In light of the foregoing analogy, the stress-intensity factors $k_0$ for an isotropic and transversely isotropic solid differ only in that $r$ is replaced by $1 - \beta/a$. Since $k_0$ and $k_2$ are independent of the shear modulus $\mu$, there is no need to redefine the constants $A$ and $B$ as it was done in equations (1) and (2) of the discussion. Furthermore, it is the opinion of the authors that the discussor did not provide the pertinent information in his work [1] for finding the expressions of $k_0$ and $k_2$. Incidentally, the derivation of $k_1$ and $k_2$ from the crack-border stress field, based on the limiting forms of the elliptical coordinates $(x, y) \to \text{polar coordinates } (r, \theta)$, is by no means an easy task.

Another point that requires clarification is concerned with the method of solution used by the discusser [1] to solve the crack problem. From the existing knowledge of the isotropic solution, he conjectures that the boundary conditions of constant shear stresses applied to the crack surfaces can be satisfied by taking

$$u \sim Z^{1/2}, \quad \text{for } z = 0, \quad z^2/a^2 + y^2/b^2 \leq 1 \quad (10)$$

where the variable $Z$ stands for

$$Z = 1 - z^2/a^2 - y^2/b^2.$$ 

This conjecture may be regarded as a particular case of the following theorem:

1. Let the displacements $u$, $v$, and $w$ on the crack plane $z = 0$, $z^2/a^2 + y^2/b^2 \leq 1$ be each given by $Z^2Q(z^2, y^2)$, where $Q(z^2, y^2)$ is a polynomial of degree $n$ in $z^2, y^2$. Then, the corresponding stresses $\tau_{xz}$, $\tau_{yz}$, and $\sigma_z$ over the elliptical region are also polynomials, $P_n(z^2, y^2)$, of the same degree in $z^2, y^2$. A detailed proof of this theorem for the transversely isotropic case follows readily from the work of Kassir and Sih [5] for isotropic elasticity. In addition, the mixed boundary-value problem, described by the four conditions in [1], can be solved directly by application of the method developed in the paper and the potential solutions in [2, 5]. The problem of specifying shear stresses on the crack surfaces as a linear function of $x$ and $y$ can also be solved explicitly. However, the results for general loading conditions, say, applied stresses given by polynomials in $x^2, y^n$ where $n \geq 2$, appear to be restrictive. A complete analysis of the three-dimensional stress distribution around a plane of discontinuity embedded in a transversely isotropic solid will be reported by the authors in another paper.

References