



Discussion

Decentralized Robust Control of Decomposed Uncertain Interconnected System¹

Shu-xin Du.² In the paper, a decentralized robust controller design method was presented. We think some aspects about derivation of theorem 1 and its corollaries presented therein are incorrect.

Let N be a set defined by

$$N(B^T) = \{x \in R^n | B^T x = 0\} \quad (1)$$

where $B \in R^{n \times m}$. Let $0 < \lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_m$ be the eigenvalues of $B^T B$, and e_1, e_2, \dots, e_m corresponding orthonormal eigenvector, and a set is defined by

$$S(B) = \text{Span}\{e_1, e_2, \dots, e_m\} \quad (2)$$

The derivation of theorem 1 and corollary 1 and corollary 2 presented in the paper was based on the assumption shown in p. 597: For any $x_i \in R^{n_i}, P_i \in R^{n_i \times n_i}, B_i \in R^{n_i \times m_i}$, there always exist $y_{1i} \in N(B_i^T), y_{2i} \in S(B_i)$ satisfying

$$P_i x_i = y_{1i} + y_{2i} \quad (3)$$

¹ By C. F. Cheng, W. J. Wang, and Y. P. Lin, published in the Dec. 1993 issue of the JOURNAL OF DYNAMIC SYSTEMS, MEASUREMENT, AND CONTROL, Vol. 115, No. 4, pp. 592–599.

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We think this assumption is unreasonable. Take $m_i = 1$ for example. $B_i^T B_i$ is a scalar since $m_i = 1$, hence the set $S(B_i) \subset R$, i.e., $y_{2i} \in R$. $N(B_i^T) \subset R^{n_i}$, that is to say, $y_{1i} \in R^{n_i}$. Therefore, for any $x_i \in R^{n_i}$ and $P_i \in R^{n_i \times n_i}$, Eq. (3) cannot hold unless $n_i = 1$. In fact, Eq. (3) holds only if $n_i = m_i$.

Additionally, the following Lemma cited by authors is also incorrect^[2].

Lemma^[1,3]: For each $x \in S(B)$, the following inequality holds

$$\lambda_1 \|x\|^2 \leq x^T B B^T x \leq \lambda_m \|x\|^2 \quad (4)$$

where $B \in R^{n \times m}$, $m \leq n$, and $\text{rank}(B) = m$, $0 < \lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_m$ are the eigenvalues of $B^T B$.

From the proof of this lemma^[1,3], x can surely be described by

$$x = \alpha_1 e_1 + \alpha_2 e_2 + \dots + \alpha_m e_m \quad (5)$$

where α_i are constants. But, for any $x \in R^n$, $e_i \in R^m$, Eq. (5) doesn't hold unless $n = m$ ^[2].

The fact that Eq. (3) doesn't hold and the cited lemma is false, means that theorem 1 and its corollaries presented in the paper, which depend on Eq. (3) and this lemma, are wrong and cannot be repaired.

References

- 1 Zak, S. H., "On the Stabilization and Observation of Nonlinear/Uncertain Dynamic Systems," *IEEE Trans. Automat. Contr.*, Vol. 35, No. 5, 1990, pp. 604–607.
- 2 Qu, Z., and Dorsey, J., "Comment On the Stabilization and Observation of Nonlinear/Uncertain Dynamic Systems," *IEEE Trans. Automat. Contr.*, V.36, No. 11, 1991, p. 1342.
- 3 Zak, S. H., "Author's Reply," *IEEE Trans. Automat. Contr.*, Vol. 36, No. 11, 1991, p. 1342.