in Fig. 5 and the model parameters given above. The visual comparison of the observed and simulated runoff hydrographs for the subwatersheds is considered to be satisfactory. Using the peak flow and the general shape of the computed hydrographs as criteria, the following observations can be made. A very good agreement was obtained for the outflow of cell No. 3 (Fig. 7). The timing and general shape of the runoff at the outlet of cell No. 7 (Fig. 8) was good, but the peak flow was overestimated. The least satisfactory results were obtained for the hydrograph computed for the outlet of cell No. 10 (Fig. 9). The peak flow was underestimated and its timing was too early, however the general

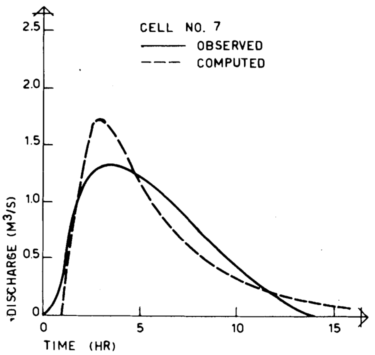

Fig. 8. Observed and computed hydrographs for the Nieuwe Molensbeek subwatershed (cell No. 7)

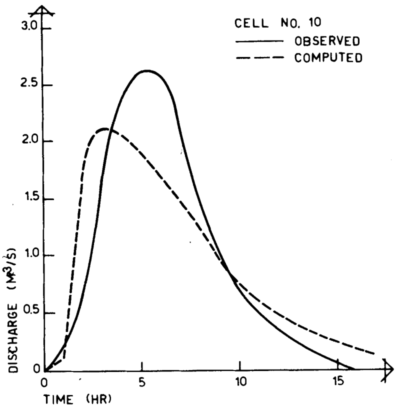

Fig. 9. Observed and computed hydrographs for the Hunslebeek subwatershed (cell No. 10)

shape of the two hydrographs is still fairly similar. The values of the mean absolute deviation between the observed and the computed hydrographs shown in Figs. 7, 8 and 9 are 0.27, 0.09 and 0.23 cu.m/sec for the three pairs of curves, respectively. Expressing these values as a percentage of the observed peak flows of the three hydrographs yielded similar values of about 6.8%. This value is roughly twice the percentage mean deviation for the hydrograph used for calibration of the model.

Conclusions and Comments

The study reported herein was concerned with the applicability of the cell model to the description of the spatially distributed process of conversion of rainfall excess to direct surface runoff. Using available data for the Bellebeek watershed it was demonstrated that the cell model can describe fairly well the production of direct surface runoff at internal points of a watershed, in addition to its ability to do so for the outlet of the entire watershed.

The comparisons of observed and computed hydrographs or surface runoff at internal points were not as good as the corresponding comparison for the outlet. The reasons for this are of course related to the fact that the outlet data were used in the calibration process while the data at internal points did not take part in this process. Better results could probably be obtained if all the available data for the four gauging stations were used in the search for optimal parameters of the model. This was not done in the present study because its purpose was to check the applicability of the cell model rather than using it as a tool to predict the runoff hydrographs as closely as possible.

Inspection of the observed and computed hydrographs compared in Figs. 6 to 9 reveals that in all cases the tail of the computed recession curves tends to be too long. This seems to be a result of the methods adopted for the separation of base flow in the observed hydrographs, and for the termination of the impulse response functions for each cell. Better results can be obtained by altering either or both of these arbitrary procedures. The problem was, however, not considered important enough to introduce any refinements at this stage.

The quality of the results is probably influenced also by the number of rain gauges available for defining the input to the cell model. A more dense network of recording rain gauges would lead to a more correct definition of the input to each cell. It could possibly give a better agreement between measured and computed hydrographs. Unfortunately this point could not be checked with the available data.

The values of the parameters obtained by the optimization procedure were of the order of magnitude expected on the basis of experience with the previous cell model. An interesting point to note is that the optimal value of the linear channel time delay *(DA)* came out to be zero. This result reflects probably the fact that the size of the cells used for the Bellebeek model is small so that the delay between cells is not significant in comparison to the channel routing coefficients *(TA)* for the cells. The occurrence of a zero value in this case does not justify, however, the elimination of the delay time feature of interior cells.

Although not directly comparable, the values obtained above for the optimal cell paramentes may be reviewed against parameters recommended by Boyd (1981) for his linear model. In that model two types of reservoirs are used. Those representing exterior cells for which the routing coefficient is given by

$$KE = 2.51\ a^{0.38} \tag{6}$$

and reservoirs representing interior cells for which

$$KI = 1.50\ a^{0.38} \tag{7}$$

Using the average area of a cell in the Bellebeek watershed, $a = 6.1$ sq.km, the last two equations yield $KE = 5.0$ hrs and $KI = 3.0$ hrs. These values are higher than, but of the same order of magnitude as the optimal parameters obtained in the present study, $KA = 3.73$ hrs. and TC = 1.31 hrs. The values cannot be expected to be equal because of the different schemes of routing cell inputs used in the two models. The difference in the topographic and climatic conditions of Belgium and NSW can also explain some of the above differences.

The main difference between the cell model used in the present study and that described previously is the different treatment of the two inputs to each cell unit. The different values obtained for the two coefficients, *KA* and *TA*, used for routing the two inputs indicate that the new model gives a better representation of the processes taking place in each unit of area of the watershed. As expected, the value of the coefficient for the routing of the channel inflow is smaller than the value of the routing coefficient for the rainfall excess input. The ratio of the values of the coefficients, which is about 1:3, points to a significant difference in their effect. In appears that the channel inflow function for each cell undergoes a much smaller change of shape than the rainfall excess input function.

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